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**The structural shift to green services**

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# The structural shift to green services\*

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## Abstract

In this paper I investigate the role of an increasingly demanded class of green services, characterised by having strong roots in local communities, high human labour intensity, flat labour productivity growth and a low impact on the environment. In order to do so, I build an endogenous growth model with a progressive manufacturing sector and a stagnant service sector. Productivity growth in the former sector is driven by the presence of a public stock of capital, representing infrastructure. The progressive sector also generates a negative externality on an open-access renewable resource that enters the households welfare function. I thus study the long-run behaviour of the economy and present a numerical example to analyse the transition during which structural change takes place. Results show the possibility of a conflict between growth and welfare by investigating the effects of a change in some relevant policy and technological parameters.

## 1 Introduction

A vibrant debate has developed in recent years over the opportunity of a transition to a “green” or “low-carbon” economy, considered by many as necessary in order to tackle the connected challenges of climate change and resources depletion. In this context, two main areas of analysis can be identified.

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The first one relates to the idea that national governments could be able to direct their economic systems on the road to sustainability through a set of public policies that include: investments in more energy-efficient infrastructure; the introduction of stricter environmental standards; the implementation of fiscal instruments such as taxes on polluting emissions or subsidies for R&D spending on green technology; and others (Barbier, 2009; OECD, 2011; UNEP, 2011). Activities such as the restoration and conservation of environment, the retrofitting of buildings to improve energy conservation, the expansion of public transportation infrastructure and the development of renewable sources of energy are considered by many as the most effective means to combine environmental sustainability with the creation of jobs and the achievement of high growth rates (Pollin et al., 2008; UNEP, 2008; Committee on Climate Change, 2010). The second one is more concerned with the behaviour of individuals, particularly in their role as members of a local socio-economic community. The “green economy” pictured in this case is associated with the idea of sustainable local economies characterised by a reduced use of energy, locally-based production and consumption, widespread skills and high levels of relational networks, rather than vast investment projects managed at a central level. This vision is well embodied in civil society by the rapidly expanding “Transition Towns” movement (Hopkins, 2011), whose objective is the creation of resilient communities capable of proposing local responses to the interconnected challenges of climate change and the increasing scarcity of resources. Although the development of a sound green economy would most probably need a balanced combination of both the approaches presented above, this paper focuses on the latter, which has only marginally been investigated by the economic literature until now.

With this aim, I investigate the dynamics of an economy with a “progressive” manufacturing sector and “stagnant” service sector having: a) a different ability to increase labour productivity; b) a different impact on an open-access asset that enters as an argument the households welfare function and that can be interpreted either as a environmental or a social common resource. The service sector is here intended to represent those activities

characterised by a strong attachment to the local community, high intensity of human labour, reduced consumption of energy and low polluting emissions, which I name “green services” (Victor and Jackson, 2011). What qualifies the stagnant sector as “green” is the existence of negative externality on the common resource generated by the output of the progressive sector, which can therefore be labelled as “dirty”. The technological index in the progressive sector is modelled to be a function of a stock of public capital (infrastructure) produced by the government and financed through the imposition of a tax on production. An increase of the stock of public capital shifts the production function upwards, thus raising the productivity of private capital. On the other hand, labour is assumed to be the only input in the production of green services; consequently, no productivity increase ever takes place in this sector.

I thus derive the balanced growth path (BGP) of the economy and show, for reasonable parameter values, its saddle-path stability. Along the BGP, the public and private stock of physical capital, as well as the output of the progressive sector, exhibit the same long-run growth rate. In order to analyse the allocation of employment between the two sectors, I present a numerical example of the transitional dynamics. To conclude, I investigate the long-run effects of a change in some relevant parameters, and discuss the possibility of a conflict between the achievement of higher growth rates and the expansion of the level of welfare.

The remainder of the paper is organised as follows: section 2 introduces and discusses the related literature; section 3 presents the structure of the model; section 4 derives the conditions to have a stable balanced growth path and offers a numerical example of the transitional dynamics; section 5 investigates the effects on long-run growth rate and welfare of a change in some relevant parameters; section 6 concludes.

## 2 Motivation and literature

### 2.1 A structural change towards green services

A wide literature exists dealing with the effects of a change in the sectoral composition of both developing and developed economies. If, on one side, the shift of production inputs from traditional sectors such as agriculture to modern manufacturing industries has been indicated as one of the main drivers of economic development<sup>1</sup>, the subsequent shrink of industries in favour of services<sup>2</sup> seems to be one of the causes of the recent decrease in developed economies growth rates, due to the slower productivity growth that takes place in the service sectors (Echevarria, 1997; Ngai and Pissarides, 2007).

Baumol (1967) has been the first to concentrate on the dichotomy, that characterises all advanced economies, between “progressive” sectors, where a continuous increase in the level of output per unit of labour occurs, and “stagnant” sectors, in which labour productivity only sluggishly increase or remains stable. As the author points out, the most relevant distinction between the two consists in the function played by human labour. In manufacturing, labour is mainly an instrument, which, joint with other inputs such as capital and energy, contributes to the production of goods. On the other hand, “there are a number of services in which labour is an end in itself, in which quality is judged directly in terms of amount of labour”. While the examples offered by Baumol (1967) are mainly extracted from the world of culture or performing arts, I apply similar considerations to what can be defined as “green services” (Jackson and Victor, 2011).

The stagnant sector in the model presented here is indeed a representation of a class of service-based economic activities characterised by two main features: a) lower material and energy intensities than the manufacturing average ones; and b) high labour intensity. Both attributes are in general present in all the service sectors<sup>3</sup> and this has already led

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<sup>1</sup>See, for instance, the seminal contribution of Lewis (1954).

<sup>2</sup>In OECD countries the services share of GDP has increased from 55% in 1970 to 73% in 2008 (World Development Indicators, available at [data.worldbank.org](http://data.worldbank.org)).

<sup>3</sup>See for example ONS (2010), Table 13.3 and 13.4.

researchers to suggest that “goods must be converted as much as possible into services” if environmental sustainability is to be achieved (Ayres, 2008). I here push these features to the extreme, modelling the service sector as having no impact whatsoever on the environment and using labour as its only input of production. Examples of such activities could be: “community energy projects, local farmers markets, slow food cooperatives, sports clubs, libraries, community health and fitness centres, gardening, local repair, maintenance and refurbishment services, craft workshops, writing centres, community music and drama, local training and skills, hairdressing, gardening, and conservation”<sup>4</sup> (Jackson and Victor, 2011). It is evident how in all these activities the labor input in the production of such services can only be reduced by a very limited extent, as it constitutes the inner value that qualifies the activity. As a consequence, the potential improvement of productivity levels is very low.

This hypothesis is supported by data regarding sectoral productivity. Van Ark and Woltjer (2008) show how in the EU15 the personal and social service sector<sup>5</sup> has exhibited a *negative* average growth rate of labor productivity during the period 1980-2005. The growth rate in this sector has been lower than the average growth rate in the whole service sector, which in its turn has been lower than the growth rate in the overall market economy<sup>6</sup>. It is also interesting to notice that, in the same period of time, the number of total hours worked in the personal and social service sector has increased more than the average of all services, which in its turn shows a higher growth rate of hours worked than the overall market

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<sup>4</sup>The classification of green services in terms of the International Standard Industrial Classification is rather complicated. A non-exhaustive list of its sectors includes: Section P (Education); Section Q (Human Health and Social Work Activities); Section R (Arts, Entertainment and Recreation); Section S (Other Service Activities, among which are included: Activities of membership organisations, Repair of personal and household goods, Other personal service activities) and Section T (Activities of Households as Employers).

<sup>5</sup>This includes: Hotel and restaurants; Other community, social and personal services; Private Households with employed persons.

<sup>6</sup>More precisely, in the period 1980-1995 the annual average growth rate of labour productivity in the personal and social service sector has been -0.5%, the growth rate in the whole service sector 1.4% and the one in the overall market economy 2.5%. In the period 1995-2005 the growth rates have been -0.4%, 1% and 1.5%, respectively.

economy<sup>7</sup>. The same pattern is confirmed by Acheson (2011) for the United Kingdom. In the period 2000-2009 the labour productivity average annual growth rates have been equal to -3.3% for the social and personal service sector and to -0.7% for the non-market service sector<sup>8</sup>, while the economy in its whole exhibits a low but positive growth rate (0.2%). Also, the annual growth rates of total hours worked in both the personal and social service sector and the non-market service sector are higher than the one in the overall economy (1.4%, 1% and 0.2%, respectively). Von Ark et al. (2008) confirms low productivity growth rates in the personal and social service sector also for the US, where its contribution to average annual labour productivity growth has been equal to zero in the period 1980-1995 and to 0.2% in 1995-2004. Finally, McMillan and Rodrik (2011) point out how the sector of community, social, personal and government services is the one with the lowest *level* of labour productivity in five out of the nine high-income countries they analyse<sup>9</sup>. Also, the sector scores the lowest average labour productivity in the whole set of 38 developed and developing countries they take into account, with the exception of agriculture.

To sum up, data seem to confirm the hypothesis that those sectors in which green services would probably be classified exhibit both low absolute values of labour productivity and low productivity growth rates, not only with respect to the manufacturing sectors but also to the rest of service sectors.

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<sup>7</sup>In the period 1980-1995 total hours worked have increased by an annual average of 1.5% in the personal and social service sector, by 1% in the service sector and have decreased by 0.3% in the overall market economy. In 1995-2005 the growth rates have been 1.4%, 1.2% and 0.4%, respectively.

<sup>8</sup>In Acheson (2011) the social and personal service sector includes: Other community, social and personal services; Private households with employed persons; Extra-territorial organisations and bodies. The non-market service sector includes: Public administration, defence, compulsory social security; Education; Health and social work.

<sup>9</sup>More precisely, the community, social, personal and government service sector is the one with the lowest labour productivity in France, Netherlands, Italy, Sweden and Denmark, while construction is the sector with lowest productivity in the US and in Spain, agriculture in Japan and whole and retail trade in the UK.

## 2.2 The role of public capital

Public infrastructure has traditionally been considered as one of the fundamental drivers of growth and development (World Bank, 1994). The public stock of physical capital increases the efficiency and productivity of the private sector by allowing firms to transport goods at decreasing costs, providing them with large amounts of energy and facilitating market access through information and communication technology (ICT)<sup>10</sup>. The development and modernisation of infrastructure has always been a crucial strategy adopted by policy-makers as a means to foster economic growth. For instance, the expansion of transport, energetic and ICT infrastructure is one of the crucial objectives of the “Europe 2020” strategy (European Commission, 2010). The American Jobs Act, the US government plan to re-launch the its economy unveiled in September 2011, indicates the modernisation of infrastructure (especially transportation one) as one of its key points<sup>11</sup>.

On a more theoretical level, the role of public capital has been extensively investigated in the economics literature (for a survey: Romp and de Haan, 2007). The seminal contribution of Barro (1990) was the first to introduce government spending in an endogenous growth framework, by inserting a flow of public expenditures as an argument in the firms production function. Such specification was easily tractable but could not be used to analyse government *investment* spending, since there was no stock to be accumulated. Subsequent contributions therefore introduced public capital<sup>12</sup> in the production function (Fugatami et al., 1993; Fisher and Turnovsky, 1998) and examined its effect on the private capital accumulation dynamics. More recently, models with two-sectors economies have been used to study the relationship between public investments and the allocation of pro-

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<sup>10</sup>A variety of channels exist through which infrastructure can affect growth rates, both in a positive and negative manner (see Agenor and Moreno-Dodson, 2006, for a detailed overview). However, the great majority of empirical tests find a beneficial effect of the presence of infrastructure on growth, which is particularly strong in low-income countries (Estache, 2007; Romp and de Haan, 2007).

<sup>11</sup>The Act is available at [www.americanjobsact.com](http://www.americanjobsact.com).

<sup>12</sup>In the real world the public stock of capital does not coincide with the overall amount of infrastructure, as private firms often provide infrastructure as well. However, as noted by Estache (2007), “ the role of the large scale private sector in the delivery of infrastructure services in energy, water or transport is far from being as widespread as many had hoped for”.

duction inputs among sectors (Agenor, 2006; Turnovsky and Pintea, 2006; Felice, 2009). The model presented here draws from this research line, as a stock of public capital is introduced into the production function of the progressive manufacturing sector so that an increase in infrastructure raises the sectoral technological index, as in Turnovsky (1997), Chatterjee et al. (2003), Felice (2009) and Agenor (2010).

The choices regarding public investments in infrastructure are also likely to have important consequences for the overall impact of the economic system on the environment. Given the strong path-dependence of infrastructure, decisions concerning the kind of investment to implement usually affect the average “cleanliness” of technology over many years. Recently, attention has been put over the role that governments could play in delivering a resource-efficient and low-carbon infrastructure network (OECD, 2011; UNEP, 2011). In order to simplify the analysis and concentrate on the green service sector, I exclude the possibility for the government to invest in “clean” infrastructure. This does not hinder the explicative power of the model regarding the environmental effects of a structural change, since the government is still allowed to invest in infrastructure enhancing the productivity of the manufacturing sector, the production of which has a negative externality on the common resource.

### **2.3 Growth as a substitution process**

The model presented here has close connections also with a third strand of research, embodied in a cluster of models that discuss the effects on economic growth and welfare of the substitution process between private market goods and open-access resources (Bartolini and Bonatti, 2002; 2006; 2008; Antoci et al., 2007). Two main features characterise these models. The first is the presence of a free open-access resource that cannot be produced and whose stock is negatively affected by the level of economic activity. The common asset can be either interpreted as an index of environmental quality or as a measure for social capital and relational networks, both of which are likely to suffer from some kind

of negative externality originated by the market economic activity<sup>13</sup>. The second feature is that the common asset enters the individuals' welfare function in combination with the consumption of privately-owned goods, in such a way that a same level of utility can be attained by substituting the flow of services obtained by using the free common resource with the consumption of more expensive market goods (and vice versa).

Therefore, when the increase in economic activity exerts its negative influence on the common resource, individuals react to its degradation by substituting it with a larger amount of private consumption<sup>14</sup>. In order to afford market goods individuals decide to increase their labour supply, thus generating growth and, consequently, greater negative externalities on the common asset. Furthermore, longer work hours gives an additional incentive to marketize a number of activities that could be produced at home, thus expanding economic activity and reinforcing the vicious cycle<sup>15</sup>.

In this paper a similar approach is adopted. A depletable common asset is introduced into the representative household utility function in combination with the consumption of goods produced by the manufacturing progressive sector. Along the balanced growth path of the economy a substitution process takes place between environmental quality and material consumption, thus leaving the level of welfare at a steady value.

### 3 The model

I consider an economy in discrete time with an infinite time horizon. The economy is populated by three types of agents: households, firms and the government. Agents expect-

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<sup>13</sup>For a review of the interactions between growth and environment, see Brock and Scott Taylor (2005) and Xepapadeas (2005). As for the potentially negative effects of economic activity on social capital, see Hirschman(1982) and Putnam (2000).

<sup>14</sup>There are many examples: degraded urban environments stimulate the demand for touristic travels; the scarcity of free public spaces accelerates the diffusion of more expensive home entertainment; polluted streets induce individuals to subscribe to private fitness centres; a decline in social capital and trust gives rise to formal mechanisms of control and enforcement (Bartolini and Bonatti, 2008); and so on.

<sup>15</sup>Pissarides (2007) refers to the differences in home production activities between European countries and the US to explain a part of the European employment gap.

tations are rational. I assume the existence of a “progressive” sector where total factor productivity is affected by the stock of public capital, and of a “stagnant” sector where labour is the only input and there is no productivity growth. The good produced in the progressive sector is used as the numeraire of the system. All markets are assumed to be perfectly competitive. In particular, it is assumed that labor is homogenous and can freely move across the two sectors.

### 3.1 Households

For simplicity and without loss of generality, I assume a constant and large number (normalised to one) of identical households who take into account the welfare and resources of their actual and prospective descendants. Following Barro and Sala-i-Martin (1995), I thus assume that the current generation maximises utility over an infinite time horizon. That is, although individuals have finite lives, the model considers the existence of immortal “dynasties”. The period utility  $U_t$  of the representative household is the following:

$$U_t = \beta \ln(X_t) + (1 - \beta) \ln(l_t), \quad 0 < \beta < 1 \quad (1)$$

where  $X_t$  represents a flow of services generated by some household activity, and  $l_t$  is the time that households devote to leisure.  $X_t$  is produced by the representative household according to the following consumer technology:

$$X_t = (R_t C_t)^\mu C_{S,t}^{1-\mu}, \quad 0 < \mu < 1 \quad (2)$$

where  $R_t$  is an open-access resource that cannot be produced,  $C_t$  is the consumption of the good produced in the progressive sector and  $C_{S,t}$  is the consumption of the good produced in the stagnant sector. Equation (2) can be interpreted in the following way: the representative household draws utility from the consumption of market private goods ( $C_t$ ), which however, in order to be enjoyed, need to be combined together with a common

resource (environment, air, relational networks, social values, etc.) of which everyone can freely make use; in its turn, this bundle of consumption services is combined with the activities originating from the green service sector. It is worth noting that the household can obtain utility from consumption if and only if  $R_t C_t$  and  $C_{S,t}$  are strictly positive.

In each period  $t$  the representative household chooses  $\{C_t\}_0^\infty$ ,  $\{C_{S,t}\}_0^\infty$ ,  $\{L_t\}_0^\infty$  and  $\{K_{t+1}\}_0^\infty$  in order to maximise:

$$\sum_{t=0}^{\infty} \theta^t U_t, \quad 0 < \theta < 1 \quad (3)$$

subject to a budget and a time constraint:

$$w_t L_t + r_t K_t + \pi_{C,t} + \pi_{S,t} + (1 - \delta)K_t \geq C_t + P_t C_{S,t} + K_{t+1} + T_t \quad (4)$$

$$l_t + L_t \leq 1 \quad (5)$$

where  $L_t$  is labour supply,  $K_t$  are the productive assets (capital) held by the representative household in  $t$ ,  $\theta$  is a time-preference parameter,  $w_t$  is the wage rate,  $r_t$  is the rental rate on capital,  $\pi_{C,t}$  and  $\pi_{S,t}$ , are, respectively, the profits that the representative household gets from the firms operating in the progressive sector and from those operating in the stagnant sector (the households are assumed to be the owners of the firms),  $\delta$  is the rate at which capital depreciates,  $P_t$  is the relative price of the green good, and  $T_t$  are the taxes paid to the government. Finally, notice that the households total time endowment is normalised to 1.

### 3.2 Production

Two goods are produced in the economy in each period  $t$ : a good  $Y_{C,t}$  produced in the progressive sector and a good  $Y_{S,t}$  produced in the stagnant sector. While  $Y_{S,t}$  is entirely consumed,  $Y_{C,t}$  can be consumed, reinvested in production or transferred to the govern-

ment. In both sectors I assume the existence of a fixed and large number (normalised to one) of identical and perfectly competitive firms. Good  $Y_{C,t}$  is produced according to the following technology:

$$Y_{C,t} = \left( \frac{K_{C,t}}{K_t} \right)^\alpha K_t L_{C,t}^\alpha, \quad 0 < \alpha < 1 \quad (6)$$

where  $K_t$  is the amount of capital rent by the representative firm operating in the progressive sector,  $L_{C,t}$  is the amount of labour used in the production of  $Y_{C,t}$  and  $K_{C,t}$  represents the public capital stock. The functional form of the production function used here is similar to Turnovsky (1997), Chatterjee et al. (2003), Felice (2009) and Agenor (2010). An increase of the stock of public capital shifts the production function upwards, thus raising the productivity of private capital, though at a diminishing rate. Transport infrastructure is a typical example: a well developed network of highways and railways facilitates the movement of both final and intermediate goods and services thus lowering production costs. Similar considerations can be applied to ICT networks and energy infrastructure. Alternatively, public capital can be considered as a third factor of production<sup>16</sup>.

In each period, the representative firm operating in the progressive sector employs labour and rents capital in order to maximise its profits  $\pi_{C,t}$ , where:

$$\pi_{C,t} = Y_{C,t} - w_t L_{C,t} - r_t K_t \quad (7)$$

Good  $Y_{S,t}$  is produced according to the following technology:

$$Y_{S,t} = L_{S,t}^\gamma, \quad 0 < \gamma < 1, \quad (8)$$

where  $L_{S,t}$  is the amount of labour employed in the stagnant sector.

As discussed in section 2.1,  $Y_{S,t}$  can be interpreted as those services for which human labour constitutes the crucial input that creates utility. These are typically services pro-

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<sup>16</sup>As argued by Romp and de Haan (2007) the two interpretations are equivalent when a Cobb-Douglas technology is used.

duced, exchanged and consumed at a community level, thus involving a relatively small deployment of both private and public physical capital. Labour productivity levels are low, as the potential productivity growth rate. Moreover, these services require a negligible use of material and energetic resources when compared to the manufacturing sector, thus allowing me to classify them as “green”. Hence, one can argue that improvements of transport and energy infrastructure have a negligible impact on the productivity of the sector producing these services, which can legitimately be treated as stagnant.

The profits  $\pi_{S,t}$  of the representative firm in the green service sector are:

$$\pi_{S,t} = P_t Y_{S,t} - w_t L_{S,t} \quad (9)$$

### 3.3 The government

The government invests a fixed fraction  $\tau$  of the value added generated by the progressive sector in the creation of public capital. This investment is entirely financed by the tax revenues. Thus, the stock of public infrastructure evolves according to:

$$K_{C,t+1} = \tau Y_{C,t} + (1 - \delta) K_{C,t} \quad (10)$$

where

$$\tau Y_{C,t} = T_t \quad (11)$$

Notice that for simplicity and without loss of generality the rate of depreciation of  $K_{C,t}$  is assumed to be identical to the depreciation rate of private capital.

### 3.4 The common resource

The representative household derives some utility from the existence of an environmental asset  $R_t$ , whose stock is negatively affected by the production of the progressive good  $Y_{C,t}$ .

In particular we specify the following functional form for the dynamics of  $R_t$ :

$$R_{t+1} - R_t = \chi R_t \left( 1 - \frac{R_t Y_{C,t}}{E_0} \right), \quad \chi > 0, E_0 > 0, R_0 \text{ given.} \quad (12)$$

where  $\chi$  represents the intrinsic growth rate of the resources and  $E_0$  is the carrying capacity of the environment. Equation (12) is a modified version of the logistic growth function, a widely used functional form to describe the dynamics of renewable resources. It is here adjusted through the introduction of the output of the manufacturing sector in a way that a higher production of  $Y_{C,t}$  negatively affects the evolution of  $R$ .

### 3.5 Market-clearing conditions

Equilibrium in the product markets requires, respectively:

$$(1 - \tau)Y_{C,t} = C_t + K_{t+1} - (1 - \delta)K_t, \quad (13)$$

and

$$Y_{S,t} = C_{S,t}. \quad (14)$$

Equilibrium in the labour market requires:

$$L_t = L_{C,t} + L_{S,t} \quad (15)$$

Equilibrium in the capital market requires:

$$K_t^s = K_t^d \quad (16)$$

where  $K_t^s$  is the capital supplied by the households in period  $t$  and  $K_t^d$  is the capital demanded by the firms operating in the progressive sector.

## 4 The balanced growth path and the transitional dynamics

### 4.1 Firms optimising behaviour

In a perfectly competitive equilibrium, the rental rate on capital  $r_t$  to capital is equal to its marginal productivity:

$$r_t = (1 - \alpha) \left( \frac{K_{C,t}}{K_t} \right)^\alpha L_{C,t}^\alpha \quad (17)$$

In equilibrium, the following condition regarding the wage rate  $w_t$  must hold:

$$w_t = \alpha L_{C,t}^{\alpha-1} \left( \frac{K_{C,t}}{K_t} \right)^\alpha K_t = P_t \gamma L_{S,t}^{\gamma-1} \quad (18)$$

Equation (18), together with the market clearing condition (15), can be used to obtain an expression for the relative price  $P_t$  along an equilibrium path:

$$P_t = \frac{\alpha \left( \frac{K_{C,t}}{K_t} \right)^\alpha K_t (L_t - L_{C,t})^{1-\gamma}}{\gamma L_{C,t}^{1-\alpha}} \quad (19)$$

### 4.2 The Balanced Growth Path

By solving the households' optimisation problem it is possible to find a system of three difference equations in  $Q_t \equiv \left( \frac{K_{C,t}}{K_t} \right)$ ,  $Z_t \equiv R_t K_t$  and  $L_t$ , that governs the equilibrium path of the economy (see Appendix A):

$$\Phi(Q_{t+1}, Q_t, L_t) = Q_{t+1} [1 + \omega(Q_t, L_t)] - t [Q_t f(L_t)]^\alpha - (1 - \delta) Q_t = 0 \quad (20)$$

$$\Lambda(Z_{t+1}, Q_t, L_t, Z_t) = \frac{Z_{t+1}}{1 + \omega(Q_t, L_t)} - Z_t - \chi Z_t \left( 1 - \frac{Z_t Q_t L_t}{E_0} \right) \quad (21)$$

$$\begin{aligned} \Psi(Q_{t+1}, L_{t+1}, Q_t, L_t) = & \frac{Q_t^\alpha (1 - L_t)}{[f(L_t)]^{1-\alpha}} \theta \{ 1 - \delta + (1 - \alpha) Q_t^\alpha [f(L_t)]^\alpha \} - \\ & - \frac{Q_{t+1}^\alpha (1 - L_{t+1})}{[f(L_{t+1})]^{1-\alpha}} [1 + \omega(Q_t, L_t)] = 0 \end{aligned} \quad (22)$$

where

$$f(L_t) = L_{C,t} = L_t - \frac{\beta\gamma(1-\mu)}{1-\beta}(1-L_t) \quad (23)$$

and

$$\omega(Q_t, L_t) = \frac{K_{t+1} - K_t}{K_t} = (1-\tau)Q_t^\alpha [f(L_t)]^\alpha - \frac{\alpha\beta\mu Q_t^\alpha (1-L_t)}{(1-\beta)[f(L_t)]^{1-\alpha}} - \delta \quad (24)$$

An equilibrium path of the economy must also satisfy the transversality condition:

$$\lim_{t \rightarrow \infty} \theta^t \frac{(1-\beta)[f(L_t)]^{1-\alpha}}{\alpha Q_t^\alpha (1-L_t)} = 0 \quad (25)$$

Along the balanced growth path (BGP) one must have  $L_{t+1} = L_t = L$ ,  $Q_{t+1} = Q_t = Q$  and  $Z_{t+1} = Z_t = Z$  in the system (20)-(22), where:

$$Q = q(L) = \left[ \frac{(1-\delta)(1-\theta)}{\theta(1-\alpha)[f(L)]^\alpha - (1-\tau)[f(L)]^\alpha + \frac{\alpha\beta\mu(1-L)}{(1-\beta)[f(L)]^{1-\alpha}}} \right]^{1/\alpha} \quad (26)$$

$$Z = z(L) = \left[ \frac{\chi + (1+\chi)\omega[q(L), L]}{1 + \omega[q(L), L]} \right] \frac{E_0}{\chi q(L)L}, \quad (27)$$

and  $L$  is implicitly defined by:

$$q(L)[1 + \omega[q(L), L]] = \tau[q(L)f(L)]^\alpha + (1-\delta)q(L) \quad (28)$$

One can show that, for reasonable parameter values, a BGP characterised by  $Q^* > 0$ ,  $Z^* > 0$ ,  $L^* > 0$  exists. By linearizing the system (20)-(22) around this BGP, one can also produce numerical examples for which  $(Q^* > 0, Z^* > 0, L^* > 0)$  is saddle-path stable (see Appendix B).

Along its BGP, the stock of private physical capital  $K^*$ , the stock of public infrastructure  $K_C^*$  and the output of the progressive sector  $Y_C^*$  all exhibit the same growth rate, equal to  $\omega^*$ . As a result, the ratio between public and private capital  $Q^*$  remains constant.

Moreover, along a BGP, the employment levels in the two sectors and the output of the green sector  $Y_{S,t}$  also remain stable. Therefore, it is not surprising that along a BGP the cost of green services in units of the good produced in the progressive sector,  $P_t$ , increases without limits.

### 4.3 Transitional dynamics: a numerical example

I study the structural adjustments taking place along the transitional path converging to a BGP by a numerical example. Let  $\alpha = 0.7$ ,  $\beta = 0.6$ ,  $\gamma = 0.75$ ,  $\mu = 0.5$ ,  $\tau = 0.1$ ,  $\theta = 0.925$  and  $\delta = 0.040657$ . In this case a BGP exists and is characterised by  $Q^* = 0.959596$ ,  $Z^* = 1.308989$ ,  $L^* = 0.55$ , entailing a growth rate of private (and public) stock of capital  $\omega^* = 0.002612$ . Linearizing the system around  $(Q^*, Z^*, L^*)$  one can find its characteristics roots:  $\lambda_1 = 0.807329$ ,  $\lambda_2 = 0.607993$  and  $\lambda_3 = -1.684844$ . Such values imply that the system is saddle-path stable. The linearized system characterizes a unique path converging to  $(Q^*, Z^*, L^*)$ , which is is governed by:

$$Q_t - Q^* = D_1 c_{11} \lambda_1^t + D_2 c_{12} \lambda_2^t \quad (29)$$

$$Z_t - Z^* = D_1 c_{21} \lambda_1^t + D_2 c_{22} \lambda_2^t \quad (30)$$

$$L_t - L^* = D_1 c_{31} \lambda_1^t + D_2 c_{32} \lambda_2^t \quad (31)$$

where

$$\begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} c_{21} \\ c_{22} \\ c_{23} \end{bmatrix} = \begin{bmatrix} -0.956617 \\ 1 \\ -0.140976 \end{bmatrix}$$

are the characteristic vectors associated with, respectively,  $\lambda_1$  and  $\lambda_2$ ; and  $D_1$  and  $D_2$  are constants whose values are to be determined. What happens during the transition to the BGP depends on the initial values of  $Q$  and  $Z$ . In particular, in the numerical

example considered, the dynamics of the employment levels in the two sectors depends on whether  $Q_0$  is higher or lower than its BGP value  $Q^*$  (see Appendix C). Consider  $Q_0 = 1.5$  and  $Z_0 = 2$ . In this case  $D_1 = 0.255922$ ,  $D_2 = -0.564911$ ,  $L_0 = 0.62963$  and  $\frac{f(L_0)}{L_0} = 0.669131 > \frac{f(L^*)}{L^*}$ . That is, for these numerical values the employment level of the manufacturing sector decreases during the transition. Simultaneously, employment in the green service sector expands.

## 5 Long run effects of a change in some relevant parameters

### 5.1 Long run effects of a change in $\tau$

I now consider a marginal increase in  $\tau$ , i.e. in the fraction of the output of the progressive sector that the government employs in the creation of public capital.

Using the equation that defines the dynamics of private capital and (22) one can find that:

$$l(Q, L, \tau, \mu) = [Qf(L)]^\alpha \left[ \theta(1 - \alpha) - 1 + \tau + \frac{\alpha\beta\mu(1 - L)}{(1 - \beta)f(L)} \right] - (1 - \delta)(1 - \theta) = 0 \quad (32)$$

Given the parameter values of section 4.3 and using the implicit function theorem it is possible to compute that:

$$\left. \frac{\partial L}{\partial \tau} \right|_{L=L^*} = - \frac{\left. \frac{\partial l(Q, L, \tau, \mu)}{\partial \tau} \right|_{L=L^*}}{\left. \frac{\partial l(Q, L, \tau, \mu)}{\partial L} \right|_{L=L^*}} > 0 \quad (33)$$

Using (33) one can find the marginal effect of an increase in  $\tau$  on the BGP values of all the other variables. In particular, one can show that  $\left. \frac{\partial \omega}{\partial \tau} \right|_{\omega=\omega^*} > 0$ . That is, in this numerical example an increase in the fraction of  $Y_{C,t}$  invested by the government has a positive effect on the long-run growth rate. By rewriting (1) as

$$U^* = \beta \ln \left\{ \left[ \frac{\alpha \beta \mu Q^* Z^* (1 - L^*)}{(1 - \beta) [f(L^*)]^{1-\alpha}} \right]^\mu \left[ \frac{\beta \gamma (1 - \mu) (1 - L^*)}{1 - \beta} \right]^{\gamma(1-\mu)} \right\} + (1 - \beta) \ln(1 - L^*) \quad (34)$$

it is also possible to show that  $\left. \frac{\partial U}{\partial \tau} \right|_{U=U^*} < 0$ <sup>17</sup>.

This example shows that a permanent increase in  $\tau$ , namely in the investment expenditures in public infrastructure, does have a positive impact on the economy long-run rate of growth, as suggested by the literature (see section 2.2). Nevertheless, at the same time the steady-state level of households welfare is *negatively* affected by such an increase because of the longer hours of work that it involves. Also, an increase in  $\tau$  expands the employment share of the dirty sector along the BGP.

## 5.2 Long run effects of a change in $\mu$

I now investigate the effects of a marginal change in  $\mu$ , i.e. the parameter that determines the relative importance of the “progressive” good with respect to the green service in generating utility for the households. Interpreting equation (2) as a household production function *à la* Becker, a change in  $\mu$  affects the relative efficiency with which the inputs ( $R_t$ ,  $C_t$  and  $C_{S,t}$ ) contribute to the production of  $X_t$ . For instance, an increase in  $\mu$  can capture a technological change making the manufactured goods relatively more efficient in satisfying consumers needs and desires.

By adopting the same steps used in section 5.1 it is possible to show that  $\left. \frac{\partial L}{\partial \mu} \right|_{L=L^*} > 0$ ,  $\left. \frac{\partial \omega}{\partial \mu} \right|_{\omega=\omega^*} > 0$  and  $\left. \frac{\partial U}{\partial \mu} \right|_{U=U^*} < 0$ <sup>18</sup>. In other words, a marginal rise in  $\mu$ , has both a positive impact on long-run growth and a negative effect on the households steady-state utility.

<sup>17</sup>More precisely,  $\left. \frac{\partial L}{\partial \tau} \right|_{L=L^*} = 0.275629$ ,  $\left. \frac{\partial \omega}{\partial \tau} \right|_{\omega=\omega^*} = 0.426771$  and  $\left. \frac{\partial U}{\partial \tau} \right|_{U=U^*} = -0.236658$ .

<sup>18</sup>More precisely,  $\left. \frac{\partial L}{\partial \mu} \right|_{L=L^*} = 0.281110$ ,  $\left. \frac{\partial \omega}{\partial \mu} \right|_{\omega=\omega^*} = 0.055143$  and  $\left. \frac{\partial U}{\partial \mu} \right|_{U=U^*} = -0.778697$ .

## 6 Conclusions

The aim of the paper was to investigate the dynamics of an economy with two productive sectors having: a) a different capability of achieving increases in labour productivity; b) a different impact on an open-access resource whose stock enters as an argument in the households welfare function. Such a dichotomy is intended to represent the increasingly evident presence of a class of goods and services characterised by a strong attachment to the local *milieu*, high intensity of human labour and dedication, reduced consumption of material resources, and low polluting emissions. These activities - which I here name “green services” (Victor and Jackson, 2011) - typically exhibit low (or negative) growth rates in labour productivity as a result of the crucial importance of non-substitutable human participation in the production process.

For this reason, I built an endogenous growth model with a “progressive” sector and “stagnant” sector producing green services. Total factor productivity in the progressive sector is modelled to be a function of a stock of public capital (infrastructure) accumulated by the government and financed through the imposition of taxes on the households. An increase of the stock of public capital shifts the production function upwards, thus raising the productivity of the inputs used in the progressive sector. On the other hand, labour constitutes the one and only input for the production of green services; consequently, no productivity increase ever takes place in this sector. What qualifies the stagnant sector as “green” is the different influence that the goods and services produced in the two sectors have on a freely accessible open resource that can be interpreted either as an environmental or a social common asset. This is assumed to behave as a renewable resource, and to be negatively affected by the production of the progressive sector.

I derived the condition to be satisfied along the economy equilibrium path and showed, for reasonable parameter values, that the balanced growth path of the economy is saddle-path stable. The public and the private stock of capital, as well as the output of the progressive sector, exhibit the same long-run growth rate, and the allocation of employment

across the two sectors stabilizes along the balanced growth path. In order to analyse the structural change that takes place along the transition path converging to the economy BGP, I presented a numerical example in which employment tends to expand in the green service sector.

Finally, I used the same numerical example to investigate the long-run effects of a change in some relevant parameters. A marginal increase in the fraction of the output of the progressive sector devoted by the government to investments in new public infrastructure, generates a positive effect on long-run growth but at the same time it is likely to affect negatively the steady-state welfare. Similarly, an increase in the technological parameter entering the household production function, that can be interpreted as an improvement in the relative efficiency of material goods in generating consumption services (for instance, through an improvement of the technological content embodied in private market goods) boosts long-run growth and simultaneously depresses the households steady-state level of utility.

## Bibliography

- ACHESON, J., (2011) Multi-factor productivity: estimates for 1970 to 2009, *Economic & Labour Market Review*, May 2011
- AGENOR, P.R. (2010) A theory of infrastructure-led development, *Journal of Economic Dynamics and Control*, **34**, 932-950
- AGENOR, P.R., MORENO-DODSON, B. (2006) Public Infrastructure and Growth: New Channels and Policy Implications, *World Bank Policy Research Working Paper*, No. 4064, The World Bank
- ANTOCI, A., GALEOTTI, M., RUSSU, P. (2007) Undesirable economic growth via agents' self-protection against environmental degradation, *Journal of the Franklin Institute*, **344**, 377-390
- AYRES, R.U. (2008) Sustainability economics: Where do we stand? *Ecological Economics*,

67, 281-310

BARBIER E. (2009) Global Governance: The G20 and a Global Green New Deal, *Economics Discussion Paper*, No. 2009-38

BARRO R.J., SALA-I-MARTIN X. (1994) *Economic Growth*, McGraw-Hill, New York.

BARTOLINI, S., BONATTI, L. (2002) Environmental and social degradation as the engine of economic growth, *Ecological Economics*, **43**, 1-16

BARTOLINI, S., BONATTI, L. (2006) The mobilization of human resources as an effect of the depletion of environmental and social assets, *Metroeconomica*, **57**, 193-213

BARTOLINI, S., BONATTI, L. (2008) Endogenous growth, decline in social capital and expansion of market activities, *Journal of Economic Behavior & Organization*, **67**, 917-926

BAUMOL, W.J. (1967) Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis, *American Economic Review*, **57**, 415-426.

BONATTI, L., CAMPIGLIO, E. (2009) Mobility Systems and Economic Growth: a Theoretical Analysis of the Long-Term Effects of Alternative Transportation Policies, *Departmental Working Papers*, 7-2009, Department of Economics, University of Trento

BONATTI, L., FELICE, G. (2008) Endogenous growth and changing sectoral composition in advanced economies, *Structural Change and Economic Dynamics*, **19**, 109-131

BROCK, W.A., SCOTT TAYLOR, M. (2005) Economic Growth and the Environment: A Review of Theory and Empirics, in *Handbook of Economic Growth*, edited by Aghion, P., Durlauf, S.N., 1749-1821

CHATTERJEE, S., SAKOULIS, G., TURNOVSKY, S.J. (2003) Unilateral Capital Transfers, Public Investments and Economic Growth, *European Economic Review*, **47**, 1077-1103

COMMITTEE ON CLIMATE CHANGE (2010) *Building a low-carbon economy the UKs innovation challenge*, London

CONRAD, J.M. (1987) *Natural Resource Economics*, Cambridge University Press, Cambridge.

- ECHEVARRIA, C. (1997) Changes in Sectoral Composition Associated with Economic Growth, *International Economic Review*, **38**, 431-452
- ESTACHE, A., FAY, M. (2007) Current Debates on Infrastructure Policy, *Policy Research Working Paper*, No. 4410, The World Bank
- EUROPEAN COMMISSION (2010) *EUROPE 2020. A strategy for smart, sustainable and inclusive growth*, Brussels
- FELICE, G. (2009) A two-sector model of public investment and growth *Departmental Working Papers*, 2009-28, Department of Economics, Business and Statistics, University of Milan
- FUTAGAMI, K., MORITA, Y., SHIBATA, A. Dynamic Analysis of an Endogenous Growth Model with Public Capital *The Scandinavian Journal of Economics*, **95**, 607-625
- HIRSCHMAN, A.O. (1982) Rival interpretations of market society: civilizing, destructive or feeble?, *Journal of Economic Perspectives*, **20**, 1463-1484
- HOPKINS, R. (2011) *The Transition Companion: making your community more resilient in uncertain times*, Green Books, London
- JACKSON, T., VICTOR, P. (2011) Productivity and work in the 'green economy'. Some theoretical reflections and empirical tests, *Environmental Innovation and Societal Transition*, **1**, 101-108.
- LEWIS, W.A. (1954) Economic Development with Unlimited Supplies of Labour, *The Manchester School*, **22**, 139-191
- MCMILLAN, M., RODRIK, D. (2011) Globalization Structural Change, and Productivity Growth, *NBER Working Paper*, No. 17143
- NGAI, R.L., PISSARIDES, C.A., (2007) Structural Change in a Multisector Model of Growth *The American Economic Review* **97**, 429-443
- OECD (2011) *Towards Green Growth*, Paris
- ONS (2010) *United Kingdom National Accounts. The Blue Book*, Newport
- PISSARIDES, C.A. (2007) Unemployment and hours of work: The North Atlantic divide

revisited, *International Economic Review*, **48**, 136

POLLIN, R., GARRETT-PELTIER, H., HEINTZ, J., SCHARBER, H. (2008). *Green Recovery: A Program to Create Good Jobs and Start Building a Low-Carbon Economy*, Center for American Progress and Political Economy Research Institute, Washington DC

PUTNAM, R.D. (2000) *Bowling alone: The Collapse and Revival of American Community*, Simon & Schuster, New York

ROMP, W., DE HAAN, J. (2007) Public Capital and Economic Growth: A Critical Survey, *Perspektiven der Wirtschaftspolitik*, **8**, 6-52

TURNOVSKY, S. (1997) Fiscal Policy in a Growing Economy with Public Capital, *Macroeconomic Dynamics*, **1**, 615-639

UNEP (2008) *Green Jobs: Towards Decent Work in a Sustainable, Low-Carbon World*, Nairobi, Kenya

UNEP (2011) *Towards a Green Economy: Pathways to Sustainable Development and Poverty Eradication*, [www.unep.org/greeneconomy](http://www.unep.org/greeneconomy)

VAN ARK, B., OMAHONY, M., TIMMER, M.P. (2008) The Productivity Gap between Europe and the United States: Trends and Causes, *Journal of Economic Perspectives*, **22**, 2544

VAN ARK, B., WOLTJER, P. (2008) *The EU KLEMS Productivity Report*, Groningen Growth and Development Centre, University of Groningen

WORLD BANK (1994) *World Development Report 1994. Infrastructure for Development*, Oxford University Press, Oxford

XEPAPADEAS, A. (2005) Economic growth and the environment, in *Handbook of Environmental Economics*, edited by Maler K. G., Vincent J. R., 1219-1271

## A Derivation of the system (20)-(22)

The problem of the representative household can be solved by maximising

$$\sum_{t=0}^{\infty} \theta^t \{ \beta \ln[(R_t C_t)^\mu C_{S,t}^{1-\mu}] + (1 - \beta) \ln(1 - L_t) + \lambda_t [w_t L_t + r_t K_t + \pi_t + (1 - \delta)K_t - C_t - P_t C_{S,t} - K_{t+1} - T_t] \} \quad (\text{A.1})$$

with respect to  $C_t$ ,  $C_{S,t}$ ,  $L_t$ ,  $K_{t+1}$  and  $\lambda_t$ , where  $\lambda_t$  is a lagrangian multiplier, thus obtaining

$$\frac{\beta\mu}{C_t} = \lambda_t \quad (\text{A.2})$$

$$\frac{\beta(1 - \mu)}{C_{S,t}} = \lambda_t P_t \quad (\text{A.3})$$

$$\lambda_t = \lambda_{t+1} \theta (1 - \delta + r_t) \quad (\text{A.4})$$

$$\frac{1 - \beta}{1 - L_t} = \lambda_t w_t \quad (\text{A.5})$$

and the budget constraint.

A transversality condition must also be respected in equilibrium:

$$\lim_{t \rightarrow \infty} \theta^t \lambda_t K_t = 0 \quad (\text{A.6})$$

Using (A.2), (A.5) and (17) one can find an expression for  $C_t$ :

$$C_t = \frac{\alpha\beta\mu Q_t^\alpha K_t (1 - L_t)}{(1 - \beta) L_{C,t}^{1-\alpha}} \quad (\text{A.7})$$

Using (A.2), (A.3), (8), (19) and (A.7) it is possible to find (23).

Substituting equations (6), (A.7) and (23) into the dynamics of private capital allows to obtain (24).

Using equations (A.2), (17), (A.7), (24) and (23) one can rewrite (A.4) as 20.

From the dynamics of public capital (10) is then possible to obtain (21).

Finally, using (6), the dynamics of the common resource (12) can be written as (22)

## B Stability of the BGP

In order to analyse the stability of system (20)-(22) one can linearize it around  $(Q^* > 0, Z^* > 0, L^* > 0)$ :

$$\begin{bmatrix} Q_{t+1} - Q^* \\ Z_{t+1} - Z^* \\ L_{t+1} - L^* \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} Q_t - Q^* \\ Z_t - Z^* \\ L_t - L^* \end{bmatrix}$$

where

$$\begin{aligned} a_{11} &= -\frac{\Phi_{Q_t}}{\Phi_{Q_{t+1}}} & a_{12} &= 0 & a_{13} &= -\frac{\Phi_{L_t}}{\Phi_{Q_{t+1}}} \\ a_{21} &= -\frac{\Lambda_{Q_t}}{\Lambda_{Z_{t+1}}} & a_{22} &= -\frac{\Lambda_{Z_t}}{\Lambda_{Z_{t+1}}} & a_{23} &= -\frac{\Lambda_{L_t}}{\Lambda_{Z_{t+1}}} \\ a_{31} &= -\frac{\Psi_{Q_t}}{\Psi_{L_{t+1}}} + \frac{\Psi_{Q_{t+1}}\Phi_{Q_t}}{\Psi_{L_{t+1}}\Phi_{Q_{t+1}}} & a_{32} &= 0 & a_{33} &= -\frac{\Psi_{L_t}}{\Psi_{L_{t+1}}} + \frac{\Psi_{Q_{t+1}}\Phi_{L_t}}{\Psi_{L_{t+1}}\Phi_{Q_{t+1}}} \end{aligned}$$

and:

$$\Phi_{Q_{t+1}} = 1 + \omega(Q_t, L_t)$$

$$\Phi_{Q_t} = Q_{t+1}\omega_{Q_t}(Q_t, L_t) - \alpha\tau Q_t^{\alpha-1}[f(L_t)]^\alpha - 1 + \delta$$

$$\Phi_{L_t} = Q_{t+1}\omega_{L_t}(Q_t, L_t) - \alpha\tau Q_t^\alpha[f(L_t)]^{\alpha-1}f_{L_t}(L_t)$$

$$\Lambda_{Z_{t+1}} = \frac{1}{1 + \omega(Q_t, L_t)}$$

$$\Lambda_{Q_t} = -\frac{Z_{t+1}\omega_{Q_t}(Q_t, L_t)}{[1 + \omega(Q_t, L_t)]^2} + \chi \frac{Z_t^2 L_t}{E0}$$

$$\Lambda_{Z_t} = 2\chi \frac{Z_t Q_t L_t}{E_0} - 1 - \chi$$

$$\Lambda_{L_t} = -\frac{Z_{t+1} \omega_{L_t}(Q_t, L_t)}{[1 + \omega(Q_t, L_t)]^2} + \chi \frac{Z_t^2 Q_t}{E_0}$$

$$\Psi_{Q_{t+1}} = -\frac{\alpha Q_{t+1}^{\alpha-1} (1 - L_{t+1})}{[f(L_{t+1})]^{1-\alpha}} [1 + \omega(Q_t, L_t)]$$

$$\Psi_{L_{t+1}} = [1 + \omega(Q_t, L_t)] Q_{t+1}^\alpha \{(\alpha - 1)(1 - L_t) [f(L_{t+1})]^{\alpha-2} f_{L_{t+1}}(L_t + 1) - f(L_{t+1})^{\alpha-1}\}$$

$$\Psi_{Q_t} = \alpha \theta (1 - \delta) \frac{Q_t^{\alpha-1} (1 - L_t)}{[f(L_t)]^{1-\alpha}} + 2\alpha \theta (1 - \alpha) (1 - L_t) [Q_t f(L_t)]^{2\alpha-1} - \frac{Q_{t+1}^\alpha (1 - L_{t+1}) \omega_{Q_t}(Q_t, L_t)}{[f(L_{t+1})]^{1-\alpha}}$$

$$\begin{aligned} \Psi_{L_t} = & \theta (1 - \delta) Q_t^\alpha \{(\alpha - 1)(1 - L_t) [f(L_t)]^{\alpha-2} f_{L_t}(L_t) - f(L_t)^{\alpha-1}\} + \\ & + \theta (1 - \alpha) Q_t^{2\alpha} \{(2\alpha - 1)(1 - L_t) [f(L_t)]^{2\alpha-2} f_{L_t}(L_t) - f(L_t)^{2\alpha-1}\} - \\ & - \frac{Q_{t+1}^\alpha (1 - L_{t+1}) \omega_{L_t}(Q_t, L_t)}{[f(L_{t+1})]^{1-\alpha}} \end{aligned}$$

where:

$$f_{L_t} = f_{L_{t+1}} = 1 + \frac{\beta \gamma (1 - \mu)}{1 - \beta}$$

$$\omega_{Q_t}(Q_t, L_t) = \alpha (1 - \tau) Q_t^{\alpha-1} [f(L_t)]^\alpha - \frac{\alpha^2 \beta \mu Q_t^{\alpha-1} (1 - L_t)}{(1 - \beta) [f(L_t)]^{1-\alpha}}$$

$$\omega_{L_t}(Q_t, L_t) = \alpha (1 - \tau) Q_t^\alpha [f(L_t)]^{\alpha-1} f_{L_t}(L_t) - \frac{\alpha \beta \mu Q_t^\alpha}{1 - \beta} [(\alpha - 1)(1 - L_t) [f(L_t)]^{\alpha-2} f_{L_t}(L_t) - [f(L_t)]^{\alpha-1}]$$

and all derivatives must be evaluated at  $(Q^* > 0, Z^* > 0, L^* > 0)$ . The characteristic equation of the system is:

$$\lambda^3 - \text{tr}(A)\lambda^2 + \text{m}(A)\lambda - \det(A) \tag{B.1}$$

where  $\text{tr}(A)$ ,  $c(A)$  and  $\det(A)$  are, respectively, the trace, the sum of the principal minors and the determinant of matrix  $A$ . The solutions  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are the eigenvalues of  $A$ .

Consider the following numerical example, where  $\alpha = 0.7$ ,  $\beta = 0.6$ ,  $\gamma = 0.75$ ,  $\mu = 0.5$ ,  $\tau = 0.1$ ,  $\theta = 0.925$  and  $\delta = 0.040657$ . In this case  $Q^* = 0.959596$ ,  $Z^* = 1.308989$ ,  $L^* = 0.55$  and the growth rate of private (and public) stock of capital is  $\omega(Q^*, L^*) = 0.002612$ . The values of the roots of the characteristic equation ( $\lambda_1 = 0.807329$ ,  $\lambda_2 = 0.607993$  and  $\lambda_3 = -1.684844$ ) implies that the system is saddle-path stable.

## C Discussion of initial values

For  $t=0$  system (29)-(31) becomes:

$$Q_0 - Q^* = D_1 c_{11} + D_2 c_{12} \quad (\text{C.1})$$

$$Z_0 - Z^* = D_1 + D_2 \quad (\text{C.2})$$

$$L_0 - L^* = D_1 c_{31} + D_2 c_{32} \quad (\text{C.3})$$

Solving the system for  $D_1$ ,  $D_2$ ,  $L_0$  gives:

$$D_2 = \frac{Q_0 - Q^* - (Z_0 - Z^*)c_{11}}{c_{12} - c_{11}} \quad (\text{C.4})$$

$$D_1 = Z_0 - Z^* - D_2 \quad (\text{C.5})$$

$$L_0 = D_1 c_{31} + D_2 c_{32} + L^* \quad (\text{C.6})$$

To have a increasing share of employment in the green service sector during the transition to the BGP one must have  $\frac{f(L_0)}{L_0} > \frac{f(L^*)}{L^*}$ . By substituting (C.4), (C.5) and (C.6) into (23), it is possible to show that this is satisfied only if  $Q_0 > Q^*$ .