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**Macroeconomic Stability and Heterogeneous
Expectations**

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Macroeconomic Stability and Heterogeneous Expectations *

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Abstract

The late 2000s financial crisis resulted in the collapse of large financial institutions, in the bailout of banks by national governments and downturns in stock markets around the world. Such a large set of outcomes put classical economic thinking under huge pressure. The 2007 crisis made many policy makers in a state of "shocked disbelief", as Alan Greenspan declared. Furthermore the recent macroeconomic literature have been stressing the role of heterogeneous expectations in the formulation of monetary policy and recent laboratory experiments provided more evidence about this phenomenon.

We use a simple model made by the standard aggregate demand function, the New Keynesian Phillips curve and a Taylor rule to deal with different issues, such as the stabilizing effect of different monetary policies in a system populated by heterogeneous agents. The response of the system depends on the ecology of forecasting rules, on agents sensitivity in evaluating the past performances of the predictors and on the reaction to inflation. In particular we investigate whether the policy makers can sharpen macroeconomic stability in the presence of heterogeneous expectations about future inflation and output gap and how this framework is able to reduce volatility and distortion in the whole system.

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1 Introduction

From the sixties onwards, *rational expectations* (RE) emerged as the dominant paradigm in economics. Nowadays RE are the main mathematical formulation of the agents' expectations. During the last decades indeed, after the works of Muth [25] and Lucas [24], the RE hypothesis has been widely applied in all the different field of economics and finance modeling, becoming the leading and more appealing paradigm. RE is a collection of assumptions regarding the manner in which economic agents exploit available information to form their expectations. In its stronger forms, RE operates as a coordination device that permits the construction of a representative agent having representative expectations.

Under the weak-form definition of RE, the concept of RE essentially reduces to an assumption that agents make optimal use of whatever information they have to form their expectations. Weak-form RE is in accordance with John Taylor's idea of an economically rational expectation in which agents' information sets are the result of cost-benefit calculations by the agents regarding how much information to obtain. On the other hand, one can also define a stronger form of RE in the sense of John Muth (1961) that places a strong restriction on the content of an agent's information sets. This definition of RE guarantees the existence of objectively true conditional expectations. This approach assumes that humans have perfect access to all information and adapt instantly and rationally to new situations, maximizing their long-run personal advantage.

Even if the RE hypothesis have the advantage of being more easily tractable, today it seems quite unrealistic to assume that agents have perfect knowledge of the whole economic system; moreover, as emphasized also by Sargent in [26], rational expectations imply not only that individuals are perfectly aware of the mechanisms moving the economy, but also that they are able to solve all the computational problems which arise in the model.

Of course real people often act on the basis of overconfidence, fear and peer pressure — topics that behavioral economics is now addressing. As stated by Hommes in [15], the characteristic of an economic system is the fact that it is an expectations feedback system, therefore expectations play a central role in all the modern macroeconomic theory. But there is a still larger problem. Even if rational expectations could be a reasonable model of human behavior, the mathematical machinery is cumbersome and requires drastic simplifications to get tractable results. The equilibrium models that were developed, by assumption, do not consider most of the structure of a real economy because this implies too much nonlinearity and complexity for equilibrium methods to be easily tractable.

The late 2000s financial crisis - considered by many economists to be the worst financial crisis since the Great Depression - resulted in the collapse of large financial institutions, the bailout of banks by national governments and downturns in stock

markets around the world. Such a large set of outcomes put classical economic thinking under huge pressure.

The notions that markets are efficient, rational agents quickly correct any mispricing or forecasting error and the idea that prices carefully reflect the underlying reality and ensure optimal allocation of resources, leading to *equilibrated* markets where crises can only be triggered by acute exogenous disturbances, are in stark contrast with most financial crashes, including the latest one. In fact the 2007 crisis made many policy-makers in a *state of shocked disbelief*, as Alan Greenspan himself declared.

An alternative approach could be modeling economy as a bottom-up system in which no individual understands the whole framework but only a very small part of the whole (see [11]). These systems work as a result of the application of simple rules by the individuals populating the system. In a model where agents are assumed to be fully rational, and where full rationality is common knowledge, no one has an incentive in getting involved in a trade operation, since both of them have the same expectation about who is going to gain and who is going to lose from the exchange. As suggested by Hommes in [16], the tremendous volume of trading operations that can be observed every day in all the real markets, reinforces the idea of heterogeneous expectations (HE) and the idea that differences of opinions among market participants are necessary for trade to occur.

In this work we study the stability of a macroeconomic model in which agents have heterogeneous expectations. We'd like to ask to the following questions: How many stable or unstable equilibria emerge if there are agents predicting future variables value using different forecasting rules? How determinacy conditions change in a framework with heterogeneous expectations? How monetary policy should be designed in order to guide the system to a stable equilibrium?

We address this questions using the following three equations system composed by the IS curve, a New Keynesian Phillips curve and a Taylor rule, as in [9]. According to the benchmark model of Branch and McGough (see [6], [7]), our setting has the same functional form as the standard formulation except for the homogeneous expectation hypothesis which is replaced with a combination of heterogeneous expectations. As a consequence, the dynamic properties of the model depend crucially on the distribution of agents. Generally most of the models introducing HE consider individuals with too many cognitive skills: they do not fully understand the underlying model due to informational inertia. For this reasons we consider a parsimonious model with simple heuristics which is able to generate endogenous waves of optimism and pessimism (animal spirits); moreover the analysis of monetary policy is conducted to investigate the role of inflation and output gap in business cycle movements.

Heterogeneous agent models mimic important observed stylized facts in asset

returns, such as fat tails, clustered volatility and long memory, as discussed e.g. in the extensive surveys of LeBaron in [22] and Hommes in [17].

Some recent examples of macro and financial models with heterogeneous expectations include Evans and Honkapohja as in [12], [13], Bullard and Mitra in [8], Hommes in [18], [19], Ascari et al. in [2].

More recently Hommes in [20], Assenza et al. in [3] and some others studies, provided evidence in favor of heterogeneous expectations using laboratory experiments with human subjects.

However, the question how to manage expectations when forecasting rules are heterogeneous has hardly been addressed. This is a model in which agents have cognitive limitations and do not understand the whole picture (the underlying model). Instead they only understand small bits of the whole model and use simple rules to guide their behavior. We introduce rationality in the model through a selection mechanism in which agents evaluate the performance of the forecasting they are following and decide to change their strategy depending on how well it performs relative to other ones. In our stylized model agents form expectations about the future rate of inflation and output using different forecasting principles. We employ the heterogeneous expectations framework of Brock and Hommes [4], where the ecology of forecasting rules is disciplined by endogenous, evolutionary selection of strategies with agents switching between forecasting rules on the basis of their past performance.

2 The model economy

The model is made up of a standard aggregate demand and supply, augmented with a Taylor rule. Heterogeneity is introduced because agents use different rules (heuristics) to forecast the future values of economic variables; moreover these rules are subjected to a learning mechanism which is able to create endogenous business cycle.

The aggregate demand is presented as

$$y_t = a_1 \tilde{E}_t y_{t+1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} \right) \quad (1)$$

where y_t and y_{t+1} are respectively the output gap in period t and $t + 1$, i_t is the nominal interest at t rate, π_{t+1} is the rate of inflation at $t + 1$. \tilde{E}_t is the expectation operator where the symbol \sim refers to expectations that are not formed in rational way.

The aggregate supply can be interpreted as a New Keynesian Phillips curve:

$$\pi_t = f_1 \tilde{E}_t \pi_{t+1} + f_2 y_t \quad (2)$$

here we present the log-linearized equation that can be derived from firms' profit maximization under sticky price assumption, inflation at time t is increasing in both output at the current period and in expected inflation.

Finally the Taylor rule is given by

$$i_t = c_1 \pi_t + c_2 y_t \quad (3)$$

we use a contemporaneous Taylor rule as a simplifying assumption even if we are aware about the critiques in that direction. We set the baseline calibration of the model according to [9] and table 1 summarizes the parameter values.

$a_1 = 1$	$a_2 = -1$
$f_1 = 0.99$	$f_2 = 0.3$
$c_1 = 1.5$	$c_2 = 0.5$

Table 1: parameters calibration

Combining (1) and (3) we have

$$y_t = a_1 \tilde{E}_t y_{t+1} + a_2 c_1 \pi_t + a_2 c_2 y_t - a_2 \tilde{E}_t \pi_{t+1}$$

and substituting (2) we get

$$y_t = a_1 \tilde{E}_t y_{t+1} + a_2 c_1 f_1 \tilde{E}_t \pi_{t+1} + a_2 c_1 f_2 y_t + a_2 c_2 y_t - a_2 \tilde{E}_t \pi_{t+1}$$

Solving for y_t

$$y_t = \frac{a_1}{(1 - a_2 c_1 f_2 - a_2 c_2)} \tilde{E}_t y_{t+1} + \frac{a_2 (c_1 f_1 - 1)}{(1 - a_2 c_1 f_2 - a_2 c_2)} \tilde{E}_t \pi_{t+1} \quad (4)$$

Then we plug (4) into (2) and after some algebra we obtain

$$\pi_t = \frac{a_1 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \tilde{E}_t y_{t+1} + \frac{f_1 (1 - a_2 c_2) - f_2 a_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \tilde{E}_t \pi_{t+1} \quad (5)$$

We want to set a map $L_\gamma(y_t, \pi_t)$

$$\begin{aligned} y_t &= g(y_{t-1}; \pi_{t-1}) \\ \pi_t &= h(y_{t-1}; \pi_{t-1}) \end{aligned} \quad (6)$$

To do so, first of all we introduce heterogeneous backward-looking expectations.

2.1 Expectations

In this part we will present the Adaptive Belief System by Brock and Hommes [4] that allows to model expectations by introducing heterogeneity among agents.

As already stressed by Simon in 1955, people have both knowledge and calculus ability that are limited, and if they want to pursue more optimal decision rules, they must bear some search costs; due to these two limitations, agents are endowed only with a bounded rationality and as a consequence, they use simple heuristics when they face a decision that entails some degree of uncertainty.

Assume that agents can form expectations choosing from H different forecasting rules. We denote with $\hat{E}_{h,t}y_{t+1}$ and $\hat{E}_{h,t}\pi_{t+1}$ the forecasts of output and inflation by h -th rule. Moreover, each rule for output and for inflation prediction can be chosen by (eventually) different number of individuals: therefore the fraction of agents using forecasting rule h to forecast output at time t is denoted by $w_{h,t}$ and the one for inflation $z_{h,t}$.

The fractions are updated according to an *evolutionary fitness measure*. The fitness measures, for output and inflation respectively, are publicly available but subject to noises and expressed in utility terms as:

$$\tilde{U}_{h,t} = U_{h,t} + \varepsilon_{h,i,t} \quad (7)$$

$$\tilde{V}_{h,t} = V_{h,t} + \varepsilon_{h,i,t} \quad (8)$$

where $U_{h,t}$ and $V_{h,t}$ are the deterministic parts (i.e. assume they can be read in a freely available newspaper) and $\varepsilon_{h,i,t}$ is the stochastic component of the fitness measures. More precisely it collects independent and identically distributed noises, idiosyncratic across each time t , each strategy $h = 1, 2, ..H$ and each agent i .

A theorem on discrete choice states that, assuming the noises $\varepsilon_{h,i,t}$ are drawn from a double exponential distribution, as the number of agents i goes to infinity, the probability that an agent picks the strategy h is given by the discrete choice fractions. The fraction of agents choosing strategy h is given by the well known discrete choice model:

$$w_{h,t} = \frac{e^{\gamma U_{h,t-1}}}{\sum_{h=1}^H e^{\gamma U_{h,t-1}}} \quad (9)$$

$$z_{h,t} = \frac{e^{\gamma V_{h,t-1}}}{\sum_{h=1}^H e^{\gamma V_{h,t-1}}} \quad (10)$$

where γ is the intensity of choice parameter and reflects the sensitivity of agents in selecting the optimal strategy.

The functions $U(\cdot)$, $V(\cdot)$ are the past squared forecast errors, as in [1].

$$U_{h,t-1} = - \left(y_{t-1} - \hat{E}_{h,t-2} y_{t-1} \right)^2 - C_h \quad (11)$$

$$V_{h,t-1} = - \left(\pi_{t-1} - \hat{E}_{h,t-2} \pi_{t-1} \right)^2 - C_h \quad (12)$$

where C_h is the information cost of predictor h . The stochastic process $\varepsilon_{h,i,t}$ is reflected into the parameter $\gamma \in [0; \infty)$ and it is inversely related to the standard deviation of σ_ε . The case $\gamma = 0$ implies $\sigma_\varepsilon = 1$, meaning that differences across the h fitness measures $\tilde{U}_{h,t}$ and $\tilde{V}_{h,t}$ cannot be observed, therefore each agent do not switch between strategies and the fractions are all constant and equal to $1/H$. Oppositely, $\gamma = \infty$ implies $\sigma_\varepsilon = 0$ and this is the case where no noise is present, the fitness measure for each strategy is perfectly observable and in each time all agents pick the forecasting rule with the higher performance in the previous period. We will see in fact that the agents have the possibility to choose among different predictors both for the output gap and for the inflation, meaning that individuals broadly know the fundamental steady state. Since they are boundedly rational, the fundamental steady state predictor for output gap and inflation requires some efforts or some information gathering costs¹ $C_h > 0$.

3 Evolutionary model with constant belief types

We consider a scenario in which agents can choose among $H+1$ ($H = 0, 1, 2, \dots, h$), H different symmetric forecasting rules, where the positive and negative biases are exactly balanced around the REE. This choice implies that the REE is among the steady states of the model. Notice however that this hypothesis is not essential for most of quantitative results to hold. The rational predictor is the only available at cost $c > 0$ whereas the others are freely observable.

The rational predictor is

$$\hat{E}_{0,t} y_{t+1} = \hat{E}_{0,t} \pi_{t+1} = 0 \quad (13)$$

¹It is here necessary to state clearly that a fundamentalist agent is different from the rational one (except in one particular case): as a matter of fact the former does not know that in the economy some disturbing, heterogeneous agents are present and so, he predicts the equilibrium thinking that all the agents are as he is; differently the "true" rational agent, must be aware of the presence of all the disturbing agents and also of their biased predictions, taking them into account while predicting the equilibrium

The other constant belief types are

$$\begin{array}{ll}
\hat{E}_{1,t}y_{t+1} = b & \hat{E}_{1,t}\pi_{t+1} = d \\
\hat{E}_{2,t}y_{t+1} = b/2 & \hat{E}_{2,t}\pi_{t+1} = d/2 \\
\vdots & \vdots \\
\hat{E}_{h/2,t}y_{t+1} = 2b/H & \hat{E}_{h/2,t}\pi_{t+1} = 2d/H \\
\hat{E}_{h/2+1,t}y_{t+1} = -2b/H & \hat{E}_{h/2+1,t}\pi_{t+1} = -2d/H \\
\vdots & \vdots \\
\hat{E}_{h-1,t}y_{t+1} = -b/2 & \hat{E}_{h-1,t}\pi_{t+1} = -d/2 \\
\hat{E}_{h,t}y_{t+1} = -b & \hat{E}_{h,t}\pi_{t+1} = -d
\end{array}$$

The market forecast is obtained as a weighted average of the H predictors:

$$\tilde{E}_t y_{t+1} = \sum_{h=0}^H w_{h,t} \hat{E}_{h,t} y_{t+1} \quad (14)$$

$$\tilde{E}_t \pi_{t+1} = \sum_{h=0}^H z_{h,t} \hat{E}_{h,t} \pi_{t+1} \quad (15)$$

Substituting (9), (11) in (14) and (10), (12) in (15), and taking also into account that the rational agents have predictor (13), we obtain

$$\tilde{E}_t y_{t+1} = \frac{\sum_{h=1}^H b_h \exp(-\gamma (y_{t-1} - b_h)^2)}{\sum_{h=1}^H \exp(-\gamma (y_{t-1} - b_h)^2) + \exp(-\gamma (y_{t-1})^2 + C)} \quad (16)$$

$$\tilde{E}_t \pi_{t+1} = \frac{\sum_{h=1}^H d_h \exp(-\gamma (\pi_{t-1} - d_h)^2)}{\sum_{h=1}^H \exp(-\gamma (\pi_{t-1} - d_h)^2) + \exp(-\gamma (\pi_{t-1})^2 + C)} \quad (17)$$

Now by inserting (16) and (17) into (4) and (5), we found the 2-D system we want to analyze

$$\begin{aligned}
y_t = & \frac{a_1}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H b_h \exp(-\gamma (y_{t-1} - b_h)^2)}{\sum_{h=1}^H \exp(-\gamma (y_{t-1} - b_h)^2) + \exp(-\gamma (y_{t-1})^2 + C)} + \\
& + \frac{a_2 (c_1 f_1 - 1)}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H d_h \exp(-\gamma (\pi_{t-1} - d_h)^2)}{\sum_{h=1}^H \exp(-\gamma (\pi_{t-1} - d_h)^2) + \exp(-\gamma (\pi_{t-1})^2 + C)}
\end{aligned} \quad (18)$$

$$\pi_t = \frac{a_1 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H b_h \exp(-\gamma (y_{t-1} - b_h)^2)}{\sum_{h=1}^H \exp(-\gamma (y_{t-1} - b_h)^2) + \exp(-\gamma (y_{t-1})^2 + C)} + \quad (19)$$

$$+ \frac{f_1 (1 - a_2 c_2) - f_2 a_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H d_h \exp(-\gamma (\pi_{t-1} - d_h)^2)}{\sum_{h=1}^H \exp(-\gamma (\pi_{t-1} - d_h)^2) + \exp(-\gamma (\pi_{t-1})^2 + C)}$$

4 Model analysis

In this section we perform an analysis of the model we have previously showed by computing the equilibria and the stability conditions of the fundamental steady state. First of all we consider the simplest scenario in which agents can choose among three different forecasting rules:

$$\begin{aligned} \hat{E}_{1,t} y_{t+1} &= b & \hat{E}_{1,t} \pi_{t+1} &= d \\ \hat{E}_{2,t} y_{t+1} &= -b & \hat{E}_{2,t} \pi_{t+1} &= -d \\ \hat{E}_{3,t} y_{t+1} &= 0 & \hat{E}_{3,t} \pi_{t+1} &= 0 \end{aligned}$$

with bias parameters $b = 1$ and $d = 1$, meaning that type 1 agents expect that inflation and output will be above its fundamental level whereas type 2 agents expect an inflation and output level lower than the fundamental value. Type 3 agents believe that output and inflation rate will be always at its RE equilibrium. Assuming these conditions, the map described in (18)-(19) always have a steady state $(y^*, \pi^*) = (0, 0)$, which is the RE equilibrium. This fundamental steady state can be locally stable or even unstable. Indeed in some case the dynamics can converge to other stable steady states.

We will provide an analysis of the dynamics which depends on parameters γ , c and the Taylor coefficient c_1 . First of all we compute the Jacobian matrix of the system at the RE equilibrium:

$$J = \begin{pmatrix} \frac{4 a_1 \gamma e^{-\gamma}}{(-a_2 c_1 f_2 - a_2 c_2 + 1) (2 e^{-\gamma} + e^c)} & \frac{4 a_1 f_1 \gamma e^{-\gamma}}{(-a_2 c_1 f_2 - a_2 c_2 + 1) (2 e^{-\gamma} + e^c)} \\ \frac{4 a_1 f_2 \gamma e^{-\gamma}}{(-a_2 c_1 f_2 - a_2 c_2 + 1) (2 e^{-\gamma} + e^c)} & \frac{4 \left(\frac{a_1 f_1 f_2}{-a_2 c_1 f_2 - a_2 c_2 + 1} + f_1 \right) \gamma e^{-\gamma}}{2 e^{-\gamma} + e^c} \end{pmatrix}$$

The trace and the determinant of this matrix are respectively given by

$$Tr(J) = \frac{4 \left(\frac{a_1 f_1 f_2}{-a_2 c_1 f_2 - a_2 c_2 + 1} + f_1 \right) \gamma e^{-\gamma}}{2 e^{-\gamma} + e^c} + \frac{4 a_1 \gamma e^{-\gamma}}{(-a_2 c_1 f_2 - a_2 c_2 + 1) (2 e^{-\gamma} + e^c)}$$

$$Det(J) = - \frac{16 a_1 f_1 \gamma^2}{(a_2 c_1 f_2 + a_2 c_2 - 1) (e^{\gamma+c} + 2)^2}$$

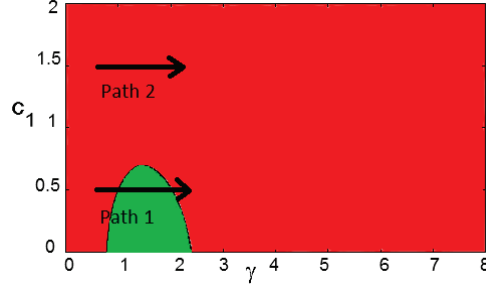


Figure 1: Stability region in the (γ, c_1) plane

The stability region of the fundamental steady state is determined by the following conditions:

$$\begin{cases} 1 - Tr(J) + Det(J) > 0 \\ 1 + Tr(J) + Det(J) > 0 \\ Det < 1 \end{cases} \quad (20)$$

In our setting the only bifurcation of the fundamental steady state that can appear is the pitchfork, whose curve is given by $1 - Tr(J) + Det(J) = 0$. We computed this curve letting the inflation coefficient and the intensity of choice vary, and keeping fixed the other parameters at the baseline calibration. Figure (1) shows the stability region in the parameter space (γ, c_1) : the RE equilibrium is locally stable for any (γ, c_1) that lie in the red region.

Moreover the fundamental steady state can also lose stability via flip bifurcation but for parameter values that have no economic meaning in our setting. Finally we are also able to show that the Neimark-Sacker bifurcation can not occur because the eigenvalues of the Jacobian matrix at the RE equilibrium are always real. Indeed let us consider the characteristic polynomial of the Jacobian matrix, which is given by

$$\lambda^2 - Tr(J)\lambda + Det(J)$$

In order to have complex eigenvalues the inequality

$$\Delta = Tr(J)^2 - 4Det(J) < 0$$

must be satisfied.

In our case the previous inequality becomes

$$\frac{16\gamma^2 e^{-2\gamma}}{(2e^{-\gamma} + e^c)} \left[\left(\frac{a_1 f_1 f_2 + a_1}{1 - a_2 c_1 f_2 - a_2 c_2} + f_1 \right)^2 - \frac{4a_1 f_1}{1 - a_2 c_1 f_2 - a_2 c_2} \right]$$

The first factor of the previous expression is always positive; the second factor, by some algebra, can be reduce to

$$\begin{aligned} & \left(\frac{a_1 f_1 f_2 - a_1}{1 - a_2 c_1 f_2 - a_2 c_2} + f_1 \right)^2 + \frac{(a_1 f_1 f_2 + a_1)^2}{1 - a_2 c_1 f_2 - a_2 c_2} - \frac{(a_1 f_1 f_2 - a_1)^2}{1 - a_2 c_1 f_2 - a_2 c_2} = \\ = & \left(\frac{a_1 f_1 f_2 - a_1}{1 - a_2 c_1 f_2 - a_2 c_2} + f_1 \right)^2 + \frac{2a_1^2 f_1 f_2}{1 - a_2 c_1 f_2 - a_2 c_2} \end{aligned}$$

Since the previous expression is always positive because of the values the parameters can assume, the Jacobian matrix at RE equilibrium has not complex eigenvalues and there are not situations in which a Neimark-Sacker bifurcation occurs².

Now we will focus on the existence of other equilibria for the map described by (18)-(19). Recall the steady states of $L_\gamma(y^*, \pi^*)$ are determined by setting

$$\begin{aligned} y^* &= g(y^*; \pi^*) \\ \pi^* &= h(y^*; \pi^*) \end{aligned}$$

From (19) we can obtain the expression for y^* , given by

$$y^* = \frac{\pi - \frac{f_1 (d_2 e^{-(\pi-d_2)^2 \gamma} + d_1 e^{-(\pi-d_1)^2 \gamma})}{e^{c-\pi^2 \gamma} + e^{-(\pi-d_2)^2 \gamma} + e^{-(\pi-d_1)^2 \gamma}}}{f_2}$$

Then by substituting the latter expression into (18) we are able to get a function $G(\pi) = 0$ which allow us to compute all the steady states of the system. Since it is very tough to find a closed expression for all the existing steady state that arise as the value of γ and c_1, c_2, C increase, we use numerical simulation to find the roots of $G(\pi)$ and, accordingly, y^* . Once we have computed the inflation equilibria values, it is possible to calculate also the corresponding output values, by plugging the latter into (19). We will consider only the case of low cost, when $C < b^2 + d^2$, which is consistent with the hypothesis of a freely available equilibrium predictor, even if we are aware that a high cost case exists, where $C \geq b^2 + d^2$. This is a less restrictive hypothesis on the agents' rationality, but this assumption does not preclude the existence of interesting dynamic properties.

Assuming $C = 0$ and a small reaction coefficient to inflation, i.e. $c_1 = 0.5$ (weak monetary policy), in Figure 2 we show the function $G(\pi)$ for small (green), medium (red) and high (blue) values of the intensity of choice γ . The intersections between

²The eigenvalues of J at the RE equilibrium (and with the parameter set at the baseline calibrations) are respectively $\lambda_1 = 7.34 \cdot 10^{-4}$ and $\lambda_2 = 0.00218$. Thus, considering a Taylor coefficient greater than 1, the RE equilibrium is locally stable, either with a small intensity of choice γ or with a bigger one. The previous computations has been obtained setting $\gamma = 10$ and $c_1 = 1.5$.

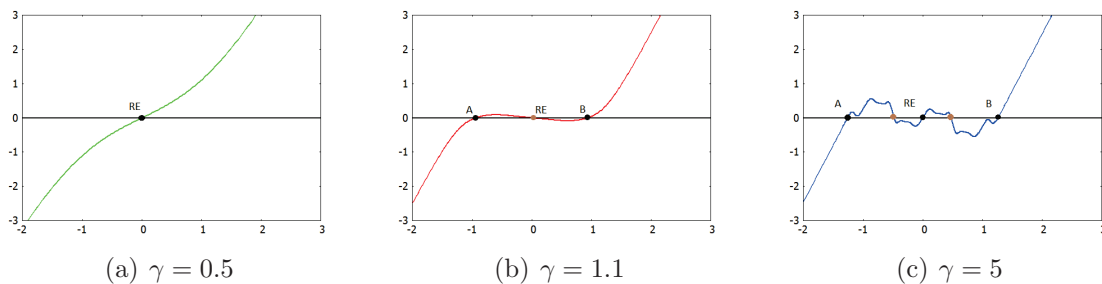


Figure 2: Weak monetary policy scenario: $G(\pi)$ in the $(\pi, G(\pi))$ plane

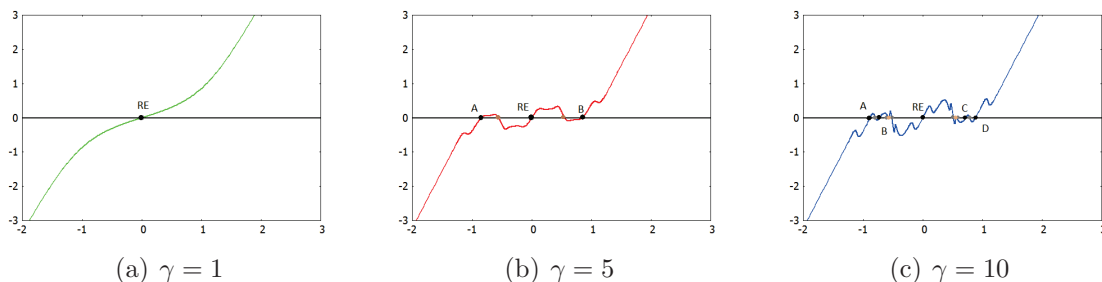


Figure 3: Moderate monetary policy scenario: $G(\pi)$ in the $(\pi, G(\pi))$ plane

the curve and the x-axis in the plane $(\pi, G(\pi))$ represent the inflation equilibrium values. When the intensity of choice is relatively low ($\gamma = 0.5$, left box), there exists only one steady state, the RE equilibrium, and this is due to the fact that agents for low γ values are roughly equally distributed over the different forecasting rule. In this case the realized inflation remains close to the fundamental steady state. As the intensity of choice increases ($\gamma = 1.1$, central box), two new steady states are created along with the RE equilibrium, which becomes a saddle. Moreover, as γ further increases ($\gamma = 5$, right box), the RE equilibrium coexists with two steady states and two saddles, as figure (2) shows. In this picture we have represented the $G(\pi)$ for small (green), medium (red) and high (blue) values of the intensity of choice γ in the weak monetary policy scenario. The distinction among fixed points and saddles will be better explained when we analyze the basins of attraction of the equilibria.

We perform the same exercise assuming $c_1 = 1.5$ (moderate monetary policy) and zero cost for the fundamental predictor. As we can see from Figure 3, when the intensity of choice is relatively small ($\gamma = 1$, left box), we have a unique steady state, which is the RE equilibrium $(y^*, \pi^*) = (0, 0)$. When the intensity of choice increases, four additional steady states appears ($\gamma = 5$, central box) along with the fundamental one. The difference between this scenario and the previous is that now the RE equilibrium is always locally stable. Finally, when γ is high ($\gamma = 10$,

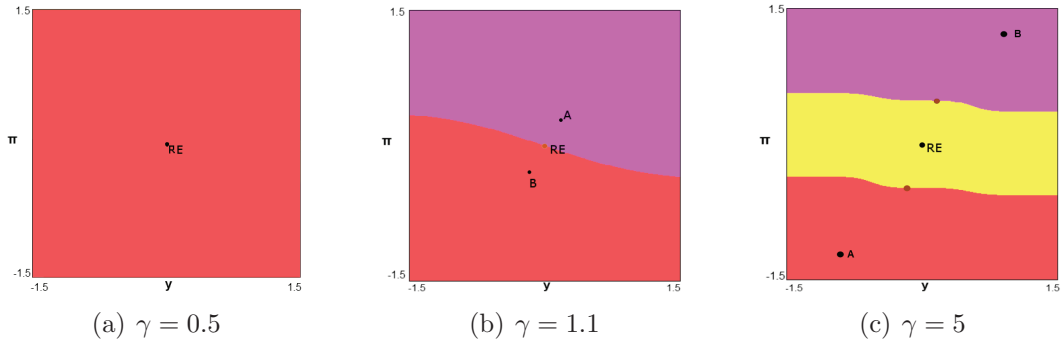


Figure 4: Basins of attraction in the low information costs scenario and weak monetary policy

right box), twelve steady states coexists with the RE equilibrium. It is worth to point out that the values for which $G(\pi) = 0$ are not all stable steady states but there are also saddles and unstable nodes. The analysis of the basins of attraction will explain better this eventuality.

In what follows we provide an analysis of the global dynamics of (18)-(19) and we show how these dynamics depend on parameters γ and c_1 .

Let us consider a first scenario (we move along *Path 1* in Figure 1) with low information costs ($C = 0$) and a weak monetary policy ($c_1 = 0.5$). Then there exist values $0 < \gamma_1^* < \gamma_2^*$ such that:

- for $\gamma < \gamma_1^*$ there exist only one steady state, the RE equilibrium, which is unique and globally stable;
- for $\gamma = \gamma_1^* \approx 1.062$ a pitchfork bifurcation appears: the RE equilibrium loses stability and two new stable steady states are created;
- for $\gamma_1^* < \gamma < \gamma_2^*$ there are two stable steady state whose basins are separated by the stable manifold of the RE equilibrium which is unstable;
- for $\gamma = \gamma_2^* \approx 1.9275$ the RE equilibrium becomes stable again via pitchfork bifurcation and two saddle cycles appears;
- for $\gamma > \gamma_2^*$ at least three locally stable steady state exist and their basins are separated by the saddles that lie on the stable manifold.

Figure (4) shows the basins of attraction of the map (18)-(19) under a weak monetary policy for small, medium and high values of the intensity of choice γ . The stable steady states are denoted with black circles, the saddles with grey points and the unstable nodes are emphasized in brown. When the intensity of

choice is relatively small the unique steady state is the RE equilibrium which is globally stable. In this scenario agents are more or less equally distributed over the different forecasting rules; hence realized inflation remains relatively close to the fundamental steady state. Recall that the case $\gamma = 0$ corresponds to the circumstance of infinite variance and difference in fitness can not be observed: so agents do not switch among predictors and all fractions are constant and equal to $1/H$. As the intensity of choice increases, the RE equilibrium loses stability and two new stable steady states arise. When γ increases further, the RE steady state is stable again, the previous steady states become saddles and two more new steady states are created. Note that the basins of attraction of these two new steady states are delimited by the stable manifolds of the saddles.

We move along *Path 2* in Figure 1 and consider a moderate interest rate rule by setting $c_1 = 1.5$ and again low information costs, i.e. $C = 0$. Then there exist values $0 < \gamma_1^* < \gamma_2^*$ such that:

- for $\gamma < \gamma_1^*$ the RE equilibrium is the only steady state which is unique and globally stable;
- for $\gamma = \gamma_1^* \approx 3.67035$ there is a saddle-node bifurcation of the RE equilibrium and two saddles and two nodes are created;
- for $\gamma_1^* < \gamma < \gamma_2^*$ there are three stable steady state and two saddle points whose stable manifolds separate the basins of the stable fixed points;
- for $\gamma = \gamma_2^* \approx 6.95$ there is another saddle-node bifurcation of the RE equilibrium which creates six new nodes (four stable and two unstable) and six saddles;
- for $\gamma > \gamma_2^*$ at least seven steady states exist: five steady states are locally stable and two other are unstable nodes. The basins of attraction of these fixed points are delimited by the stable manifolds of the saddle points.

Figure (5) shows the basins of attraction of the map (18)-(19) under a moderate monetary policy for small, medium and high values of the intensity of choice γ . As in the previous case, when the intensity of choice is low there is a unique and globally stable steady state, the RE equilibrium $(y^*, \pi^*) = (0, 0)$. When the value of γ increases, the RE equilibrium remains locally stable and two additional steady states are created: unlike the weak interest rate rule, with a moderate monetary policy the RE equilibrium remains locally stable. As a matter of fact, when the reaction to inflation in a neighborhood of the RE steady state is relatively high, the dynamics converge to the fundamental equilibrium. On the other hand, when

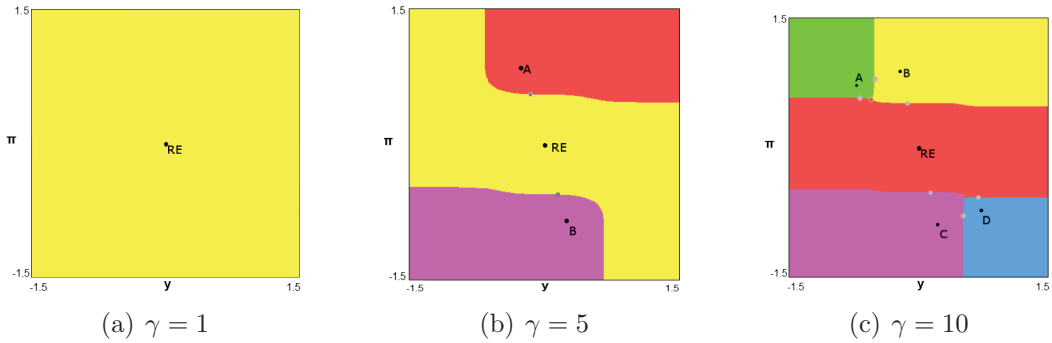


Figure 5: Basins of attraction in the low information costs scenario and moderate monetary policy

the intensity of choice is higher and inflation and output gap are out of the basin of attraction of the RE equilibrium, the implemented policy is not able to lead the economy to the fundamental steady state. Hence more agents will adopt the positive (negative) bias driving the system to the positive (negative) non-fundamental steady states.

Finally we consider the limit case when the Central Bank implements an aggressive monetary policy. By numerical investigation, we found that, fixing the inflation reaction coefficient of the Taylor rule equal to $c_1^* = 4.425$, the RE equilibrium is the only steady state of the system which is also stable for high values of γ . Figure 6 shows this situation: in particular the left panel displays the existence of a unique solution for the equation $G(\pi) = 0$ when γ is set equal to 1000 and $c_1 = c_1^*$. The right panel shows the basin of attraction of the map in the plane $(y, \pi) \in (-5, 5)$. Indeed when we look at the limit case $\gamma \rightarrow +\infty$, the Jacobian matrix has an eigenvalues equal to zero with multiplicity two. Therefore the RE equilibrium is always locally stable. The economic intuition behind this result can be found in the heterogeneous framework of expectations. The Central Bank has to react aggressively to an inflation deviation from its RE level in order to send correct signals for the evolutionary selection of the strategies, generating stable dynamics that settle down to the RE equilibrium. Our result differs from [1], where they found evidence of global stability in a 1-D map for inflation dynamics if the Taylor rule coefficient was greater than 2. Employing a standard 3-equations New-Keynesian model, we conclude that the monetary authority has to react more aggressively in order to have the unique RE equilibrium to be the only steady state of the system. This difference can be related to the presence of the output, whose dynamics is influenced by inflation. The aggressiveness of the policy is needed to break the reinforcing process that arise between inflation and output.

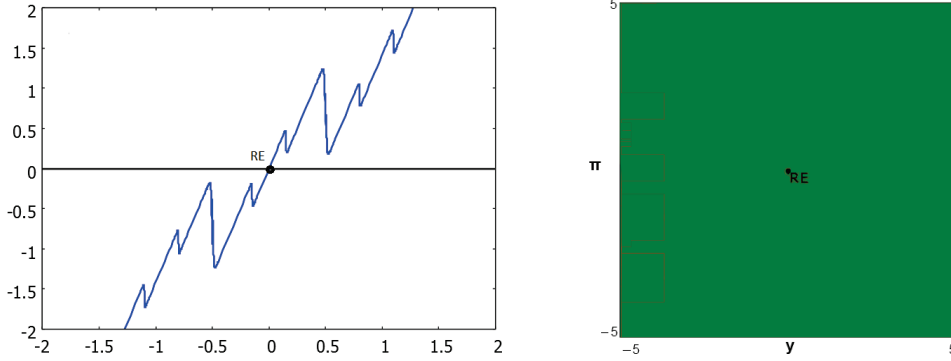


Figure 6: Lhs: $G(\pi) = 0$ for $\gamma = 1000$ and $c_1 = c_1^*$. Rhs: Basin of attraction in aggressive monetary policy scenario

5 Many beliefs types

The aim of this section is to address the following question: "What happens when the number of constant forecasting rules increases and approaches to infinity?". Previously we showed that in an economy populated by only 3 types of agents, the fundamental steady state $y^* = \pi^* = 0$ can be locally stable or even unstable and the dynamics can converge to other stable steady states. When the intensity of choice γ increases, meaning that the agents can easily switch between predictors, the system can reach other steady states in which the population minimizes its forecasting error at that steady state. A Taylor coefficient greater than one can help in stabilizing the economy but, as shown before, it is not sufficient when γ is relative large.

What happens if there is a continuum of forecasting rules representing all constant beliefs? Suppose there exist H different belief types for output b_h and H for inflation d_h all available at zero costs. Then the map $L_\gamma(y_t, \pi_t)$

$$\begin{aligned} y_t &= g(y_{t-1}; \pi_{t-1}) \\ \pi_t &= h(y_{t-1}; \pi_{t-1}) \end{aligned}$$

becomes

$$\begin{aligned} y_t &= \frac{a_1}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H b_h \exp(-\gamma (y_{t-1} - b_h)^2)}{\sum_{h=1}^H \exp(-\gamma (y_{t-1} - b_h)^2)} + \\ &+ \frac{a_2 (c_1 f_1 - 1)}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H d_h \exp(-\gamma (\pi_{t-1} - d_h)^2)}{\sum_{h=1}^H \exp(-\gamma (\pi_{t-1} - d_h)^2)} \end{aligned} \quad (21)$$

$$\begin{aligned}\pi_t = & \frac{a_1 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H b_h \exp\left(-\gamma (y_{t-1} - b_h)^2\right)}{\sum_{h=1}^H \exp\left(-\gamma (y_{t-1} - b_h)^2\right)} + \\ & + \frac{f_1 (1 - a_2 c_2) - a_2 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\sum_{h=1}^H d_h \exp\left(-\gamma (\pi_{t-1} - d_h)^2\right)}{\sum_{h=1}^H \exp\left(-\gamma (\pi_{t-1} - d_h)^2\right)}\end{aligned}\quad (22)$$

In order to study the dynamics of the system as long as the number of predictors become large, we apply the concept of Large Type Limit (LTL) developed in [5].

Assume that at $t = 0$ the beliefs about output $b = b_h \in \mathbb{R}$ are drawn from a common distribution with density $\psi(b)$, and that the predictors for inflation $d = d_h \in \mathbb{R}$ are also drawn from a common initial distribution with density $\omega(d)$. In order to replace the sample mean with the population mean, we divide both numerator and denominator of the expectation operators in the previous equations by H , obtaining:

$$\begin{aligned}y_t = & \frac{a_1}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\int b \exp\left(-\gamma (y_{t-1} - b)^2\right) \psi(b) db}{\int \exp\left(-\gamma (y_{t-1} - b)^2\right) \psi(b) db} + \\ & + \frac{a_2 (c_1 f_1 - 1)}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\int d \exp\left(-\gamma (\pi_{t-1} - d)^2\right) \omega(d) dd}{\int \exp\left(-\gamma (\pi_{t-1} - d)^2\right) \omega(d) dd}\end{aligned}\quad (23)$$

$$\begin{aligned}\pi_t = & \frac{a_1 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\int b \exp\left(-\gamma (y_{t-1} - b)^2\right) \psi(b) db}{\int \exp\left(-\gamma (y_{t-1} - b)^2\right) \psi(b) db} + \\ & \frac{f_1 (1 - a_2 c_2) - a_2 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{\int d \exp\left(-\gamma (\pi_{t-1} - d)^2\right) \omega(d) dd}{\int \exp\left(-\gamma (\pi_{t-1} - d)^2\right) \omega(d) dd}\end{aligned}\quad (24)$$

When H becomes large, the LTL map expressed by the previous two equation 23-24 is a good approximation of 21-22 and with high probability the steady states and the local stability conditions are the same for both maps.

Now assuming that both distributions for predictors are normal, $\psi(b) \sim N(m, s^2)$ and $\omega(d) \sim N(n, q^2)$, the LTL map denoted by $L_\gamma(y, \pi)$ becomes

$$y = \frac{a_1}{1 - a_2 c_1 f_2 - a_2 c_2} \frac{m + 2\gamma s^2 y}{1 + 2\gamma s^2} + \frac{a_2 (c_1 f_1 - 1)}{1 - a_2 c_1 f_2 - a_2 c_2} \frac{n + 2\gamma q^2 \pi}{1 + 2\gamma q^2}\quad (25)$$

$$\pi = \frac{a_1 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{m + 2\gamma s^2 y}{1 + 2\gamma s^2} + \frac{f_1 (1 - a_2 c_2) - a_2 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \frac{n + 2\gamma q^2 \pi}{1 + 2\gamma q^2}\quad (26)$$

where m and n represent the mean of the distributions, while s^2 and q^2 are the variances. We also assume that both distribution functions are centered around zero mean $n = m = 0$ giving the unique steady state $y^* = \pi^* = 0$. Normality and zero mean are simplifying assumptions, indeed the main results hold also

for positive distributions and asymmetric predictors. Obviously the beliefs are symmetric with respect to the RE equilibrium only under the previous hypothesis, helping us in investigating the stability properties of the fundamental steady state. Now we compute the Jacobian matrix of the system given by 25-26 to investigate the global stability of the RE equilibrium. If $n \neq m \neq 0$ the following results and the critical values of γ do not change even if the steady state of the LTL map is not the RE equilibrium.

$$J = \begin{pmatrix} -\frac{2s^2\gamma a_1}{(2s^2\gamma+1)(a_2c_1f_2+a_2c_2-1)} & \frac{2q^2\gamma}{2q^2\gamma+1} \frac{a_2-a_2c_1f_1}{a_2c_1f_2+a_2c_2-1} \\ -\frac{2s^2\gamma a_1 f_2}{(2s^2\gamma+1)(a_2c_1f_2+a_2c_2-1)} & \frac{2q^2\gamma}{2q^2\gamma+1} \left(f_1 + \frac{a_2f_2-f_2a_2c_1f_1}{a_2c_1f_2+a_2c_2-1} \right) \end{pmatrix}$$

The trace is

$$Tr(J) = \frac{2\gamma q^2 \left(f_1 + \frac{a_2f_2-f_2a_2c_1f_1}{a_2c_1f_2+a_2c_2-1} \right)}{2\gamma q^2 + 1} - \frac{2\gamma s^2 a_1}{(2\gamma s^2 + 1)(a_2c_1f_2 + a_2c_2 - 1)}$$

The determinant is

$$Det(J) = \frac{-4q^2 s^2 \gamma^2 a_1 f_1}{(2\gamma q^2 + 1)(2\gamma s^2 + 1)(a_2c_1f_2 + a_2c_2 - 1)}$$

The stability conditions in a 2-D system, as we have already shown, are given by (20). Substituting our parametrization we can find a relation between the intensity of choice γ and the variances of the two distributions, s^2 and q^2 . By numerical investigation, we can exclude the existence of flip or Neimarck-Sacker bifurcation for the parameter values reported in Table 1. The pitchfork bifurcation, whose curve is given by $H_{c_1}(\gamma) = 1 - Tr(J) + Det(J) = 0$, is the only bifurcation that can occur in our setting.

Hence, one can ask if the RE equilibrium exhibits local stability or not. In order to answer to this question we investigate numerically the possibility for the fundamental steady state to be locally unstable, assigning different values at q^2 , s^2 and γ . We found that the space in which the RE equilibrium is unstable implies that the intensity of choice has to be high and the Taylor principle not satisfied. On the other hand, the RE equilibrium is locally stable if the reaction coefficient for inflation in the Taylor rule is bigger than 1, regardless the values that γ assumes. Indeed, fixing $s^2 = q^2 = 1$ we can compute

$$\lim_{\gamma \rightarrow +\infty} H_{1.5}(\gamma) = 0.0795$$

$$\lim_{\gamma \rightarrow +\infty} H_{0.5}(\gamma) = -0.0879$$

Figure 7-(a) represents the stability/instability region in the space (s^2, q^2) if $c_1 = 0.5$ and $\gamma = 10$. The stable part is marked in red whereas blue color represents

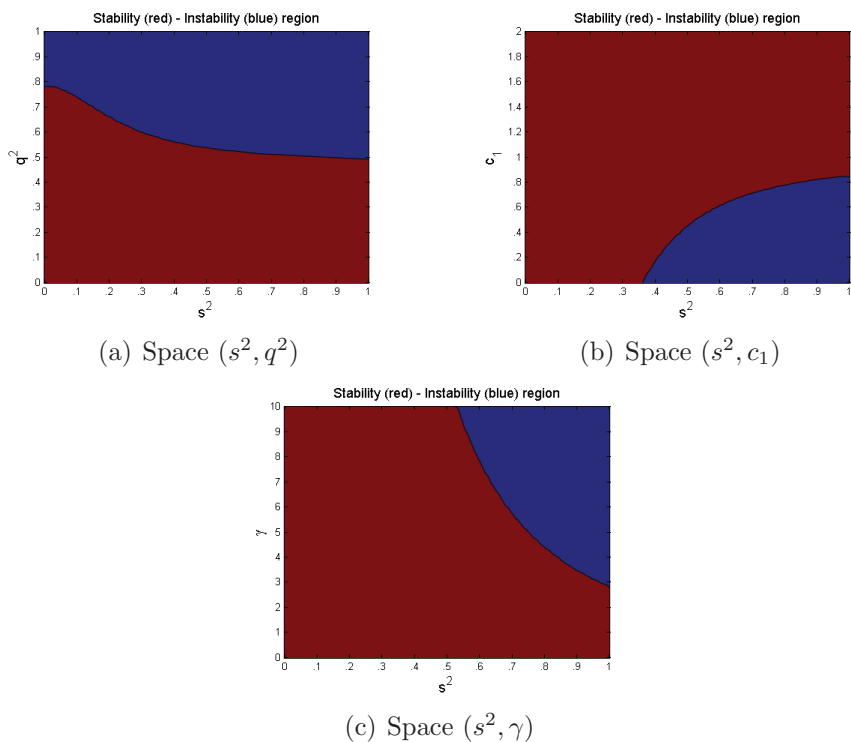


Figure 7: Stability/instability region of the RE equilibrium in different parameter spaces

the unstable configuration. It has to be noticed that with these calibrations, the stability conditions can be achieved only if the variances for the predictors of both output and inflation are not large, meaning that the agents can choose among a continuum of forecast that are not *too much distant* from the RE predictor. This underlines the importance of the spread of the initial beliefs as reported in [1]. As suggested before, an interest rate rule that satisfies the Taylor principle can prevent unstable behaviors. For sake of precision we have also to emphasize the fact that instability can occur if the variance is sufficiently large and for small values of the inflation reaction coefficient c_1 . In order to have a better understanding on the conditions under which the system can be stable, we assigned the same variance to both output and inflation distribution. Figure 7-(b) displays the stability region in the parameter space (s^2, c_1) . According to this plot, if the variance is not too large, the RE equilibrium always exhibits local stability for any c_1 value.

Finally Figure 7-(c) shows the stability/instability region in the plane (s^2, γ) . There exists an inverse relation between the variance and the intensity of choice. This result remarks the importance of the spread among predictors: thus if s^2 is not too large, the system is locally stable because the adopted forecasts are equally distributed around the RE predictor. Therefore the fractions will remain

almost constant. On the other hand, if s^2 is large enough, the system is unstable only if γ is sufficiently large: this means that agents can easily switch among different predictors which are not close to the RE equilibrium, leading the system to converge to other steady states.

In the previous analysis we have assumed a normal distribution with zero mean of initial beliefs for both output and inflation. As shown in [21] similar conclusions can be derived for fixed strictly positive distribution functions of initial beliefs. To get some intuition for this result, it is useful to look at the limiting case $\gamma = \infty$. If there exists a continuum of beliefs, the best predictor in every period, measured according to the past forecast error, will be the forecast that exactly coincides with the last period's realization of both output and inflation, $b_h = y_{t-1}$ and $d_h = \pi_{t-1}$. For $\gamma = \infty$, the fitness measure for each strategy is perfectly observable and in each period all agents pick the forecasting rule with the higher performance in the previous period, switching to the optimal predictor. Therefore, for the case $\gamma = \infty$, the economy can be represented taking into consideration only one representative naive agent. In this case we have that

$$\lim_{\gamma \rightarrow +\infty} L_\gamma(y, \pi) = (y, \pi)$$

Therefore the system is given by:

$$y_t = \frac{a_1}{1 - a_2 c_1 f_2 - a_2 c_2} y_{t-1} + \frac{a_2 (c_1 f_2 - 1)}{1 - a_2 c_1 f_2 - a_2 c_2} \pi_{t-1}$$

$$\pi_t = \frac{a_1 f_2}{1 - a_2 c_1 f_2 - a_2 c_2} y_{t-1} + \frac{f_1 (1 - a_2 c_2) - a_2 f_2}{(1 - a_2 c_1 f_2 - a_2 c_2)} \pi_{t-1}$$

Following [14] we can rewrite the system in matrix notation obtaining

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = A \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} \quad (27)$$

$$A = \Omega \begin{bmatrix} a_1 & a_2 (c_1 f_2 - 1) \\ a_1 f_2 & f_1 (1 - a_2 c_2) - a_2 f_2 \end{bmatrix}$$

where

$$\Omega = \frac{1}{1 - a_2 c_1 f_2 - a_2 c_2}$$

is a parameter aggregation and, under the usual assumptions it is always positive. The solution $y^* = \pi^* = 0$ always satisfies the system (27) which is globally stable if the trace and the determinant of A satisfy the usual stability conditions shown in (20).

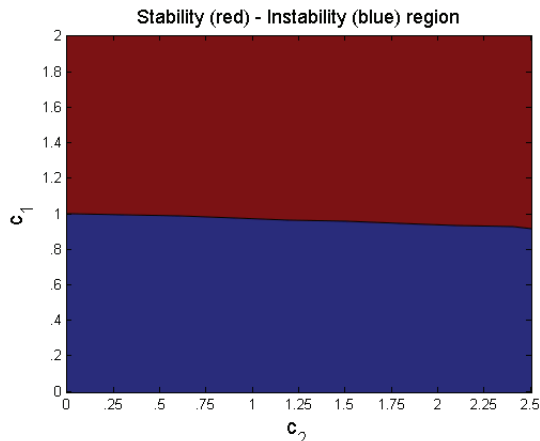


Figure 8: Stability and instability region for the case $\gamma = \infty$

The dynamics of (27) depends on the policy coefficients (c_1, c_2) , in addition to the non-policy parameters. Let us restrict our attention to the case of rules for which $c_1, c_2 > 0$. Then necessarily the second and the third condition hold.

Under the assumption of non-negative values for (c_1, c_2) , a necessary and sufficient condition for $y^* = \pi^* = 0$ to be globally stable is given by

$$(a_1 + a_2c_2 - a_1a_2c_2 - a_1a_2c_1f_2 - 1)f_1 + a_2f_2 - a_1 + 1 > 0 \quad (28)$$

Therefore in an environment with only one representative naive agent the monetary authority should respond to deviations of inflation and output from their target levels by adjusting the nominal rate satisfying the Taylor principle: at least in the long run, nominal interest rates should rise by more than the increase in the inflation rate. Indeed figure (8) illustrates graphically the regions of parameter space for c_1, c_2 associated with determinate and indeterminate equilibria, as implied by condition (28). Thus, the equilibrium will be unique under interest rate rule (3) whenever c_1 and c_2 are sufficiently large enough to guarantee that the real rate eventually rises in the face of an increase in inflation.

6 Policy analysis

Could the Central Bank stabilize the economy in an environment of heterogeneous agents? Is the Taylor principle sufficient to make the economy converge at the rational expectations solution or at least can it help in achieving a more stable economic environment? To answer these question we follow Lengnick and Wohltmann [23] introducing two indexes. The first is based on the distortion, calculated

as:

$$Dist(x) = \frac{1}{T} \sum_{t=1}^T |x_t - x_{RE}|$$

with $x = y, \pi$.

$Dist(x)$ measures the distortion of the time series, that is the mean of the deviation of the relevant variable from its steady state. We do not use the standard deviation because it considers the distortion as the dispersion of the time series from its mean, while the mean of x_t is not the steady state.

The second index, called volatility index, denotes the rate of change of the time series and it is calculated as:

$$Vol(x) = \frac{1}{T-1} \sum_{t=2}^T |x_t - x_{t-1}|$$

with $x = y, \pi$.

To calculate the distortion and the volatility we add a white noise term ε_t to equation (2). This component, as in [9], can be interpreted as a *cost push shock* and it can affect marginal costs due to, for example wage stickiness. We run the model for 1000 quarters with different values of the Taylor rule coefficients, c_1 and c_2 . We perform Monte Carlo simulations using 1000 different realizations of the pseudo random number generator for each c_1 as well as each c_2 , and then taking the mean. In so doing we also vary the intensity of choice coefficient and, to have a larger overview, we compute the analysis for three values of γ , namely $\gamma = [1, 3, 5]$.

First of all we present the results obtained letting the inflation coefficient in the Taylor rule vary, setting $c_2 = 0.5$ and fixing the number of agents equal to 11 because results with only three agents are not robust enough and display an unpredictable behavior.

Figures 9 (top left and bottom left boxes) present the distortion and the volatility of the output as long as c_1 increases from 0.5 to 1.5. The three curves are computed for different values of the intensity of choice parameter γ . Increasing the rationality parameter, the generated time series deviate much more from the rational expectations equilibrium, displaying also a greater volatility. Output distortion and volatility reach their minimum at $\hat{c}_1 = 1.02$. It has to be noticed that, even if the Taylor principle is only *weakly* satisfied, it minimizes both distortion and volatility of output. On the other hand, both inflation distortion and volatility monotonically decrease as c_1 increases (see figures 9 top right and bottom right boxes). Moreover the series are steeper as long as γ increases, meaning that the Taylor principle mostly affects an economy populated by more reactive agents.

The fact that the curve of output distortion and volatility is not monotonically decreasing means that there exists a value \hat{c}_1 such that for $c_1 < \hat{c}_1$ inflation targeting is able to reduce both inflation and output distortion and volatility. Indeed for

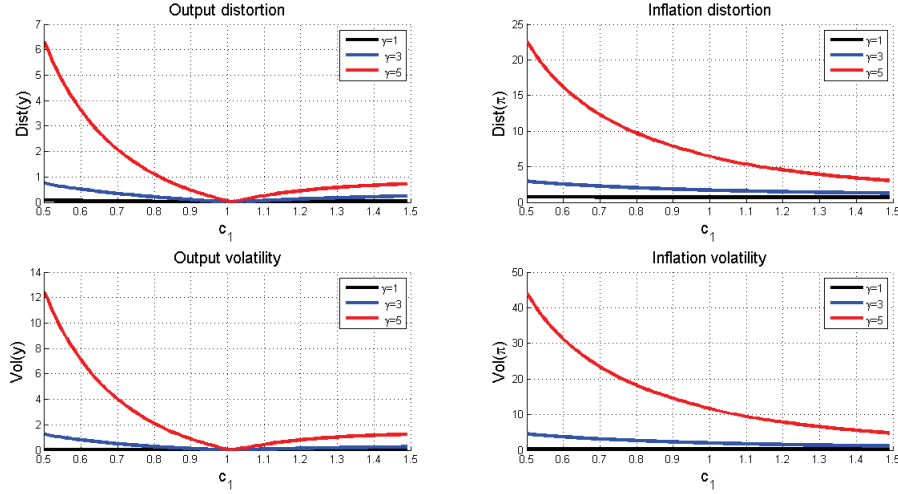


Figure 9: The influence of c_1 on volatility and distortion indexes

$c_1 > \hat{c}_1$ there exists a trade-off between output and inflation, the lower are $Dist(\pi)$ and $Dist(y)$, the higher are $Vol(y)$ and $Vol(\pi)$. Inflation targeting, of course, does not exclude the role of output stabilization. DSGE modelers underline that price rigidities provide a rationale for output stabilization by Central Bank (see [9] and [14]) or for a flexible inflation targeting [27]. Because of the existence of rigidities, when sufficiently large shocks occur, leading inflation to depart from its target, the Central Bank should follow a strategy of gradual return of inflation to its target. Since too abrupt attempts to bring back inflation to its target, would require such high increases in the interest rate implying too strong declines in output.

What would happen in a world of heterogeneous agents if the Central Bank shifted its target from inflation to output?

To give an answer to this question we perform a similar exercise, fixing $c_1 = 1.5$ and letting the output coefficient of the Taylor rule vary from 0.1 to 2.1. Results suggest that there exists a trade-off between inflation and output distortion (figure 10 top left and top right panels). Output distortion monotonically decreases as long as the reaction coefficient to output gap increases. On the contrary, inflation distortion increases. It has to be noticed that the rise in the inflation distortion is bigger (in absolute value) than the reduction in output distortion.

On the other hand, by increasing the parameter c_2 , we observe a reduction in output volatility and an increase in inflation volatility, as figure 10 shows (bottom left and right panels). It is also worth to notice that in figure 10 (bottom left) output volatility increases for $c_2 < 0.5$ and a sufficiently high value of γ . These

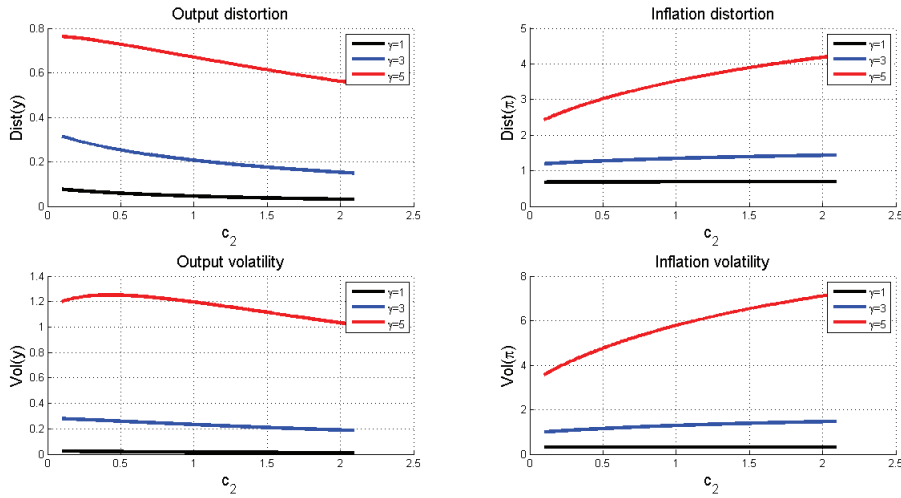


Figure 10: The influence of c_2 on volatility and distortion indexes

trends makes an interpretation less clear: indeed there exists a trade-off between output and inflation volatility for $c_2 > 0.5$, if agents switch sufficiently fast between predictors. As in the previous analysis, the rationality parameter affects the policy influence on the economy: if agents hardly switch among predictors, the monetary policy loses most of its influence but distortion and volatility of the two variables are cut down by this occurrence.

To summarize our findings, if the Central Bank is keen in inflation targeting with a moderate monetary policy ($c_1 < \hat{c}_1$), it is possible to reduce both output and inflation variability. The relation is non-linear and, with a too high inflation stabilization parameter, there exist a trade-off where lower inflation variability is obtained at the cost of increased output variability. Even with a Taylor rule coefficient that could lead to a multiplicity of equilibria, as we have previously shown, the variability of the two variables can be lowered at the same time. Moreover some output stabilization is good because it reduces both output and inflation variability by preventing too large switches in forecasting behavior.

7 Conclusions

In this work we have presented a standard 3-equations New Keynesian model to investigate the role of heterogeneous expectations about future inflation, output and the role of monetary policy on the dynamics of the business cycle. We have adopted the heterogeneous expectation framework of [4], where the evolution of

forecasting rules is governed by an endogenous dynamic selection of strategies where agents switch among different predictors towards the most efficient strategy.

The business cycle dynamics depends on the expectations environment and the coefficients of an interest rate rule (e.g. Taylor rule). Since heterogeneous agents try to learn from their past forecasts, if the monetary policy reacts weakly to inflation, a cumulative process of rising inflation and output appears. Signals from the market lead the economy to non-fundamental steady states, reinforced by self-fulfilling expectations of high inflation. On the contrary, when the response to inflation is moderate, the heterogeneous expectations can be managed in order to correct past forecast error and to conduct the economy towards the RE equilibrium. Even with an aggressive monetary policy, the monetary authority is able to send correct signals to agents and can induce stable dynamics settling down to the fundamental steady state. In rational expectations model, by following the Taylor principle, the Central Bank stimulates explosive inflation dynamics. In contrast, this setting is able to avoid explosive paths and also to guarantee uniqueness of the RE equilibrium. It is also worth to point out that even if the Taylor principle is sufficient to guarantee convergence to the fundamental steady state, it is no longer enough to avoid multiple equilibria. Indeed the monetary policy rule must be sufficiently aggressive to guarantee a proximity between the realized inflation and the RE equilibrium.

We have also to highlight that, in the case of many beliefs types (a *continuum of beliefs*), a monetary policy rule that reacts aggressively to current inflation can fully stabilize the system. If the policy rule is not aggressive enough and the intensity of choice is large, the cumulative process of inflation and output appear again.

Finally, to get some policy outcomes, we considered two summary indexes (i.e. volatility and distortion) that link the impact of the Taylor rule coefficients to distortion and volatility of the fundamental variables. In this heterogeneous agents framework, policy makers can reduce volatility and distortion of output and inflation with a sufficient degree of reaction. If the Central Bank is keen in inflation targeting with a moderate monetary policy, it is possible to reduce both output and inflation variability. The relation is non-linear and, with a too high inflation stabilization parameter, there exist a trade-off where lower inflation variability is obtained at the cost of increased output variability. Moreover some output stabilization is good because it reduces both output and inflation variability by preventing too large switches in forecasting behavior.

Depending on the target of the monetary authority, inflation volatility and distortion can be minimized but also output stabilization can be taken into account. Indeed, if the central Bank shifted its target from inflation to output, results sug-

gest that there exists a trade-off between inflation and output distortion but, a strong reaction to output is more likely to stabilize the economy.

Increasing the number of agents, the results connected to the reaction to inflation are quite similar to the scenario with a small number of agents. On the other hand, the enhancement observed in inflation volatility with a small population is no longer true as the reaction to output increases. Thus there is a possibility to decrease inflation volatility as c_2 increases, and to decrease the distortion of both variables when the Central Bank is keen in output stabilization.

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