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nonlinear demand function

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Information exchange in a Cournot duopoly with nonlinear demand function

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Abstract

We study information sharing in a Cournot duopoly with isoelastic demand function, when the elasticity is uncertain. This is one of the first attempts to analyze the role of nonlinearity in such a framework. We found important results about the profitability of sharing informations when marginal costs are high and/or the variance between elasticity values is low. From the point of view of welfare considerations, not sharing information seems to be the best scenario.

Keywords: Information exchange, Cournot duopoly, Hyperbolic demand

JEL Classification: D43, D82, L13

1. Introduction

There exists a burgeoning theoretical literature dealing with the problem of oligopolies where firms are characterized by uncertainty about some relevant feature of the market: demand, costs, number of competitors, and so on. The main question behind these studies is the following: what are the conditions under which firms endowed with private informations about the uncertain market feature prefer to collaborate and share their information? A seminal contribution is due to Novshek and Sonnenschein (1982), who found that in a Cournot duopoly with uncertain linear demand's intercept, for the firms it is more profitable to keep private their informations. This result has been later confirmed by Clarke (1983), Vives (1984), Gal-Or (1985), Li (1985), Kirby (1988), among the others. If we limit our analysis to demand uncertainty in Cournot oligopolies, the first case in which for firms it is preferable to share their private information is due to Vives (1984). He found that firms expect to gain a higher profit by sharing their information if products are differentiated and relatively poor substitutes. Kirby (1988) found conditions favourable to information exchange under the assumption of steep marginal costs. Malueg and Tsutsui (1996) explore the case in which the slope of the linear demand function is uncertain, and they found that

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firms prefer to collaborate if the signal they receive is sufficiently accurate and the ratio between the two possible demand slopes is high enough (i.e. volatility is high). Recently, further conditions that permit the sharing of information in Cournot competitions with linear demand can be found in Chokler et al. (2006) and Lagerlöf (2007).

The assumption of linear demand function is in general justified by practical reasons. In fact, models with uncertainty in the demand's intercept are analytically tractable and permit to obtain unique Nash equilibria in closed form and also permit to explicit the expected profit. As Malueg and Tsutsui (1996) note, things are more complicated when the slope of the demand function is uncertain. Nevertheless, limiting the analysis to a Cournot duopoly and assuming that the uncertain demand parameter may only assume two possible values, they are still able to obtain analytical results.

One limit of the assumption of linearity about the demand function is that in some cases, prices and outputs may become negative. Malueg and Tsutsui (1998) solve this problem by imposing a non-negativity constraint. In other words they assume a piecewise-linear demand function. A similar assumption is made by Lagerlöf (2007) who stresses the fact the multiple equilibria may arise by relaxing the linearity assumption about the demand function.

To the best of our knowledge the case with a purely nonlinear demand function has not been still investigated. This paper is a first attempt of filling this gap. We deepen the case of iso-elastic demand that, among the possible nonlinear shapes of the demand function, is one of the most common (see microeconomics textbooks such as Frank 2009 and McAfee and Lewis, 2009). The properties of Cournot oligopolies with general isoelastic demand function have been recently analyzed in Beard (2013) and Collie (2004). With this nonlinear demand function we ride out the problem of negative price and quantities and moreover, in its unit-elasticity version, it is micro-founded on the basis of Cobb-Douglas utility functions of the consumers. This subcase is often used in dynamic oligopolies with boundedly rational players (see Puu, 1991, Agliari et al. 2000, Tramontana, 2010, and the book by Bischi et al. 2010).

On the other hand, reaction functions deriving from such a nonlinear demand function are usually not monotonic and problems of existence and unicity of Nash equilibria may arise. As we will see, this nonlinearity excludes the possibility of obtaining an explicit analytic expression of reaction functions and Nash equilibria, nevertheless we can obtain implicit equations relating the symmetric equilibrium quantities and we can numerically approximate their solutions and use them to compare expected profits. Probably, the hard analytical tractability of models involving nonlinearities is one of the main reasons behind the lack of research work on this topic. Nevertheless, we think that nowadays the velocity and the accuracy of modern mathematical softwares permit to adequately face these problems, obtaining numerical results that can be quite robust if confirmed for large portions of the parameters' space. We find that sharing information is profitable if marginal costs are high enough and the variability between the possible elasticity values is low. Differently from the already existing literature, information sharing drastically reduces consumer surplus and

total welfare.

The paper is organized as follows: in Section 2 we introduce the general duopoly model with isoelastic demand and without uncertainty. In Section 3 we study the effect of an uncertain demand elasticity and we show our numerical results. Section 4 concludes the paper.

2. The model without uncertainty

We consider a duopolistic market where two firms produce homogeneous goods, and compete in a market characterized by a nonlinear demand function. In particular, we consider an isoelastic (or hyperbolic) inverse demand, given by:

$$P(Q) = Q^{-\eta} \quad (1)$$

where $Q = q_1 + q_2$ is the total output and $\eta > 0$ is the constant inverse of the elasticity of substitution characterizing the demand function.

Firms adopt the same technology characterized by fixed marginal costs (c).

Thus, the profit of firm 1 is given by

$$\Pi_1 = \frac{q_1}{(q_1 + q_2)^\eta} - cq_1 \quad (2)$$

and the first order condition for profit maximization gives:

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \implies c(q_1 + q_2)^{\eta+1} = q_1 + q_2 - q_1\eta \quad (3)$$

Similarly, we get the f.o.c. related to the maximizing problem of firm 2:

$$\frac{\partial \Pi_2}{\partial q_2} = 0 \implies c(q_1 + q_2)^{\eta+1} = q_2 + q_1 - q_2\eta \quad (4)$$

All the vectors of positive outputs (q_1^*, q_2^*) satisfying both (3) and (4) are feasible Cournot-Nash equilibria of the game. The nonlinearity in the demand function causes the nonlinearity of the f.o.c. and consequently existence and uniqueness of the equilibrium are no more ensured. Anyway, given the symmetry between the duopolists, we prefer to limit our analysis only to symmetric equilibria (i.e. those with $q_1^* = q_2^* = q^*$)¹.

After simple algebraic manipulations, we obtain that the system of equations (3) and (4) is solved by a symmetric equilibrium only when its coordinates are both equal to

$$q^* = \frac{1}{2} \left(\frac{2 - \eta}{2c} \right)^{\frac{1}{\eta}} \quad (5)$$

¹An example of multiple equilibria in presence of nonlinear reaction functions can be found in Bischi and Kopel (2001) and Bischi et al. (2009).

In order to be sure that q^* is a local maximum of the profit function, we assume that $0 < \eta \leq 1^2$.

Simple comparative statics reveals that the output at equilibrium is negatively correlated with the level of marginal cost:

$$\frac{\partial q^*}{\partial c} = -\frac{q^*}{\eta c} < 0 \quad (6)$$

while the relation between optimal output and the demand's elasticity is more complicated. The partial derivative of the optimal output is the following:

$$\frac{\partial q^*}{\partial \eta} = -\frac{q^*}{\eta} \left[\ln(2q^*) + \frac{1}{2-\eta} \right] < 0 \iff q^* > \frac{1}{2e^{\frac{1}{2-\eta}}} \quad (7)$$

In other words, if equilibrium quantities are not too small, to a lower demand elasticity it corresponds a lower symmetric Nash equilibrium³.

Let us now look at how the optimal profit changes by varying costs or demand's elasticity. The optimal profit is given by:

$$\Pi^* = q^* \left(\frac{\eta c}{2-\eta} \right) \quad (8)$$

whose partial derivative with respect to c is:

$$\frac{\partial \Pi^*}{\partial c} = q^* \left(\frac{\eta-1}{2-\eta} \right) \quad (9)$$

which is negative if $0 < \eta < 3/2$, that is also under our restriction. In other words, profits diminish if marginal costs increase (and the opposite). This result does not sound strange. Instead the sign of the partial derivative with respect to the elasticity of substitution is not univocal, so we skip it.

Finally, let us consider the Consumer Surplus (CS henceforth), that is given by

$$CS = \int_0^{Q^*} (Q^*)^{-\eta} dQ - P(Q^*)Q^* \quad (10)$$

where $Q^* \equiv 2q^*$. CS is convergent if $\eta < 1$, and under such a restriction it becomes:

$$CS = \frac{\eta}{1-\eta} (Q^*)^{1-\eta} = \frac{\eta}{1-\eta} \left(\frac{2-\eta}{2c} \right)^{\frac{1-\eta}{\eta}} \quad (11)$$

²If $0 < \eta \leq 1$, then $\frac{\partial^2 \Pi_1}{\partial q_1^2} = \frac{\eta[q_1(\eta-1)-2q_2]}{(q_1+q_2)^{\eta+2}} < 0$.

³From the expression inside the square brackets, it is clear that $q^* > 1/2$ is a sufficient (but not necessary) condition for a negative correlation between Nash equilibrium and demand elasticity.

3. The model with uncertainty

Let us assume now that the elasticity parameter η is not known by the duopolists. In particular, it is a random parameter that may assume two values: an high value (η_H) and a low one (η_L), with $1 > \eta_H > \eta_L > 0^4$.

Before making their output decisions, each firm observes a private signal s_x^i , with $i = 1, 2$ and $x = L, H$. The two signals are assumed to be conditionally independent and their accuracy is given by the parameter $\sigma > 1/2$, that is:

$$Pr(s_x^i = s_L^i | \eta = \eta_L) = Pr(s_x^i = s_H^i | \eta = \eta_H) = \sigma .$$

The duopolists play a game made up by three stages:

- Stage 1: firms decide if sharing or not the signal they will receive. Agreements are binding so we avoid focusing on the case in which firms may deviate from the agreement;
- Stage 2: firms receive signals and share them with the competitor if at stage 1 they decided to do so, otherwise they keep them private;
- Stage 3: firms compete à la Cournot, using the informations they have gathered at the previous stage.

At the beginning of the game, firms do not have any reason to consider one case more probable than the other, so each signal has the same probability of been received $Pr(s_x^i = s_L^i) = Pr(s_x^i = s_H^i) = 1/2$.

All these features are common knowledge between duopolists.

Note that the order of the three stages is important because by inverting stages 1 and 2 we would permit firms to decide if sharing information or not after they received the signal, so it would be possible that they share information with a signal and not with the other one. With our order firms decide if sharing or not the information, a priori, independently of the signal they will receive.

3.1. First order conditions

According to our assumptions, when firm i decides its output, it may have two kinds of informative set (I^i). If it have decided to share its signal with the rival, its informative set (I_s^i) is made up by the two signals. If it does not share its information, the informative set (I_{ns}^i) only contains the private signal:

$$\begin{array}{r}
 I^i \begin{array}{l} \nearrow \\ \searrow \end{array} \\
 \begin{array}{l} I_s^i = \{s_x^i, s_y^i\} \\ I_{ns}^i = \{s_x^i\} \end{array}
 \end{array}
 \begin{array}{l}
 i, j = 1, 2 \quad i \neq j \\
 x, y = L, H
 \end{array}
 \quad (12)$$

⁴We limit our analysis to the case of elastic demand, i.e. elasticity higher than one. As we have proved in the previous section, such restriction permits to obtain a finite value for the consumer surplus.

Given that the firms are in a situation of strategic interdependence, it is important to note that when they share their information, they reduce the degree of uncertainty, because they don't need to make an expectation about the rival's production. We will come back to this point later, when we talk about the Bayesian equilibrium without information sharing in sec. 3.2.

Duopolist i produces the amount of output that maximizes her expected profit, given the informative set I^i :

$$E(\Pi_i | I^i) = E \left[\frac{q_i}{(q_i + q_{-i})^\eta} - cq_i \mid I^i \right] \quad (13)$$

The first order condition for profit maximization is the following::

$$q_i(I^i) - q_i(I^i)E[\eta | I^i] + E[q_j | I^i] = cE \left[(q_i(I^i) + q_j)^{\eta+1} \mid I^i \right] \quad (14)$$

that implicitly defines the equilibrium output $q_i(I^i)$.

In order to find the Cournot equilibrium, equation (14) must be solved with both $i = 1, 2$, obtaining a pair of optimal outputs, given the informative set.

In the next subsections we analyze the case without information exchange and the case in which duopolists let the rival know their private signals.

3.2. Bayesian equilibrium without information exchange

If firms decide to keep private the information about their signals, then they face one of the two following scenarios: a firm either receives a signal suggesting that the demand elasticity parameter is low or that it is high.

Let us consider the problem of firm 1. *Ceteris paribus* a similar reasoning also works for firm 2. In the first scenario she maximizes her expected profit given the signal s_L^1 . From the first order condition (14) we know that she does not only need to calculate the expected value of the elasticity η and the rival's output q_2 , but she must also calculate the expected value of the term $(q_{1,L} + q_2)^{\eta+1}$, where $q_{1,L}$ denotes the output that maximizes the expected profit of firm 1, given the signal s_L^1 .

We only focus on symmetric equilibria (i.e. $q_{1,L} = q_{2,L} = q_L$ and $q_{1,H} = q_{2,H} = q_H$).

So the optimal quantity given the signal s_L^1 solves the following equation:

$$\begin{aligned} q_L - q_L [\eta_L Pr(\eta_L | s_L^1) + \eta_H Pr(\eta_H | s_L^1)] + q_L Pr(s_L^2 | s_L^1) + q_H Pr(s_H^2 | s_L^1) = \\ = c \left[(2q_L)^{\eta_L+1} Pr(\eta_L, s_L^2 | s_L^1) + (q_L + q_H)^{\eta_L+1} Pr(\eta_L, s_H^2 | s_L^1) + \right. \\ \left. + (2q_L)^{\eta_H+1} Pr(\eta_H, s_L^2 | s_L^1) + (q_L + q_H)^{\eta_H+1} Pr(\eta_H, s_H^2 | s_L^1) \right] \end{aligned} \quad (15)$$

We need the following conditional probabilities:

$$\begin{aligned}
Pr(\eta_L | s_L^1) &= \sigma & ; & \quad Pr(\eta_H | s_L^1) = 1 - \sigma \\
Pr(s_L^2 | s_L^1) &= \sigma^2 + (1 - \sigma)^2 & ; & \quad Pr(s_H^2 | s_L^1) = 2\sigma(1 - \sigma) \\
Pr(\eta_L, s_L^2 | s_L^1) &= \sigma^2 & ; & \quad Pr(\eta_H, s_L^2 | s_L^1) = (1 - \sigma)^2 \\
Pr(\eta_L, s_H^2 | s_L^1) &= \sigma(1 - \sigma) & ; & \quad Pr(\eta_H, s_H^2 | s_L^1) = \sigma(1 - \sigma)
\end{aligned} \tag{16}$$

and by using them into (15) we get the following implicit equation:

$$\begin{aligned}
q_L [1 - \eta_H + \sigma (\eta_H - \eta_L) + \sigma^2 + (1 - \sigma)^2] + 2q_H \sigma(1 - \sigma) &= \\
= c \left[(2q_L)^{\eta_L+1} \sigma^2 + (q_L + q_H)^{\eta_L+1} \sigma(1 - \sigma) + \right. & \\
\left. + (2q_L)^{\eta_H+1} (1 - \sigma)^2 + (q_L + q_H)^{\eta_H+1} \sigma(1 - \sigma) \right] &
\end{aligned} \tag{17}$$

With a similar procedure applied to the case in which the private signal received is s_H , we get a second implicit equation:

$$\begin{aligned}
q_H [1 - \eta_L + \sigma (\eta_L - \eta_H) + \sigma^2 + (1 - \sigma)^2] + 2q_L \sigma(1 - \sigma) &= \\
= c \left[(2q_H)^{\eta_H+1} \sigma^2 + (q_L + q_H)^{\eta_H+1} \sigma(1 - \sigma) + \right. & \\
\left. + (2q_H)^{\eta_L+1} (1 - \sigma)^2 + (q_L + q_H)^{\eta_L+1} \sigma(1 - \sigma) \right] &
\end{aligned} \tag{18}$$

Symmetric equilibria are pairs of strictly positive values of the components of the vector (q_L, q_H) that solve both eq. (17) and (18). Unfortunately, they cannot be solved analytically and we are not even certain that they have a unique solution. However, we can numerically solve the system of equations and considering a large set of parameters' combinations, we find that in the most of the cases there exists only one symmetric equilibrium, so multiple equilibria arise only in a small region of the parameters' space⁵.

3.3. Bayesian equilibrium with information exchange

If firms opt for sharing their private informations, three cases may happen: they can both receive the s_L signal; they both receive the s_H signal or eventually they may receive opposing signals.

We want to identify a vector of symmetric equilibria for each possible case, (q_{LL}, q_{HH}, q_{LH}) .

By rearranging the first order condition (14) we turn it into:

$$2q_{xy} - q_{xy} E [\eta | s_x^1, s_y^2] = c E \left[(2q_{xy})^{\eta+1} | s_x^1, s_y^2 \right] \tag{19}$$

⁵See Lagerlöf (2007) for examples of more relevant multiple equilibria caused by a convex expected demand

Now, the two outputs coincide and in this sense, the degree of uncertainty is lower with respect to the case in which duopolists do not collaborate.

The expected values that appear in (19) must be explicated in one of the following ways depending on the signals' combination:

$$\begin{aligned}
E[\eta | s_L^1, s_L^2] &= \eta_L Pr(\eta_L | s_L^1, s_L^2) + \eta_H Pr(\eta_H | s_L^1, s_L^2) \\
E[\eta | s_H^1, s_H^2] &= \eta_H Pr(\eta_H | s_H^1, s_H^2) + \eta_L Pr(\eta_L | s_H^1, s_H^2) \\
E[\eta | s_L^1, s_H^2] &= \eta_L Pr(\eta_L | s_L^1, s_H^2) + \eta_H Pr(\eta_H | s_L^1, s_H^2)
\end{aligned} \tag{20}$$

and:

$$\begin{aligned}
E[(2q_{LL})^{\eta+1} | s_L^1, s_L^2] &= (2q_{LL})^{\eta_L+1} Pr(\eta_L | s_L^1, s_L^2) + (2q_{LL})^{\eta_H+1} Pr(\eta_H | s_L^1, s_L^2) \\
E[(2q_{HH})^{\eta+1} | s_H^1, s_H^2] &= (2q_{HH})^{\eta_H+1} Pr(\eta_H | s_H^1, s_H^2) + (2q_{HH})^{\eta_L+1} Pr(\eta_L | s_H^1, s_H^2) , \\
E[(2q_{LH})^{\eta+1} | s_L^1, s_H^2] &= (2q_{LH})^{\eta_L+1} Pr(\eta_L | s_L^1, s_H^2) + (2q_{LH})^{\eta_H+1} Pr(\eta_H | s_L^1, s_H^2)
\end{aligned} \tag{21}$$

while the conditional probabilities we need are given by:

$$\begin{aligned}
Pr(\eta_x | s_x^1, s_x^2) &= \frac{\sigma^2}{\sigma^2 + (1-\sigma)^2} \quad ; \quad Pr(\eta_y | s_x^1, s_x^2) = \frac{(1-\sigma)^2}{\sigma^2 + (1-\sigma)^2} \\
Pr(\eta_x | s_x^1, s_y^2) &= Pr(\eta_y | s_x^1, s_y^2) = \frac{1}{2}
\end{aligned} \tag{22}$$

The following equations are obtained by combining (19-22) and implicitly define the symmetric equilibrium output strategies:

$$\begin{aligned}
2q_{LL} - q_{LL} \left[\frac{\eta_L \sigma^2 + \eta_H (1-\sigma)^2}{\sigma^2 + (1-\sigma)^2} \right] &= c \left[\frac{(2q_{LL})^{\eta_L+1} \sigma^2 + (2q_{LL})^{\eta_H+1} (1-\sigma)^2}{\sigma^2 + (1-\sigma)^2} \right] \\
2q_{HH} - q_{HH} \left[\frac{\eta_H \sigma^2 + \eta_L (1-\sigma)^2}{\sigma^2 + (1-\sigma)^2} \right] &= c \left[\frac{(2q_{HH})^{\eta_H+1} \sigma^2 + (2q_{HH})^{\eta_L+1} (1-\sigma)^2}{\sigma^2 + (1-\sigma)^2} \right] \\
4q_{LH} - q_{LH} (\eta_L + \eta_H) &= c \left[(2q_{LH})^{\eta_L+1} + (2q_{LH})^{\eta_H+1} \right]
\end{aligned} \tag{23}$$

3.4. Comparisons between expected profits

In order to decide which strategy to adopt, firms compare expected profits.

Before receiving the signal firms consider each signal's realization equiprobable, so in the case in which duopolists do not share their information, the expected profit is the average between the expected profits given each possible signal, that is:

$$\Pi_{ns}^i = E_{ns} [\Pi_i | I^i] = \frac{1}{2} E [\Pi_i | s_L^i] + \frac{1}{2} E [\Pi_i | s_H^i] \tag{24}$$

The two expected profits are given by:

$$\begin{aligned}
E[\Pi_i | s_L^i] &= \left[\frac{q_L}{(2q_L)^{\eta_L}} - cq_L \right] Pr(\eta_L, s_L^j | s_L^i) + \left[\frac{q_L}{(q_L + q_H)^{\eta_L}} - cq_L \right] Pr(\eta_L, s_H^j | s_L^i) + \\
&\quad + \left[\frac{q_L}{(q_L + q_H)^{\eta_H}} - cq_L \right] Pr(\eta_H, s_H^j | s_L^i) + \left[\frac{q_L}{(2q_L)^{\eta_H}} - cq_L \right] Pr(\eta_H, s_L^j | s_L^i) \\
E[\Pi_i | s_H^i] &= \left[\frac{q_H}{(2q_H)^{\eta_H}} - cq_H \right] Pr(\eta_H, s_H^j | s_H^i) + \left[\frac{q_H}{(q_L + q_H)^{\eta_H}} - cq_H \right] Pr(\eta_H, s_L^j | s_H^i) + \\
&\quad + \left[\frac{q_H}{(q_L + q_H)^{\eta_L}} - cq_H \right] Pr(\eta_L, s_L^j | s_H^i) + \left[\frac{q_H}{(2q_H)^{\eta_L}} - cq_H \right] Pr(\eta_L, s_H^j | s_H^i)
\end{aligned} \tag{25}$$

where we have inserted the symmetric equilibria q_L and q_H implicitly defined in (17) and (18), into the profit equation (13). By substituting the two expected profits given the signals in (24) and using the conditional probabilities in (16) we obtain:

$$\begin{aligned}
\Pi_{ns}^i = E_{ns}[\Pi_i | I^i] &= \frac{1}{2} \left[\frac{q_L}{(2q_L)^{\eta_L}} + (q_L + q_H)^{1-\eta_L} + \frac{q_L}{(2q_L)^{\eta_H}} + \right. \\
&\quad \left. + \frac{q_H}{(2q_H)^{\eta_H}} + (q_L + q_H)^{1-\eta_H} + \frac{q_H}{(2q_H)^{\eta_L}} - 4c(q_L + q_H) \right]
\end{aligned} \tag{26}$$

Differently, if firms share their information the expect profit is obtained as follows:

$$\begin{aligned}
\Pi_s^i = E_s[\Pi_i | I^i] &= Pr(\eta_L, s_L^1, s_L^2) \left[\frac{q_{LL}}{(2q_{LL})^{\eta_L}} - cq_{LL} \right] + Pr(\eta_H, s_H^1, s_H^2) \left[\frac{q_{HH}}{(2q_{HH})^{\eta_H}} - cq_{HH} \right] + \\
&\quad Pr(\eta_L, s_H^1, s_H^2) \left[\frac{q_{HH}}{(2q_{HH})^{\eta_L}} - cq_{HH} \right] + Pr(\eta_H, s_L^1, s_L^2) \left[\frac{q_{LL}}{(2q_{LL})^{\eta_H}} - cq_{LL} \right] + \\
&\quad + [Pr(\eta_L, s_H^1, s_L^2) + Pr(\eta_L, s_L^1, s_H^2)] \left[\frac{q_{LH}}{(2q_{LH})^{\eta_L}} - cq_{LH} \right] + [Pr(\eta_H, s_H^1, s_L^2) + Pr(\eta_H, s_L^1, s_H^2)] \left[\frac{q_{LH}}{(2q_{LH})^{\eta_H}} - cq_{LH} \right]
\end{aligned} \tag{27}$$

With the symmetric equilibria implicitly defined in (23) and the total probabilities given by:

$$Pr(\eta_x, s_x^1, s_x^2) = \frac{\sigma^2}{2} \quad ; \quad Pr(\eta_y, s_x^1, s_x^2) = \frac{(1-\sigma)^2}{2} \tag{28}$$

$$Pr(\eta_x, s_x^1, s_y^2) = Pr(\eta_y, s_x^1, s_y^2) = \frac{\sigma(1-\sigma)}{2}$$

we can finally obtain the expected profit of collaboration:

$$\begin{aligned}
\Pi_s^i = E_s[\Pi_i | I^i] &= \frac{\sigma^2}{2} \left[\frac{q_{LL}}{(2q_{LL})^{\eta_L}} + \frac{q_{HH}}{(2q_{HH})^{\eta_H}} - c(q_{LL} + q_{HH}) \right] + \\
&\quad \frac{(1-\sigma)^2}{2} \left[\frac{q_{HH}}{(2q_{HH})^{\eta_L}} + \frac{q_{LL}}{(2q_{LL})^{\eta_H}} - c(q_{LL} + q_{HH}) \right] + \\
&\quad + \sigma(1-\sigma) \left[\frac{q_{LH}}{(2q_{LH})^{\eta_L}} + \frac{q_{LH}}{(2q_{LH})^{\eta_H}} - 2cq_{LH} \right]
\end{aligned} \tag{29}$$

For symmetry reasons, the expected profits of sharing and not sharing information is the same for both $i = 1$ and $i = 2$, so from now on we simply denote them by Π_s and Π_{ns} , respectively.

Given the impossibility of analytically determine whenever the expected profit without information sharing (26) is higher/lower than the expected profit with information sharing (29), we have numerically obtained the symmetric output equilibria in both cases and we used them to compare the expected profits.

The results are in figures 1-3. In each row of panels, we keep fixed the two possible values of constant elasticities (η_L and η_H) and the signals' accuracy (σ) and let the marginal cost vary. In each row, in the left panel expected profits ($\Pi_{n,s}$ and Π_s) are compared, while in the right panel, consumer surplus in both cases are drawn.

Under our parameters' settings condition (7) is always fulfilled, so we have that $q_H < q_L$ and $q_{HH} < q_{LH} < q_{LL}$. Looking at the pictures we can see that expected expected profits diminish by increasing the marginal cost (this is a just a confirm of eq. (6) and (9)).

Apart from that confirmations of already known features of the model, the first and most important numerical evidence lies in the left panels of the each row, and it can be formalized as follows:

Numerical Evidence 1: *if the marginal cost is low enough then the firms' expected profit at the equilibrium are higher if they don't share their private informations. At the opposite, if marginal costs are high enough, firms found convenience in sharing information.*

The threshold value of the marginal cost is denoted by \tilde{c} .

Differently from an analytical result, our numerical evidence does not exclude that we could find parameters' combinations that contradict such a result. We stress that we performed a lot of numerical investigations with a lot of parameters' combinations, so if such contradiction exists, it should be present in a quite minimal region of the parameters' space.

This result can only be compared with the result of Kirby (1988), who analyze a Cournot oligopoly with linear demand and quadratic costs. He find that information sharing can be the most profitable choice wiewh the quadratic parameter of the cost function is large enough. Our result goes in the same direction but we don't need to assume a quadratic cost function. Like Kirby, we can interpret such a result by stating that when marginal costs are high, also the costs of an erroneous production are high, making more valuable sharing information. Figs. 2 and 3 are obtained with higher values of the signal's accuracy ($\sigma = 0.7$ and 0.9 , respectively) and their panels (c) confirm this result.

By comparing the three sets of panels within each figure (obtained with different values of the lower possible elasticity parameter η_L), we reach a second numerical result:

Numerical Evidence 2: *the value of \tilde{c} decreases by reducing the difference between the two possible values of the demand elasticity.*

In fact, in all the figures, the lower value of \tilde{c} is obtained with $\eta_L = 0.7$ (remember that η_H is mantained fixed at 0.9). So the smaller is the difference between the two possible elasticity values, the more is convenient for the firms to collaborate and share their private informations. We can look at the same result from a different perspective and we can interpret it as a relation between the average expected elasticity and the profitability of the information sharing. From such a point of view we can state that with lower expected elasticity the range of marginal costs favouring collaboration is larger.

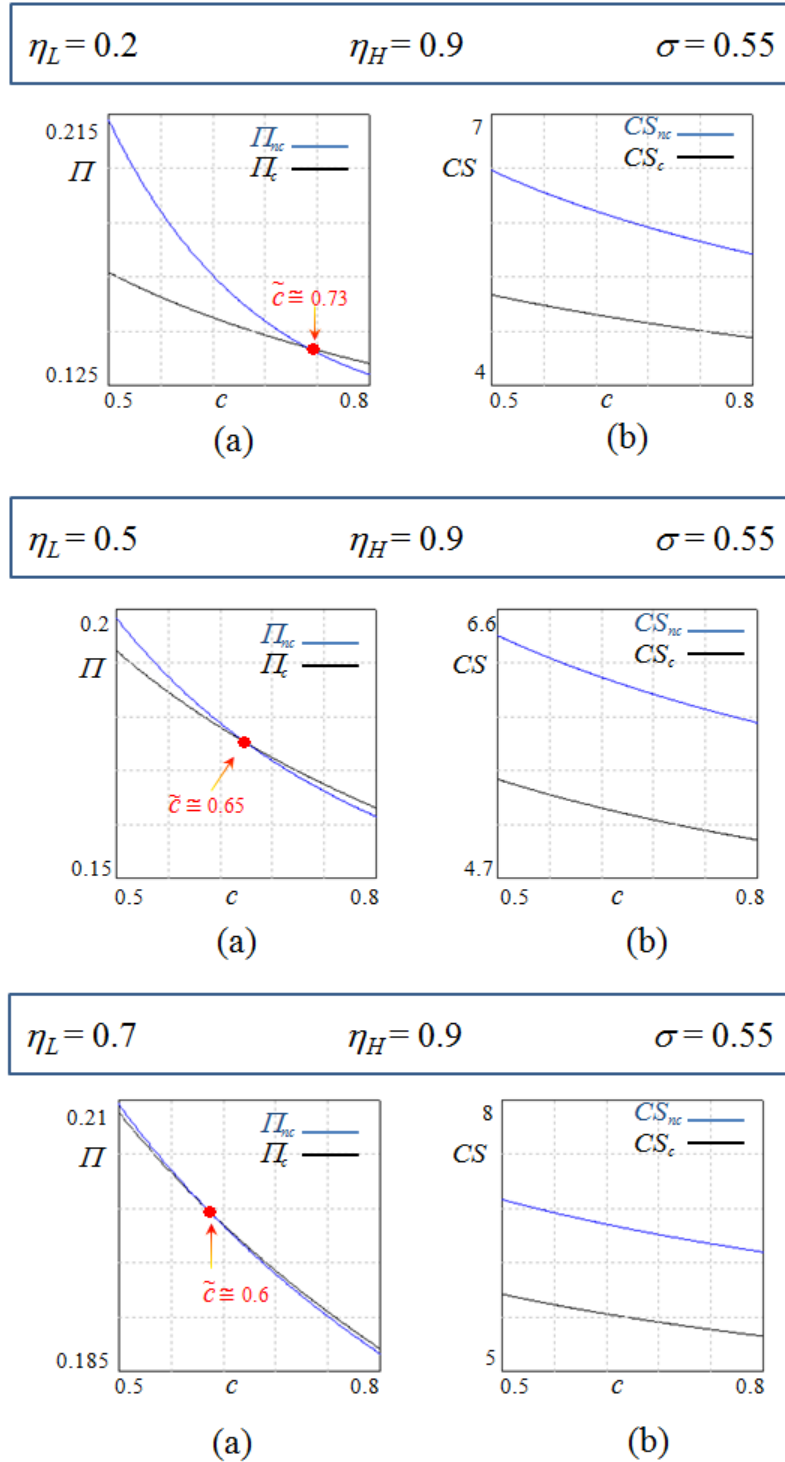


Figure 1:
 Expected profits and Consumer Surplus obtained by keeping fixed $\sigma = 0.55$ and $\eta_H = 0.9$. The panels of the first row are obtained with $\eta_L = 0.2$, those of the second row with $\eta_L = 0.5$, while in the third row $\eta_L = 0.7$.

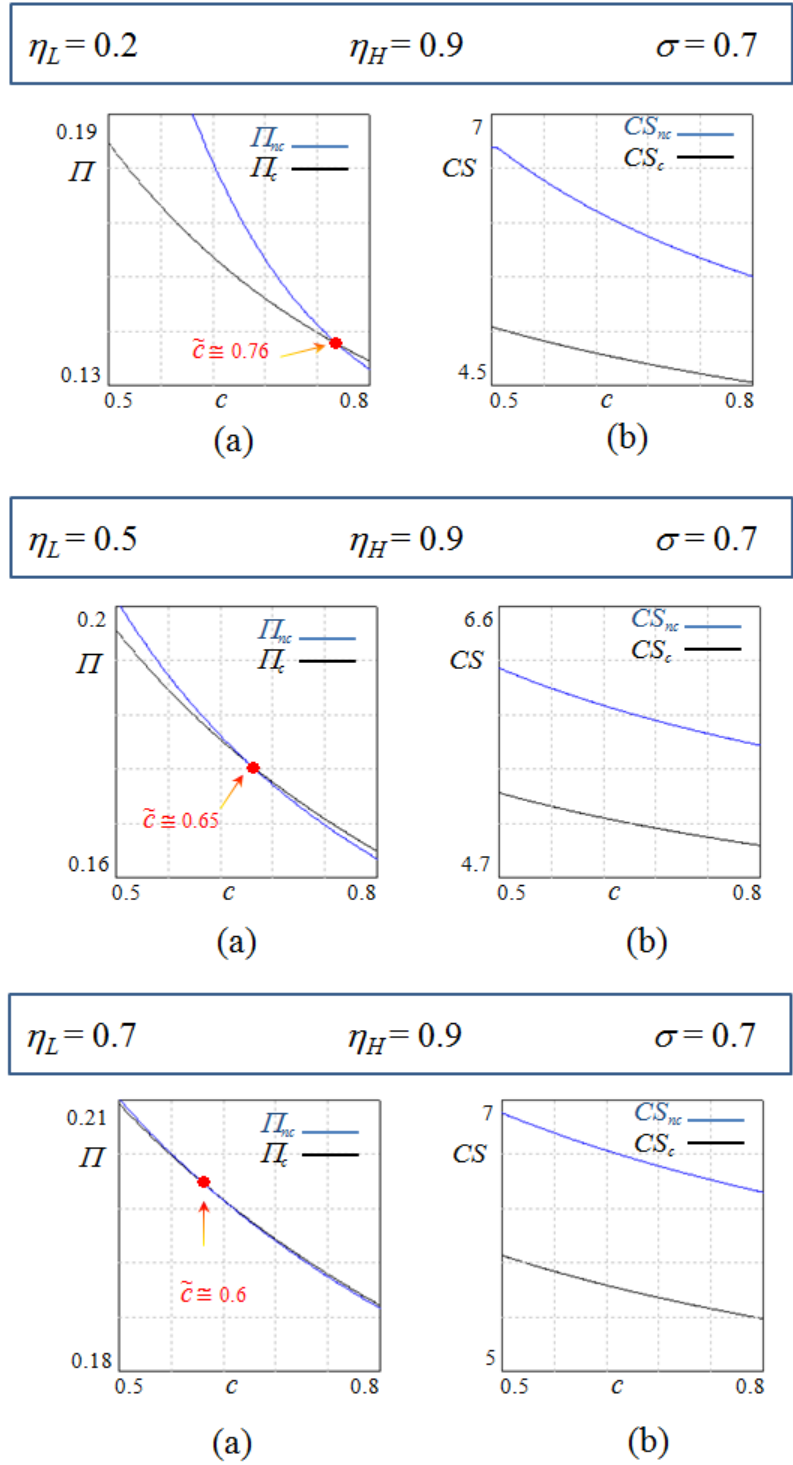


Figure 2:
 Expected profits and Consumer Surplus obtained by keeping fixed $\sigma = 0.7$ and $\eta_H = 0.9$. The panels of the first row are obtained with $\eta_L = 0.2$, those of the second row with $\eta_L = 0.5$, while in the third row $\eta_L = 0.7$.

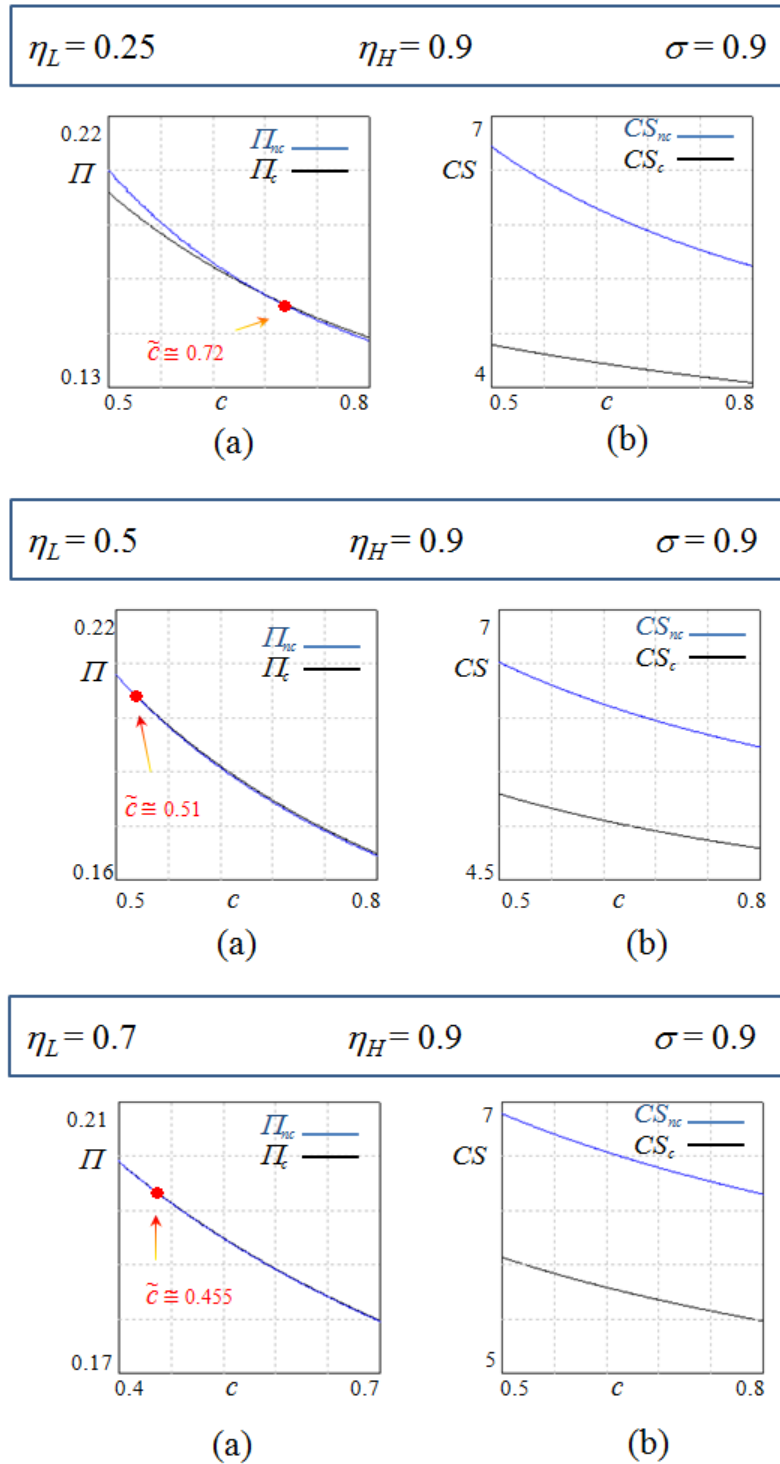


Figure 3:

Expected profits and Consumer Surplus obtained by keeping fixed $\sigma = 0.9$ and $\eta_H = 0.9$. The panels of the first row are obtained with $\eta_L = 0.25$, those of the second row with $\eta_L = 0.5$, while in the third row $\eta_L = 0.7$.

Let us now try to say something about the role of the signal's accuracy. In order to do that we have numerically identified the threshold values of the marginal cost (\tilde{c}) in several cases. Keeping fixed again the higher possible value of the demand elasticity parameter at 0.9, we let the values of σ and η_L vary and the results are summarized in the following table:

		η_L					
		\tilde{c}	0.2	0.26	0.35	0.5	0.7
σ	0.55		0.728	0.712	0.689	0.65	0.6
	0.63		0.744	0.723	0.696	0.652	0.605
	0.7		0.7635	0.738	0.698	0.6515	0.6
	0.8		0.787	0.740	0.691	0.63	0.583
	0.85		0.784	0.734	0.662	0.583	0.534
	0.9		0.72	0.707	0.617	0.5065	0.455

Table 1

One further result is thus the following:

Numerical Evidence 3: *by taking a value of the signal's accuracy (σ) close to 0.5 and starting increasing it without moving the other parameters, initially the threshold value \tilde{c} increases, it reaches a maximum and then it decreases faster and faster.*

A good way to graphically represents this result is by using a concave function in the (σ, \tilde{c}) parameter plane. We have isolated three cases, showed in Fig. 4.

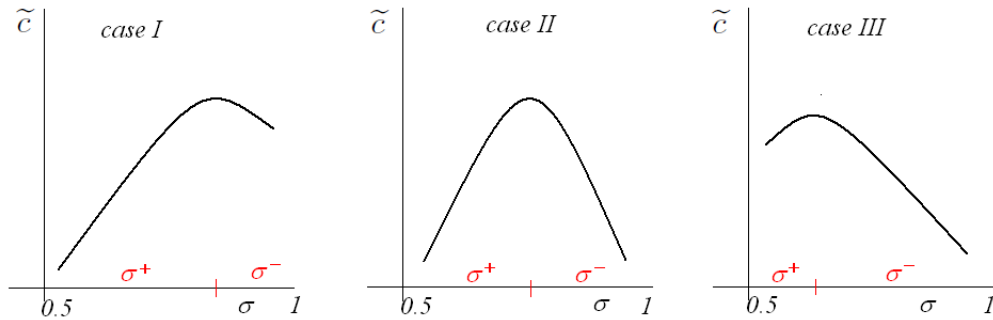


Figure 4:

Possible relations between \tilde{c} and σ .

If we label σ^+ (resp. σ^-) the interval of accuracy's values denoting a positive (resp. negative) correlation between \tilde{c} and σ , then the three cases can be distinguished by the length of the two intervals. In Case I, σ^+ is larger than

σ^- and this means that only for values of the accuracy close to 1 (i.e. perfect signal) a better signal implies a lower value \tilde{c} . This case is numerically verified with low values of η_L . For intermediate values of η_L , Case II holds. That is the sets σ^+ and σ^- have almost the same length. Finally, when η_L is quite close to η_H , then the role of σ is better explained by Case III, where the length of σ^- is quite larger than the length of σ^+ .

Unfortunately, an economic explanation of these results seems to be not trivial at all.

Finally, the panels displaying Consumers Surplus permit us to obtain a further result concerning welfare:

Numerical Evidence 4: *information sharing drastically reduces Consumer Surplus with respect to the case of no collaboration between firms and this holds for total welfare, too.*

In fact, the level of profits are negligible with respect to the level of Consumer Surplus (see the vertical axis of c and d panels) and the differences in Consumer Surplus related to the decision of sharing information or not are much relevant for the total welfare with respect to the differences in profits.

This result is opposite with respect to the corresponding results already existing in the literature. We think that this result is related to the particular shape of our demand function, characterized by an extremely large consumer surplus compared to the size of profits.

4. Concluding remarks

We consider a Cournot duopoly and study the consequences for information sharing of uncertainty related to a particular case of nonlinear demand function: the isoelastic case. Despite the limits in the analytical tractability of the model, we numerically explore the most of the parameters' plane and we are able to obtain some general result. It is our firm belief that if we want to study the consequences of nonlinearities in this and other research frameworks, we must look at numeric results as an important source of information. Specifically, we show that high marginal costs and low variance in the uncertain elasticity parameter, favour a sharing of private information. From the point of view of welfare, information sharing reduces the consumer surplus in a way that it is always a better scenario the one in which firms do not collaborate.

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