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**Boom and Burst in Housing Market with
Heterogeneous Agents
(New Version)**

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Boom and Burst in Housing Market with Heterogeneous Agents*

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Abstract

We study the housing market using a partial “dis”-equilibrium dynamic model in which the rational expectations hypothesis is relaxed in favor of chartist-fundamentalist mechanism to allows for the endogenous development of bubbles. Our model is able to replicate the recent house price dynamics in the US, with the preference shock being the main forcing variable. We also analyze the role of the interest rate policy. Our model supports the view that anchoring the interest rate to the change in house price would have reduced the volatility and the distortion in the price dynamics.

Keywords: Heterogeneous agents, house price, agent-based model
JEL: E3, E4

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1 Introduction

The economies of almost every countries have recently been hit by a turmoil in the financial markets. The financial crisis demonstrated that developments in financial markets can have a large impact on real economy. For this reason, a lot of recent research has been devoted to investigate the interdependencies between real and financial markets in macroeconomics.

Shiller [31] was the first who emphasized the fact that the movements in the stock prices were much more marked with respect to the relative changes in dividends. From that moment a huge amount of literature was focused on asset price movements and irrational exuberance as well as fundamentals as driving forces of price dynamics (see [13], [4], [19], [20], [22], [18]). Furthermore Shiller emphasized the fact that the same forces of human psychology driving financial markets could also have the potential to affect other markets: this seems to be true for housing market. Indeed some recent example of macro models with heterogeneous expectations include [10], [14], [15], [6], [7].

The recent boom-burst development appears to be anything never seen before, since the late 1990s a dramatic increase in housing price has been observed all over the world. For example, London real house price tripled during the period 1996-2008, also in the United State the housing prices increased by 85 percent roughly during the same period. It is quite a challenge to explain this phenomena merely on the basis of rational behavior, because fundamentals such as real rents or construction costs do not match up with this incredible price boom. The speculative thinking and the use of non rational expectations deriving from market psychology could be elements that play an important role in determining house prices.

The method of Agent-Based computational (ABC) simulation could then provide a valuable modelling framework in this respect. It drops the assumption of rational expectations, homogeneous individuals, perfect ex-ante coordination in favor of adaptive learning and simple interaction of heterogeneous agents. One strength of the ABC method is that it allows for the endogenous creation of bubbles, typically observed in housing market.

In this paper we connect a simple ABC model with some typical elements of New Keynesian Dynamic Stochastic General Equilibrium (DSGE) analysis. Houses are seen as assets that can be driven by fundamentals and by *animal spirits*. Starting from this point, the possibility to predict the future changes in house prices and the deviations between housing prices and fundamentals create the opportunity of large gains. The aim is to investigate if the behavioral approach and bounded rationality could explain the recent boom and burst in house prices. Note that a large empirical evidence shows that human agents generally act in a bounded rational way (see [25], [21]).

Before the 2007 crisis the literature about house price dynamics was poor, except for Iacoviello [24], who developed a business cycle model with houses as collateral, and Lustig and van Nieuwerburgh [29], who focused on the role of housing collateral on stocks pricing. The recent fluctuations in housing market have increased the interest of researchers in this field but it is still difficult to

explain the large and rapid rise and fall in housing price using a purely rational model.

Some recent papers use models of learning to explain the observed phenomena. Burnside, Eichenbaum and Rebelo [11] adopted a model in which agents have heterogeneous expectations about long-run fundamentals but are also influenced by infectious optimism, a *social dynamics*, that vanishes as soon as people become certain about fundamentals. Adam, Marcet and Kuang [3] developed a model able to replicate quantitatively the house price dynamics from 2001 to 2008 in the G7 economies as well as the associated current account, relaxing the rational expectations hypothesis and allowing households to be uncertain about how house prices are related to the economic fundamentals. To reach this goal, they use the concept of *internal rationality*, previously developed by Adam and Marcet (see [1], [2]), where utility maximizer agents do not fully understand how prices are formed, so that their subjective probability distribution about prices may not exactly be equal to the true equilibrium distribution.

We borrow the model from Adam, Marcet and Kuang (2011), but we employ a different mechanisms for expectations formation. Our approach is inspired by recent works on Agent-Based¹ financial market models in which the dynamics of financial market depends on the expectation formation of boundedly rational heterogeneous interacting agents (see in particular, Lengnick and Wohltmann [28]). Households are maximizing agents: they can be either chartists, believing the house price trend to continue, or fundamentalists, expecting mispricing will be corrected by the market. Agents thus use adaptive learning strategies and continuously evaluate those strategies according to past performance. According to this evaluation, agents endogenously switch between the fundamentalist and the chartist strategies. This leads to changes in the size of the different groups and finally to the price dynamics. When chartists dominate the market, house price can sharply deviate from the underlying fundamental value but, if the *animal spirits* change, the market will be dominated by fundamentalists and the price will converge to the fundamental value.

Given our model, we are able to discuss three features that the literature suggests as potential sources of the recent boom and burst in US house prices. First, Adam, Marcet and Kuang [3] discuss the role of the interest rate during this crisis: the house price boom would be caused by the persistent reductions in the interest rate. They suggest that for the U.S. economy the boom would have been largely avoided if the interest rate had fallen by less at the beginning of the 2000's. Second, house prices are usually also connected to credit availability, as in Favilukis, Ludvigson and Van Nieuwerburgh [17]. They developed an overlapping generation model in which heterogeneous households face limited risk-sharing opportunities as a result of incomplete financial markets. They focus on the macroeconomic consequences of systemic changes in housing finance, with an emphasis on how these factors affect risk premia in housing markets, and how risk premia, in turn, affect house prices. Their results show that credit

¹To have a survey on the Agent-Based Computational models visit the Tesfatsion website, www.econ.iastate.edu/tesfatsi/ace.htm

tightness can be a driving force of boom and bursts on house price because it has a huge impact on risk premia. On the contrary, the interest rate does not have such an influence in their analysis. Third, our model is well equipped to analyse the role of households' beliefs and the psychological variables during the recent housing boom. As stressed in Piazzesi and Schneider [30], who present evidences from the Michigan Survey of Consumers², the percentage of households believing it was a good time to buy a house (because expecting the price to raise further) increased towards the end of the boom. The mechanism of chartism and fundamentalism is one of the simplest method to be able to pickup this kind of expectations.

It turns out that our model identifies the shock to the preference rate for houses as the main driving force behind the recent behavior of house price in the US. Using the Michigan Survey of Consumers, the model captures quite well the persistence and hump shaped behavior of the boom and burst in house price. On the contrary, the interest rate and the credit tightness have only minor effects on house price movements.

Finally, we can also show that an interest rate policy that reacts to the deviations of house price from steady state or to the change in the house price can substantially stabilize the house price.

The paper is constructed as follows. The model is developed in section 2 where we explain the Agent-Based price formation and derive the optimality conditions for households and house builders and describe the time of actions and the interactions between demand and supply. The study of the simulations and the impulse response functions is done in section 3. Section 4 shows the capability of the model to replicate the recent house price dynamics taking into consideration the real interest rate dynamics, the credit tightness and a preference shock on house demand. In section 5 we use a particular monetary policy rule to study the impact of a policy that links the interest rate to the house price. Section 6 concludes.

2 The Model

The model reflects the one of Adam, Marcet and Kuang [3] but it moves away from it in the type of expectations we adopt. Households use backward-looking expectations to infer about future house price. We consider house like an asset and, in so doing, it seems suitable to express the expectations starting from the financial Agent-Based literature³. Households are utility maximizers belonging to two different groups: agents thinking the trend on house price services will continue in the next periods and others thinking the market will lead the price in the direction of the *perceived* fundamental value. The perceived fundamental value is the long run house price but it can be different from its steady state value. How it is formed will be specified later on.

²<http://www.sca.isr.umich.edu/>

³See Hommes [22], LeBaron [27] or Westerhoff [32]

In the model the size of the two groups is not fixed but it changes across time according to past strategy performance and this mechanism is able to generate endogenous waves of chartism and fundamentalism that could move the price away from its fundamental value.

Another important difference with Adam, Marcet and Kuang [3] is the timing of actions: demand and supply are not simultaneous and the house price does not emerge from the equality between supply and demand. Regarding demand, households solve their problem daily, which we suppose to be the smallest fraction of time for the real economy. Regarding supply, house builders are subject to time-to-build, and take decisions on a quarterly basis using as a reference price the average of the past quarter house price. They can also influence future price, changing the perceived fundamental value in the households mind. This particular choice for timing is due to the necessary time that elapses during the construction of a house. Therefore the price is *demand-driven* even if the demand of the fundamentalist households is influenced by supply. This model is a model of *dis*-equilibrium: the price reacts to the difference between demand and supply increasing in time where the demand is larger than supply and decreasing on the other hand.

2.1 An Agent-Based approach to House Price

In this section we analyze the price mechanism formation following Lengnick and Wohltmann [28]. The house price, expressed in deviation from its steady state, \hat{Q}_t is driven by the different expectations of agents. Chartists (*c*) and fundamentalists (*f*) influence the price formation through their demand, which is determined by solving the daily households maximization problem. The total amount of agents using a certain type of expectations is not fixed but it varies over time according to the evaluation of past performances. This mechanism creates an endogenous environment with booms and bursts.

The law of motion for the house price is given by:

$$\hat{Q}_{t+1} = \hat{Q}_t + a(W_t^c \hat{h}_t^{d,c} + W_t^f \hat{h}_t^{d,f}) + \varepsilon_t^Q, \quad (1)$$

and it can be interpreted as a market maker scenario, where prices are adjusted according to excess demand.

\hat{Q}_{t+1} is the price percentage deviation from its steady state at $t + 1$ and it is driven by the past price deviation \hat{Q}_t , and by the deviation of chartists $\hat{h}_t^{d,c}$ and fundamentalists $\hat{h}_t^{d,f}$ demands from their respective steady state values. The demand functions are obtained below by solving a maximization problem. Moreover, the demand of each group is a function of their respective expectations (see below).

W_t^c and W_t^f are the fractions of agents adopting the two strategies, and they endogenously vary over time. The total amount of the population is normalized to one. a is a parameter that governs the impact of the demands on the price formation. The noise term ε_t^Q is i.i.d. normally distributed with standard

deviation σ_Q^2 and it captures the fact that the two strategies are not the only possible strategies that exist into the market.

2.2 The Households' Problem

Households are maximizing agents that consume and invest. They are allowed to borrow from banks subject to a borrowing constraint as in Kiyotaki and Moore [26].

The economy is populated by a unit mass of households with identical preferences but different beliefs, they can be chartists $E_t^c(\cdot)$ or fundamentalists $E_t^f(\cdot)$ concerning expectations about future house price. They take daily decisions (t stands for days) and maximize an inter-temporal utility function:

$$E_t^{c/f} \sum_{t=0}^{\infty} \delta^t (c_t + j_t \log h_t), \quad (2)$$

where $c_t > 0$ is the daily consumption of goods, h_t is the consumption of house services, $\delta \in (0, 1)$ is the discount factor, and j_t is a shock that reflects a preference shock for house demand.

The household maximization is subject to the following budget constraint:

$$c_t + [h_t - (1 - d)h_{t-1}]Q_t + R_t b_{t-1} + k_t = y_t + b_t + k_{t-1}p_t. \quad (3)$$

We denote the house price at time t with Q_t , $d \in [0, 1)$ is the daily rate at which house depreciate, b_t is the households' new loans and R_t is the gross real interest rate maturing on loan b_{t-1} . Until section 5, we consider the steady state value of the interest rate as the daily transformation of the 30-Year Conventional Mortgage Rate in Q1-2004⁴ and R_t as the percentage deviation from it. In section 5 we modify this assumption in order to give some policy suggestions. In addition we use an exogenous process for the income y_t . $k_t \geq 0$ is the capital sold to house builders who use it as an input to produce new houses. Capital fully depreciates in one period and its remuneration is p_t .

Following Adam, Marcet and Kuang [3] households have the possibility to borrow from banks, but they are subject to the following borrowing constraint

$$b_t \leq \theta \frac{Q_t}{R_t} h_t \quad (4)$$

The parameter θ represents the share of assets that can be collateralized, it is fixed and cannot exceed the house value after the depreciation: hence $\theta \in (0, 1 - d]$. As in Kiyotaki and Moore [26] a value of θ lower than one reflects the cost the lenders would suffer in case of default. When the house price increases, the collateral constraint is relaxed implying that the households will have greater access to credit.

⁴Data are taken from <http://www.stlouisfed.org/>

2.2.1 The Solution of Households' Problem

In this section we show the solution of households' maximization problem assuming that the utility from consumption is bounded for high level of c . The first order conditions are necessary and sufficient conditions to achieve a maximum due to the linearity of the constraint in the households' choice variable and the concavity of the objective function.

Households maximize their utility function (2) subject to the budget and borrowing constraints (3-4).

The maximization problem is:

$$\max_{\{c_t, h_t, b_t, k_t\}} E_t \sum_{t=0}^{\infty} \delta^t \left\{ \begin{array}{l} (c_t + j_t \ln(h_t)) + \\ -\lambda_t (c_t + (h_t - (1-d)h_{t-1})Q_t + R_t b_{t-1} + k_t - y_t - b_t - k_{t-1}p_t) + \\ + \gamma_t (\theta Q_t h_t - R_t b_t) + \mu_t c_t + \kappa_t k_t \end{array} \right\},$$

where p_0, k_{-1}, b_{-1} are given initial conditions.

The first order conditions with respect to c_t, h_t, b_t and k_t are:

$$(\partial c_t) : 1 - \lambda_t + \mu_t = 0 \quad (\mu_t \geq 0; \mu_t c_t = 0) \quad (5)$$

$$(\partial h_t) : \frac{j_t}{h_t} - \lambda_t Q_t + (1-d)\delta E_t^{c/f} \lambda_{t+1} Q_{t+1} + \gamma_t \theta Q_t = 0 \quad (6)$$

$$(\partial b_t) : \lambda_t - R_t \delta E_t^{c/f} \lambda_{t+1} - \gamma_t R_t = 0 \quad (\gamma_t \geq 0; \gamma_t (\theta Q_t h_t - R_t b_t) = 0) \quad (7)$$

$$(\partial k_t) : -\lambda_t + \kappa_t + \delta E_t^{c/f} \lambda_{t+1} p_{t+1} = 0 \quad (\kappa_t \geq 0; \kappa_t k_t = 0). \quad (8)$$

Assuming that the non-negativity of consumption holds ($\mu_t = 0$) and $R_t \delta < 1$, households will borrow as much as possible: hence the borrowing constraint is binding ($\gamma_t > 0$). From equation (5) $\lambda_t = 1$; therefore from (7) $\gamma_t = \frac{1}{R_t} - \delta > 0$. Using these results, from equation (6) it is possible to derive the demand for new houses:

$$h_t^d = j_t \left[\left(1 + \delta \theta - \frac{\theta}{R_t} \right) Q_t - (1-d)\delta E_t^{c/f} Q_{t+1} \right]^{-1}. \quad (9)$$

The demand functions for each type of household will be specified in the following section.

The optimal level of borrowing can be derived from the borrowing constraint and is equal to:

$$b_t = \frac{\theta Q_t h_t}{R_t}. \quad (10)$$

The capital offered by the consumers to house builders is only restricted to satisfy:

$$(1 - \delta p_{t+1})k_t = 0,$$

so that either $p_t = \delta^{-1}$ or $k_t = 0$. This means that if the non-negativity constraint is non-binding, capital and consumption are not uniquely determined

and agents are indifferent between increasing slightly the capital sold to firms at time t in exchange for δ^{-1} more units of consumption at $t + 1$. Since firms have a positive demand for k , market clearing occurs at

$$p_t = \frac{1}{\delta}, \quad (11)$$

with capital supply offered by consumers being perfectly elastic, so that k_t is determined by firm's demand. Finally consumption can be obtained residually plugging (11) into the flow budget

$$c_t = y_t + b_t - (h_t - (1 - d)h_{t-1})Q_t - b_{t-1}R_t - k_t - k_{t-1}\delta^{-1}. \quad (12)$$

2.2.2 Housing Supply

House builders operate quarterly (q). The difference in action timing among households and house builders reflects the time that elapses in creating new houses. The house builders technology exhibits decreasing return to scale to capital which is the only factor of production:

$$u = (\alpha\delta)^{-1}k_q^\alpha. \quad (13)$$

k_q is the sum over a quarter of the daily capital received from household and $\alpha \in (0, 1)$. We also assume that the market for input is always in equilibrium, thus the price of capital is $p_t = p_q = \delta^{-1} \forall t, q$.

The house builders' maximization problem is:

$$\max_{\{k_q \geq 0\}} E_q (uQ_{q+1} - \delta^{-1}k_q).$$

The first order condition is:

$$k_q = (E_q Q_{q+1})^{\frac{1}{1-\alpha}}. \quad (14)$$

House builders maximize profits but they do not know the future demand. We formulate house builders expectations in a stationary manner $E_q[Q_{q+j}] = Q_q \forall j = 1, 2, \dots$. Therefore the profit-maximizing input choice becomes:

$$k_q = (Q_q)^{\frac{1}{1-\alpha}} \quad (15)$$

substituting k_q into the production function (13), we obtain the quarterly house supply:

$$h_q^s = (\alpha\delta)^{-1}Q_q^{\frac{\alpha}{1-\alpha}}. \quad (16)$$

Finally, the housing stock evolves according to:

$$h_q = (1 - d)h_{q-1} + S_q^h,$$

where h_{q-1} is the existing housing stock in the previous quarter.

2.3 Expectations

Chartists expect the price trend will continue, consequently their expectation is:

$$E_t^c[\hat{Q}_{t+1}] = \hat{Q}_t + l^c(\hat{Q}_t - \hat{Q}_{t-1}), \quad (17)$$

where the parameter l^c governs the expected trend and hatted variables are deviations from steady state as above.

Fundamentalists, instead, believe that the price reverts towards a fundamental value and that in every period the price moves to partially correct the actual mispricing, that is, the difference between the current price and the fundamental price:

$$E_t^f[\hat{Q}_{t+1}] = \hat{Q}_t + l^f(\hat{Q}_t^{fd} - \hat{Q}_t), \quad (18)$$

where the parameter l^f defines the amount of the expected mispricing correction in the next period. \hat{Q}_t^{fd} is the perceived fundamental value in deviation from long-run equilibrium. Following Lengnick and Wohltmann [28], we assume that the fundamental price perceived by the fundamentalist moves over time and it does not coincide with the steady state value. In a non-rational expectation context as ours, it is natural to assume that households are not able to determine the true fundamental price. Instead, they proxy the fundamental price from a function of the current supply. More precisely, the *perceived* fundamental price by fundamentalists is given by:

$$Q_t^{fd} = (h_q^s)^z \quad q = \text{floor}\left(\frac{t-1}{64}\right), \quad z > 0. \quad (19)$$

The function $\text{floor}(\cdot)$ rounds its argument to the nearest integer less than or equal to the argument itself. The intuition for this choice is that, in this way, the perceived fundamental price becomes a long term variable but it is also biased in the direction of the most recent real economic activity, that is if output is high (low) the fundamental house price is perceived to lie above (below) its true counterpart.

The fraction of agents using a certain type of expectation is not fixed. Households are allowed to learn about the past, changing their beliefs according to previous performances. Therefore each group evaluates the attractiveness of an action using the following rule:

$$A_t^i = [\exp(\hat{Q}_t) - \exp(\hat{Q}_{t-1})]\hat{h}_{t-2}^{d,i} + \eta A_{t-1}^i \quad i = c, f. \quad (20)$$

The parameter $0 \leq \eta \leq 1$ is a memory parameter that defines the strength with which agents discount past actions. The fraction of agents that adopt a particular strategy is updated thanks to the Gibbs Probability. In so doing we employ the framework of Adaptive Belief System proposed by Brock and Hommes (see [8], [9]).

$$W_t^i = \frac{\exp(eA_{t-1}^i)}{\sum_i \exp(eA_{t-1}^i)} \quad i = c, f. \quad (21)$$

The more attractive a strategy is, the higher is the fraction of agents using it. The parameter e , called the rationality parameter, reflects the intensity of choice. The higher is e , the greater will be the change in the size of agents that adopt the strategy with the highest attractiveness.

2.4 The Log-Linearized Model

In steady state the timing of actions does not matter. Equalizing demand and supply in a timeless fashion we can find the true fundamental value for the house price and then the percentage deviations of the price from its steady state.

The main steady state equations for our purposes are:

$$h^d = \frac{j}{Q \left(1 + \delta\theta - \frac{\theta}{R} - (1-d)\delta\right)}, \quad (22)$$

$$h^s = \frac{1}{\alpha\delta} Q^{\frac{\alpha}{1-\alpha}}. \quad (23)$$

Equalizing (22) and (23), and solving for Q we obtain the true fundamental value for house price:

$$Q = \left(\frac{j\alpha\delta}{1 + \theta\delta - \frac{\theta}{R} - (1-d)\delta} \right)^{1-\alpha}. \quad (24)$$

Given the steady state, it is easy to obtain the following log-linearized equations for demand, supply and the perceived fundamental price:

$$\hat{h}_t^d = \hat{j}_t + \frac{Qh^d}{j} \left[(1-d)\delta E_t^{c/f} \hat{Q}_{t+1} - \left(1 + \delta\theta - \frac{\theta}{R}\right) \hat{Q}_t - \frac{\theta}{R} \hat{R}_t \right], \quad (25)$$

$$\hat{h}_q^s = \frac{\alpha}{1-\alpha} \hat{Q}_q, \quad (26)$$

$$\hat{Q}_t^{fd} = z \hat{h}_q^s. \quad (27)$$

The demand function in (25) depends positively from the expected future price and from the preference shock for houses but negatively from the current price and from the interest rate. The supply is a positive function of the quarterly price and finally the fundamental perceived price is positively related to the supply.

Inserting (17) and (18) into (25) we can write the chartists' and the fundamentalists' demand functions as:

$$\hat{h}_t^{d,c} = \hat{j}_t + \frac{Qh^d}{j} \left[(1-d)\delta \left(\hat{Q}_t + l^c (\hat{Q}_t - \hat{Q}_{t-1}) \right) - \left(1 + \delta\theta - \frac{\theta}{R}\right) \hat{Q}_t - \frac{\theta}{R} \hat{R}_t \right] \quad (28)$$

$$\hat{h}_t^{d,f} = \hat{j}_t + \frac{Qh^d}{j} \left[(1-d)\delta \left(\hat{Q}_t + l^f(\hat{Q}_t^{fd} - \hat{Q}_t) \right) - \left(1 + \delta\theta - \frac{\theta}{R} \right) \hat{Q}_t - \frac{\theta}{R} \hat{R}_t \right] \quad (29)$$

The log-linearized stock evolution is:

$$\hat{h}_q = (1-d)\hat{h}_{q-1} + S_q^{rh}(Q_q) \quad (30)$$

Finally, we need to define the exchange quantity as the short side of the market, i.e., the minimum between the sum of the chartists' and fundamentalists' demands in the correspondent quarter and the relative existing stock:

$$G = \min \left\{ \sum_{t=64(q-1)+1}^{64q} \left(\hat{h}_t^{d,f} + \hat{h}_t^{d,c} \right); \hat{h}_q \right\}. \quad (31)$$

2.5 The Time of Actions

The actions of households and house builders are not synchronized. The formers operate daily and the latters quarterly, because a house needs time to be built. Demand and supply thus run on different time scale. We assume that a quarter is composed by 64 days. Therefore households solve their maximization problem and find their demand for houses 64 times within one increment on house supply's time index q . The model is implemented as follows. First, we run the daily demand for a quarter. Then we take the mean of the daily price over that quarter to find the quarterly price. Finally, we insert it into the supply equation to find the reaction of house builders. Note that the average of house price is determined at the end of each quarter.

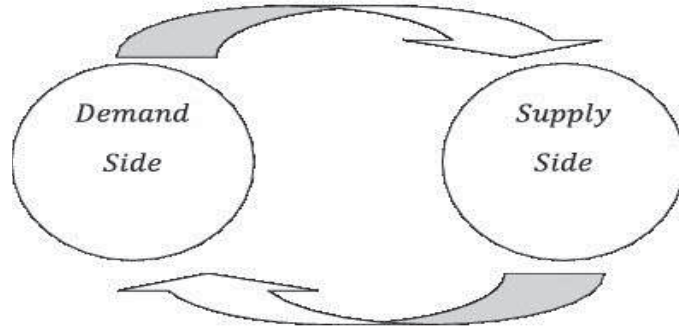
A quarter is defined to contain days $64(q-1)+1, \dots, 64q, q=1, 2, \dots$. The quarterly price is thus:

$$\hat{Q}_q = \frac{1}{64} \sum_{t=64(q-1)+1}^{64q} \hat{Q}_t. \quad (32)$$

Therefore, house supply for the next quarters, \hat{h}_q^s , is given by (26) and the fundamentalists' perceived fundamental value by (27). Note that \hat{h}_q^s is the end-of period supply that will remain fixed for the next quarter.

According to this mechanism the demand and supply side influence each other (see Figure 1) Households influence house builders via the daily price formation generated by the chartist/fundamentalist dynamics in (32), while house builders affect the demand via the perceived fundamental value in (27).

$$\hat{Q}_q = \frac{1}{64} \sum_{t=64(q-1)+1}^{64q} \hat{Q}_t$$



$$\hat{Q}_q^{fd} = z \left(\hat{S}_q^h \right)$$

Figure 1: Channels between demand and supply side

3 Model Simulation

In this section we simulate the model and analyze its performance. We are interested in investigating the capability of the model to generate endogenous waves of chartism and fundamentalism driving the price up and down. Moreover, we study the impulse response functions of the system to an increase in house preferences.

The parameter calibrations are reported in the table below

Macro parameter	Agent-based parameters
$\alpha = 0.5$	$a = 0.001$
$\delta = 0.96$	$l^c = 0.04$
$\theta = 0.55$	$l^f = 0.04$
$d = 4618 \cdot 10^{-8}$	$\eta = 0.975$
$z = 1$	$e = 100$

Tab 1: Calibration of the model

The parameter values are set according to the baseline calibration used in Adam, Marcet and Kuang [3] and in Lengnick and Wohltmann [28]. The rational parameter e is lower than the one presented in Westerhoff [32]. Since we have no clear calibration guidance for the intensity of choice e , we calibrated it to minimize the distance between the real data and the model output. However our results are quite robust to alternative calibrations of e , as also showed by

Table 2 in Section 4. Indeed we can think about financial market as a system populated by a large number of sophisticated professional investors with great rationality, whereas the housing market encompass a bigger class of participants which is consistent with a lower rationality parameter.

Another difference with Westerhoff [32] is the parameter a that links the demand for housing and its price. This parameter reflects the fact that to have a considerable change in house price the excess demand has to be high. The daily depreciation rate is set in a way to be consistent with the quarterly depreciation rate in [3]. The variance of the shock to the house price equation (1) σ_Q^2 will be calibrated so that the variance of our simulated quarterly house price series matches the variance of the real quarterly house price, collected by Federal Housing Finance Agency⁵. Finally we choose the parameter that relates the house supply to the perceived fundamental value to be $z = 1$.

3.1 Waves of Chartism and Fundamentalism

Following Lengnick and Wohltmann [28], we simulate a *representative* run for a period consisting in 40 quarters to show the action that the two types of expectations exert on the house price. We have two shocks in the model: the noise term on the house price and the preference for houses. In this part of the paper we focus on the first shock and thus we set \hat{j}_t fixed at zero. In this way we analyze only the response of the system to a repeated draw realization of the noise term ε_t^Q .

Figure 2 shows the dynamics of the relevant variables: the top left panel displays the quarterly house prices; the top right shows the evolution of housing stock. The daily house price along with the perceived fundamental value are exhibited in the middle of the plot whereas waves of chartism (green) and fundamentalism (yellow) are presented below. Finally the bottom left panel displays the exchanged house quantity and the bottom right panel exposes the optimal demand (the last variable is founded as a weighted sum of the daily chartists and fundamentalists demand).

⁵<http://www.fhfa.gov/>

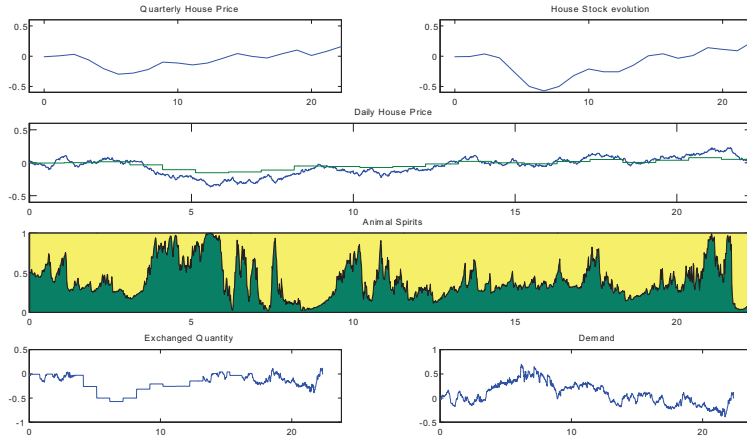


Figure 2: Representative run of the relevant variables

The two strategies dominate the market from time to time, but the continuous evaluation of past results and the endogenous competition among them assure that none dominates forever. It is evident that fundamentalists dominate for most of the time but in some particular periods the optimal strategy becomes chartism. When chartists prevail, the house price departs from its perceived fundamental value, in particular this movement is strong for $q = 20 - 22$ with a positive increment in price (a boom) or for $q = 5 - 7$. In the latter case chartists expect a negative trend and hence they create a burst. In phases dominated by fundamentalists, on the contrary, the house price tends to go back to its fundamental value, which is evident for $q = 8 - 10$. The quarterly house price is smoother than the daily one. Given (26), the supply follows the path of the quarterly house price and so does the evolution of existing stock of houses. Moreover, when the daily house price is lower (higher) than its fundamental value, the demand for housing services tends to increase (decrease) and the exchanged quantity of housing services is dominated by supply (demand). Note that being the exchanged quantity the minimum between demand and the existing housing stock, when the former is greater than the latter the time series for the exchanged quantity is more volatile. On the contrary, when supply prevails the time series is represented by a broken line because this variable changes only at the end of each quarter.

3.2 Impulse Response Functions

In this section we analyze how house price reacts to a positive exogenous preference shock via impulse response function. We try to isolate the impact of the preference shock in the following way:

1. Generate the model dynamic with $\varepsilon_t^j = 0 \forall t$
2. Generate the same dynamic with the identical realizations of ε_t^Q and with $\varepsilon_t^j = 1$
3. Calculate the difference between the trajectories of step 1 and 2 which gives the isolated effect of the preference shock.

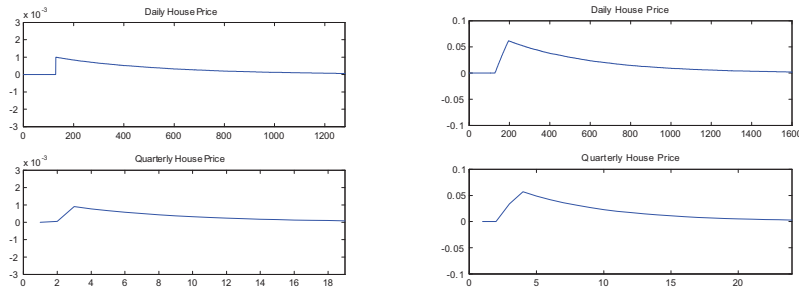


Figure 3: (a) Daily shock to house preferences and (b) Quarterly shock to house preferences

First, we look at the impact of a daily preference shock, setting $\varepsilon_{128}^j = 1$ for $t = 128$, and $\varepsilon_{128}^j = 0$ otherwise. Figure 3(a) shows the resulting responses of the daily and the quarterly house price. A daily shock on house preferences increases the daily house price on impact and the effect persists about 20 quarters although the size is very small. Looking at the quarterly variable the rise in price is not instantaneous, and this is due to the way we have defined this variable, which is an average of daily prices. This exercise reveals that the quarterly variable shows more persistence than the daily one: indeed, the impact of a positive preference shock in $t = 128$ (i.e. the last day of the second quarter) increases the quarterly price but the maximum level is reached in the next quarters. This process can be explained by the endogenous mechanism of the Agent-Based model, and in particular with the backward-looking expectations inside the model: the positive effect of the shock does not vanish immediately but it influences agents' behavior for more periods. While a daily shock seems not to have a particular economic meaning, this exercise is instructive about the propagation properties of our model.

The second experiment extends the length of the shock to a quarter, setting $\varepsilon_t^j = 1$ for $t = 128, \dots, 192$ and 0 otherwise. Three main differences emerge from Figure 3(b) with respect to the previous one:

- The daily variable increases but the maximum is reached at $t = 192$ when the shock vanishes.

- The size of the change in the price is much bigger, roughly from 0.001 to 0.05, both because the shock is more prolonged and because it is propagated by the backward-looking expectations hypothesized in the model.
- The movement of the variable toward the steady state is much more prolonged (it takes more than 20 quarters).

Also this second exercise shows the role of backward-looking expectations, because the persistence of the shock is amplified due to a learning mechanism.

4 Matching Real Data

The aim of this section consists in explaining what are the main driving forces acting on house price dynamics. Favilukis, Ludvigson and Van Nieuwerburgh [17] studies the link between house prices and interest rates or credit availability, showing that credit tightness can be a driving force of boom and bursts on house price, whereas the interest rate does not have such an influence. Other contributions, are able to match the data quite well by means of non-rational expectations (e.g., Adam, Kuang, Marcet, 2011).

In our model, we can consider three factors as possible driving forces of the recent boom and burst in the US housing market: the interest rate, credit tightening and the exogenous preference shock for housing. We first look at each of these factors in turn and then consider them all at once. In our model the changing in agents' believes can amplify these forces playing a significant role on the resulting housing dynamics.

Our attempt is to match quarterly house price for the period going from Q1-2004 to Q1-2009. We use the technique adopted in constructing the impulse response function, that is we look at the difference between the response of the system to the various driving forces and its value in Q1-2004. The data, Seasonally Adjusted Purchase-only Index, are taken from the Federal Housing Finance Agency and to make our approach consistent, we compute the percentage deviation of the real house price with respect to its value in Q1-2004.

First, we consider the path of the interest rate feeding into the model the percentage change in 30-Year Conventional Mortgage Rate. In so doing we define the steady state value of the interest rate as the daily transformation of the 30-Year Conventional Mortgage Rate in Q1-2004 and we look at the percentage deviation from it, as Figure 4 shows. The percentage deviation is quite high from 2004 to 2006 but it moves down in the following years, and it sharply increases in 2009. Noteworthy that the percentage change in this variable is small. We fit this series into the demand function to check how the price reacts. Results are shown in Figure 5: the reaction (red) is very small compared with real data (blue); moreover in the small box, where the fitted prices are shown more clearly, it is possible to note that the series is always increasing.

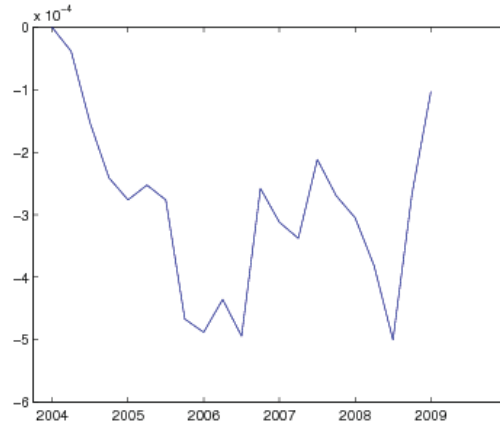


Figure 4: Percentage deviation of 30-Year Conventional Mortgage Rate

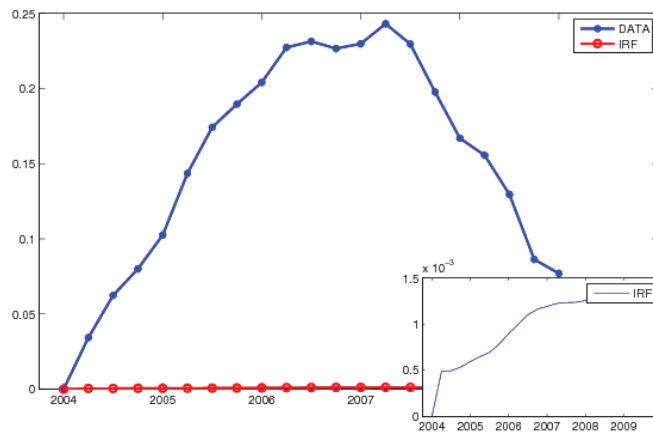


Figure 5: Model response to the interest rate path

Second, concerning the tightness of credit, we take the parameter θ in the borrowing constraint as a proxy for it. We calibrate this parameter initially to be equal to Iacoviello [24]: $\theta = 0.55$. Then we consider *The January 2012 Senior Loan Officer Opinion Survey on Bank Lending Practices*⁶, especially the Net percentage of banks reporting tightening credit standards for US. (see Figure 6). From 2004 to the third quarter of 2006 access to credit has remained stable, whereas from that date on credit tightness increased sharply.

⁶<http://www.federalreserve.gov/boarddocs/snloansurvey/201201/default.htm>

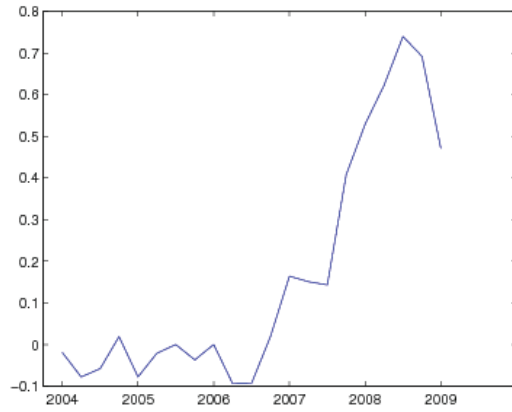


Figure 6: Net percentage of banks reporting tightening credit standard

We define θ from Q1-2004 to Q1-2009 as follows: $\theta = 0.55 - 0.55 * tight\ credit$, because the credit availability is an increasing function of θ . Results (see Figure 7) do not match the data because the size of change in fitted price is much smaller than real house price. The small box reproduces this dynamics which is very high up to the second quarter of 2004 and then continuously decreases. From Figure 7 it is evident that our model rejects this element as the driving force of house price dynamics.

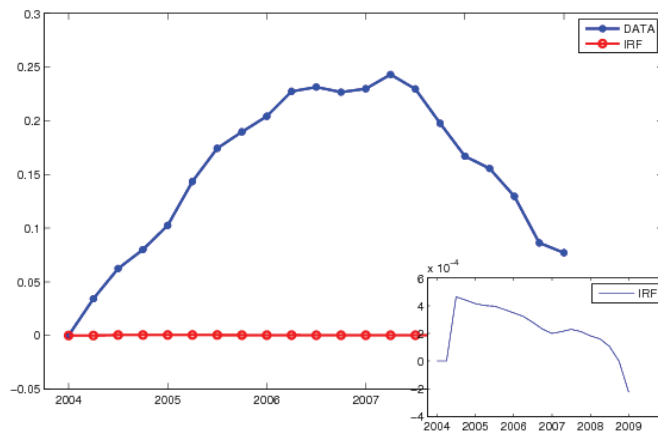


Figure 7: Model response to a credit tightening

Finally we consider the shock on house preferences \hat{j}_t . We look at the *Michigan Consumers Surveys*, more specifically at the quarterly table showing the Buying Condition for Houses as a proxy to calibrate the demand shock for houses. Figure 8 summarizes the answers at the following question according to the Survey: *Generally speaking, do you think now is a good time or a bad time to buy a house?* We focus on the percentage of positive answers, transforming the series in a way to have figures in the subset $(-1, 1)$. Then we fitted the normalized series into the model via the parameter \hat{j}_t .

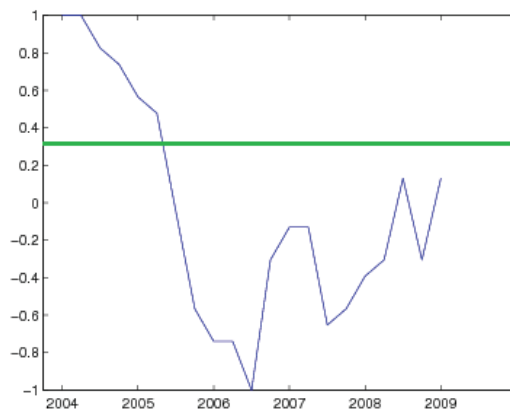


Figure 8: Preference shock

As we can see from Figure 8, values of the series higher than 0.3063 mean a positive preference shock, whereas we observe the contrary for lower values. In particular the positive trend goes from 2004 to the first half of 2006 and the negative trend starts in the second half of 2006 and it lasts until 2009.

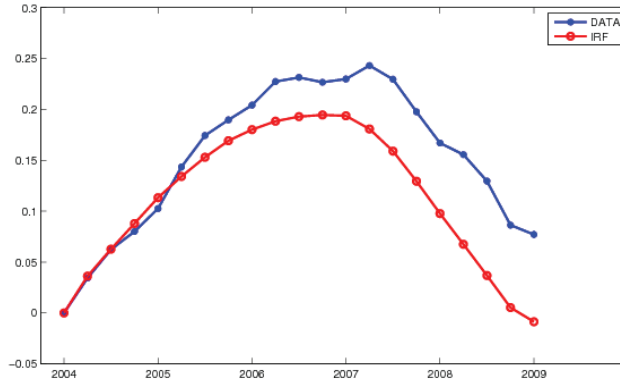


Figure 9: Model response to preference shock

Figure 9 shows the response of the system to an exogenous preference shock calibrated using the Michigan Consumers Surveys. Our model economy is able to replicate quite well the real price dynamics. The two series follow the same path during the first year, the maximum percentage deviation of house price is reached in 2007, nevertheless after this year the fitted series has a steeper decrease than the Seasonally Adjusted Purchase only Index. Our model seems capable of reproducing the hump-shaped behavior in the house price series, thanks to the endogenous inertia typically generated by boundedly rational model where agents do not fully understand the nature of the shock or its transmission mechanism, and hence they apply trial and error learning rules.

A crucial parameter for the result in Figure 9 is the rational parameter (intensity of choice) e , for which we have no clear calibration guidance. We calibrated e to minimize the distance between the real data and the model output. However notice that the results are quite robust as long as the parameter value increases.

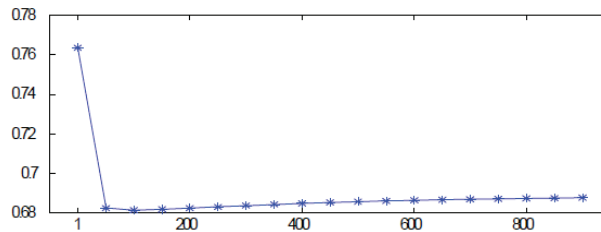


Figure 10: Sum of the absolute value of the difference between the data and the model

e	1	100	300	400	500	700	900
$\sum data - output $.7636	.6815	.6839	.6850	.6859	.6871	.6879

Tab 2: Robustness check of the parameter e

Figure 10 shows a robustness analysis, plotting the sum of the absolute value of the deviation between real data and the generated time series. This is minimized for $e = 100$ after a sharp reduction, but it then remains within a limited interval. Our results are thus quite robust to alternative calibration of the intensity of choice parameter, as also showed by Table 2.

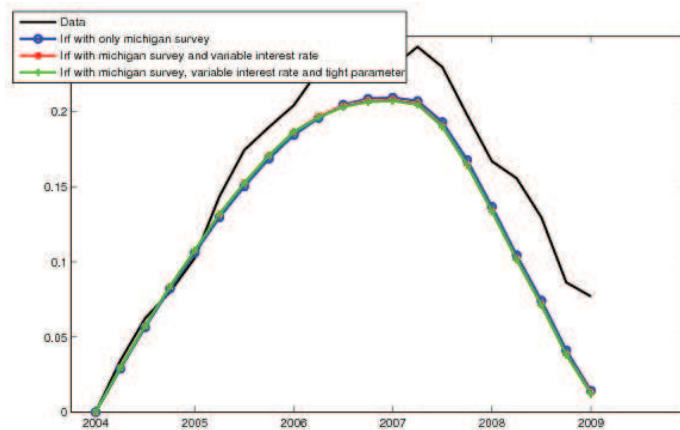


Figure 11: Model response to the three effects

The final exercise consists in taking together the three effects (see Figure 11): as the blue line shows, the most of the dynamics is generated by the preference shock. Adding the interest rate and the credit tightness effects have basically no effect on the price movement.

Our analysis emphasizes the importance of the behavioral approach and the selection mechanism among different expectation rules as determinant factors of the boom and bursts cycle in the housing market.

The model matches the data quite well, in particular it captures the moment of maximum percentage increase of the house price starting from psychological studies and empirical surveys. Our model thus supports the view that it is important to incorporate behavioral features in macroeconomic models, though this is still rather simple in our model. On the contrary, a perfect rational agent would have anticipated and discounted any future movement in price.⁷

⁷For recent critiques to rational expectation hypothesis see [5], [12] and [16]

5 Policy suggestions

In this section we try to give an answer to the question: "Could the boom in house price have been avoided if the interest rate had been increased?" The question arises for a simple reason: the growth in U.S. house price coincides with a fall in the ex-ante real interest rate. Moreover, the model in Adam, Kuang and Marcet [3] predicts that the recent house price dynamics would have been avoided and the current account deficit would have been considerably smaller, if the interest rate had fallen by less at the beginning of the 2000's⁸.

To answer the question we let the real interest rate to react to the house price. We then use this simple rule to minimize two different measures that capture the fluctuations of the house price: the volatility and the distortion of the house price, as in Lengnick and Wohltmann [28].

The distortion measures the difference between the variable and its true steady state (implicitly set to zero):

$$dis(Q) = \frac{1}{T} \sum_{t=1}^T |\hat{Q}_t|. \quad (33)$$

The volatility represents the rate of change in the value of the simulated time series:

$$vol(Q) = \frac{1}{T-1} \sum_{t=2}^T |\hat{Q}_{t-1} - \hat{Q}_t|. \quad (34)$$

$dis(Q)$ measures the distortion of the time series, that is the mean of the deviation of the house price from its steady state. We do not use the standard deviation because it considers the distortion as the dispersion of the time series from its mean, while the mean of \hat{Q}_t is not the steady state.

For this exercise, we adopt two very simple policy rules. The first links the real interest rate to the quarterly house price:

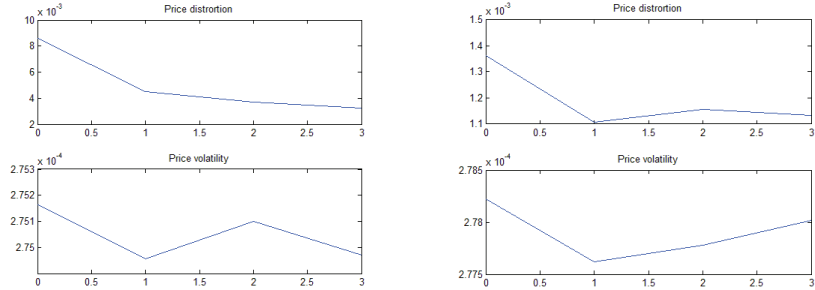
$$\hat{R}_q = r^q \hat{Q}_q. \quad (35)$$

The second rule modifies the target making the interest rate to respond to the difference in house price between two subsequent periods:

$$\hat{R}_q = r^q (\hat{Q}_q - \hat{Q}_{q-1}). \quad (36)$$

We then set the preference shock j_t equal to zero and perform Monte Carlo simulations using 1000 different realizations of the pseudo random number generator (ε_q^Q) for each r^q and that taking the mean.

⁸See also Himmelberg et al. [23]



(a) Interest rate rule (35)

(b) Interest rate rule (36)

Figure 12: (a) House price distortion and volatility with the interest rate reacting at house price and (b) House price distortion and volatility with the interest rate reacting at difference in house price

Figure 12(a) upper panel summarizes the results on price distortion using (35): on the x-axis we have the value of the r^q parameter and on the y-axis the value of the distortion.

We can see that the distortion is minimized for $r^q = 3$ meaning that the interest rate has to react three times a percentage point deviation of the house price. Other experiments, implying higher values of r^q , show that the distortion is minimized for the maximum value of this parameter and this means a negative correlation between the distortion and the interest rate. Our exercise shows also that most of the distortion is removed by only taking into account a one-to-one reaction of the interest rate to a movement in the house price. Figure 12(a) upper, in fact, shows a broken line with the highest slope in the segment between zero and one meaning that a one-to-one response of the interest rate is sufficient to have a huge decrease in price distortion.

Figure 12(a) lower panel shows the change in the volatility of prices to different values of the reaction parameter in the interest rate rule (35). The values representing the volatility on the y-axis are lower than the ones of the distortion, because the first gives the discrepancy between two subsequent house prices whereas the second depicts the distance between the house price and its steady state.

The result is quite difficult to read: again we have a broken line but composed by three different segments with different slopes. The volatility decreases in the segment between zero and one reaches its minimum exactly in one then increases until two and then decreases again until three. The minimum is reached for $r^q = 1$ meaning that, taking into account a one-to-one response of the interest rate to a change in price, the government should be able to minimize at the same time volatility and cancel out most of the distortion in house price.

Figure 12(b) shows the results about distortion and volatility using (36). Now the volatility measure is minimized for $r^q = 1$ and the distortion exhibits the same behavior of the volatility in Figure 12(a). Noteworthy that the distortion in Figure 12(b) is lower than the one in Figure 12(a) whereas the volatility

results to be approximately of the same size. Using the interest rate to respond one-to-one to the difference in house price the government could minimize both volatility and distortion on house price.

Although we know the limits of our results a policy suggestion emerges clearly from this simple experiments. Taking into account the possibility to use the interest rate to influence the house prices, the government could avoid some dangerous movements at the heart of booms and bursts.

6 Conclusion

We have developed a model to study the housing market starting from an Agent-Based perspective. Relaxing the rational expectation hypothesis and allowing households to have a backward-looking behavior, the model endogenously generates long-last deviation of house price from its fundamental steady state, that we interpret as boom and burst. The chartist-fundamentalist mechanism matches real data quite well. The model simulations point to the exogenous preference shock, calibrated using the Michigan Consumers Surveys, as the main driving force behind the recent boom and burst in house prices in the US. On the contrary, the interest rate and the credit tightness do not contribute much to the house price dynamics.

The heterogeneous framework gives the right persistence in the house price dynamics. The self fulfilled mechanism induced by the backward looking expectation amplifies and prolongs the impulses of shocks producing an hump shape response of the house price series.

In contrast, our behavioral model is able to reproduce the inertia in the price time series starting from psychological factors without imposing transmission lags.

The model also provides a clear policy suggestion. It shows that anchoring the interest rate to house price could reduce the distortion and the volatility of house price.

We know this model is still rather simple in incorporating a really psychological foundation of expectations but the mechanism of chartism and fundamentalism is sufficient to create endogenous movement in house price due to the different sizes of this groups and this simple interactive dynamics has a huge influence on the economic system.

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