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# **Learning the Fiscal-Monetary Interaction under Trend Inflation**

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# Learning the Fiscal-Monetary Interaction under Trend Inflation

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## Abstract

What are the effects of a higher inflation target on the determinacy properties under alternative monetary/fiscal policy mixes in New Keynesian models? Would it be more difficult for the central bank to stabilize inflation expectations if the inflation target is raised? What role for central bank transparency? We find that trend inflation does not affect determinacy as long as monetary policy is passive. Conversely, an active central bank should fight inflation more strongly with higher trend inflation, in order to guarantee the determinacy of the AM/PF equilibrium. Furthermore, this equilibrium, if determinate, is always E-stable under transparency. In the AF/PM case the equilibrium is always determinate and E-stable under both transparency and opacity. We find the degree of price stickiness to be a crucial structural parameter. In particular, in a low price rigidity country, say the United States, adhering to the Taylor principle is a sufficient condition for equilibrium determinacy under the AM/PF regime, irrespective of the level of trend inflation. Still, the central bank must be transparent to stabilize expectations. On the contrary, in a high price rigidity economy, say Europe, to have determinacy under AM/PF mix, the inflation target cannot be larger than 2% but the central bank needs not to be transparent to stabilize expectations. Furthermore, high rigidity makes non-Ricardian policies less E-stable.

*JEL classification:* E5. *Keywords:* Trend Inflation, Learning, Monetary Policy, Transparency.

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# 1 Introduction

The bulk of the recent theoretical literature on monetary policy considers models where governments are ready to passively change taxes to cover public debt. Assuming an always satisfied government budget constraint is equivalent to leaving fiscal policy in the background. The Great Recession, with the joint stimulus of aggregate demand by both monetary and fiscal policies and the coordination problems among the two, has renewed attention on the “fiscal theory of the price level”, initiated by Sargent and Wallace (1981), Sims (1994), Leeper (1991) and Woodford (1996). This literature brings fiscal policy on the foreground together with monetary policy since it finds that the determination of the price level cannot abstract from their interactions.<sup>1</sup> In particular, the price level can be pinned down under rational expectations only in two different cases. In the first, the central bank implements a policy that complies with the Taylor principle while the government simply limits itself to satisfy its budget constraint - this situation has been dubbed as the active-monetary/passive-fiscal (AM/PF) case by Leeper (1991) or as the Ricardian case by Woodford (1996). In the second case, the government independently decides the level of the budget surplus (or deficit) while the central bank is required to adjust monetary policy in order to satisfy the government budget constraint through price level changes - the so-called active-fiscal/passive-monetary (PM/AF) or non-Ricardian case. The most recent literature on the argument (see, e.g., Bianchi 2012, Davig and Leeper 2011) employs Markov switching models to account for regime shifts in policy rules that take place through the years in a given country: next to the conventional case where an active monetary rule is associated to a passive rule, estimates document periods of passive monetary and active fiscal rules but even of double active or double passive policies.

This paper examines how the equilibrium properties of a New Keynesian model with monetary-fiscal interactions are modified when trend inflation is accounted for. Studying whether and how a variation in the inflation target affects the equilibrium properties of the model is particularly important in light of the recent proposals (Rogoff, 2008; Blanchard et al., 2010; Ball, 2013) to increase the inflation target in order to exit from the zero lower bound region where monetary policy is ineffective. What are the effects of

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<sup>1</sup>For a review of the existing literature on this issue see Canzoneri et al. (2011).

a higher inflation target on the determinacy properties under alternative monetary/fiscal policy mixes in New Keynesian models? Would it be more difficult for the central bank to still retain a tight grasp of inflation expectations once the inflation target is raised? What role for central bank transparency? To provide an answer to these questions we study, beyond equilibria determinacy, even expectational stability (E-stability). In order to do so, we need to drop the assumption of rational expectation and let private agents form their forecasts according to a recursive learning rule *à la* Evans and Honkapohja (2001). The analysis is undertaken assuming both an opaque central bank that does not communicate its policy to private agents and a transparent one, in light of the work of Eusepi and Preston (2010). Unlike the most recent literature, we do not explicitly model regime changes but, rather, we set a best-case scenario with a stationary model environment: if agents result unable to learn under this static context they, *a fortiori*, will find it more difficult to learn with policy regime shifts.

The rest of the paper is organized as follows. After having devoted the next subsection to highlight similarities and differences from related literature, Section 2 presents the model and the methodology. Section 3 contains the determinacy results under zero (3.1) and positive trend inflation (3.2), the learning analysis (3.3) and the impulse response functions derived from the model (3.4). Section 4 provides some robustness checks and Section 5 concludes.

## 1.1 Related literature

This paper can be considered as an extension of Ascari et al. (2012) to a setting with an explicit role for fiscal policy.<sup>2</sup> Beside studying determinacy and learning under opacity and transparency in a New Keynesian model with trend inflation, we also provide an analysis of the impulse response function of the model. This should be of interest from a double perspective. First, it shows if and how the transmission mechanism of economic shocks changes when alternative policy mixes are in place. Second, it reveals how high levels of trend inflation affect the results.

In order to better organize the exposition we divide the related literature distinguishing among papers that deal, respectively, with determinacy, learning and impulse

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<sup>2</sup>In this section we will just report the literature dealing with monetary-fiscal interactions. We refer the interested reader to Ascari et al. (2012) for a more general review of the literature on learning and trend inflation.

response analyses under monetary-fiscal interactions.

**Determinacy.** The pivotal paper studying determinacy under both active and passive monetary and fiscal rules is Leeper (1991). The author, employing a flexible price model with contemporaneous policy rules and lump-sum taxes, shows that there is a unique stable equilibrium (determinacy) only if the fiscal and monetary authorities coordinate their policies (AM/PF or PM/AF). On the other hand, a lack of coordination returns multiple equilibria (indeterminacy) in case of a double passive mix or lack of equilibrium (explosiveness) with a double active mix. Branch et al. (2008) extend the paper by Leeper (1991) to include determinacy analysis for different monetary policy rules under the same (flexible price) model. A work closer to ours is instead Rossi (2009) who studies the equilibrium determinacy of a New Keynesian model with trend inflation and public debt. However, Rossi does not account for non-rational expectations and employs distortionary taxation, while our study deals with learning and maintains Leeper's (1991) assumption of lump-sum taxes.

**Learning.** Evans and Honkapohja (2007) study the learning proprieties of Leeper's (1991) flexible price model by finding that determinate equilibria are also E-stable. Our analysis, as for learning is concerned, is an extension of theirs along three dimensions: we consider a model with price rigidity, we include trend inflation and we look at expectational stability under both monetary policy transparency and opacity. Apart from the inclusion of trend inflation, our model is close to Eusepi and Preston (2012) who study learning dynamics under uncertainty about monetary and fiscal conditions and find that, in this case, stabilization policies are more difficult than under rational expectations. To enlarge the set of policies consistent with E-stability, the expectations about monetary and fiscal policies should be anchored through transparency. Our paper abstracts from transparency considerations on fiscal policy though we consider it a fruitful point for future research.

**Impulse response analysis.** Canzoneri et al. (2011), following Kim (2003), use impulse response functions to show the different effects of policy innovations according to the monetary/fiscal regime in place in a cash and credit good model.<sup>3</sup> While in the Ricardian case shocks have the conventional effects, in the non-Ricardian one the presence of wealth effects modifies the impulse responses. By introducing trend inflation

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<sup>3</sup>Kim (2003) employs a money-in-utility model.

we can show that the impulse responses of our model change not only depending on the policy mix but also on the level of the inflation target.

## 2 Model and methodology

### 2.1 The Model

The model we use is based on Ascari and Ropele (2009), that extends the basic New Keynesian (NK) framework (e.g., Galí, 2008, and Woodford, 2003) to allow for positive trend inflation. Fiscal policy is added following Bhattarai et al. (2013) introducing a simple backward looking fiscal rule with lump-sum taxes and one period government bonds. Details are presented in Appendix A1. Log-linearizing the model around a generic positive inflation steady state yields the following equations:

$$\hat{y}_t = E_{t-1}^* \hat{y}_{t+1} - E_{t-1}^* (\hat{R}_t - \hat{\pi}_{t+1}), \quad (1)$$

$$\hat{\pi}_t = \beta \bar{\pi} E_{t-1}^* \hat{\pi}_{t+1} + \lambda_{\bar{\pi}} E_{t-1}^* [(1 + \sigma_n) \hat{y}_t + \sigma_n \hat{s}_t] + \eta_{\bar{\pi}} E_{t-1}^* [(\theta - 1) \hat{\pi}_{t+1} + \hat{\phi}_{t+1}], \quad (2)$$

$$\hat{\phi}_t = \alpha \beta \bar{\pi}^{(\theta-1)} E_{t-1}^* [(\theta - 1) \hat{\pi}_{t+1} + \hat{\phi}_{t+1}], \quad (3)$$

$$\hat{s}_t = \xi_{\bar{\pi}} \hat{\pi}_t + \alpha \bar{\pi}^{\theta} \hat{s}_{t-1}, \quad (4)$$

$$\hat{R}_t = \phi_{\pi} E_{t-1}^* \hat{\pi}_t + \varepsilon_{m,t}, \quad (5)$$

$$\hat{\tau}_t = \gamma \hat{b}_{t-1} + \varepsilon_{\tau,t}, \quad (6)$$

$$\hat{b}_t = \beta^{-1} \hat{b}_{t-1} + \bar{b} \beta^{-1} (\hat{R}_{t-1} - \hat{\pi}_t) - \hat{\tau}_t. \quad (7)$$

Hatted variables denote percentage deviations from steady state<sup>4</sup>, apart from  $\hat{y}$ , which is the usual New Keynesian output gap defined as deviation from the flexible-price output level. The structural parameters and their convolutions ( $\lambda_{\bar{\pi}}$ ,  $\eta_{\bar{\pi}}$  and  $\xi_{\bar{\pi}}$ ) are described in Table 2.  $\varepsilon_{m,t}$  and  $\varepsilon_{f,t}$  are the monetary and fiscal policy shocks that follow the autoregressive processes  $\varepsilon_{m,t} = \rho_m \varepsilon_{m,t-1} + u_{m,t}$  and  $\varepsilon_{f,t} = \rho_f \varepsilon_{f,t-1} + u_{f,t}$ , where  $u_{m,t}$  and  $u_{f,t}$  are i.i.d noises and  $0 < \rho_m, \rho_f < 1$ .

Equation (1) is the standard Euler equation for consumption. Equations (2) and (3) describe the evolution of the inflation rate in presence of trend inflation.  $\hat{\phi}_t$  is just

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<sup>4</sup>We use the definitions  $\hat{b}_t = \frac{b_t - b^*}{y^*}$  and  $\hat{\tau}_t = \frac{\tau_t - \tau^*}{y^*}$ , where we indicate the steady state values with a star.

an auxiliary variable (equal to the present discounted value of future expected marginal revenue) that allows writing the model in recursive way. Equation (4) describes the evolution of price dispersion,  $s_t$ . In contrast to the zero inflation steady state case, in presence of positive average inflation price dispersion affects inflation dynamics at first-order approximation and thus has to be taken into account. Equations (2)-(4) are the counterparts of the New Keynesian Phillips curve for the standard zero inflation steady state case. Equation (5) is the simplest standard contemporaneous Taylor rule. Equation (6) is the fiscal policy rule that sets taxes according to outstanding real debt, while equation (7) is the flow budget constraint of the government (where  $\bar{b}$  is the steady state debt over GDP ratio). By plugging the fiscal rule into the budget constraint we obtain

$$\hat{b}_t = (\beta^{-1} - \gamma)\hat{b}_{t-1} + \bar{b}\beta^{-1}(\hat{R}_{t-1} - \hat{\pi}_t).$$

We deviate from Ascari and Ropele (2009), by following Evans and Honkapohja (2001) and much of the related literature on learning, by assuming that agents have non-rational expectations, that we denote with  $E^*$ . Furthermore, we assume that expectations are formed on the basis of period  $t - 1$  information set (see also Bullard and Mitra, 2002). According to Evans and Honkapohja (2001), this assumption is more natural in a learning context, since it avoids simultaneity between expectations and current values of endogenous variables.

## 2.2 Methodology

We are interested in analyzing both determinacy and learnability conditions. As for determinacy, we refer the reader to Sections 3.1 and 3.2 where we derive analytically the determinacy conditions, respectively, under a zero inflation target and under positive trend inflation (for the case with indivisible labor and no price indexation  $\sigma_n = \varepsilon = 0$ ). In case of positive trend inflation, we provide additional numerical simulations to investigate the relevance of our analytical results. Section 3.3 deals, instead, with numerical simulation under learning.

### 2.2.1 Learnability

When agents do not possess rational expectations, the existence of a determinate equilibrium does not ensure that agents coordinate upon it. As from the seminal contribution of Evans and Honkapohja (2001), we assume agents do not know the true structure of the economy. Rather, they behave as econometricians and learn adaptively, using a recursive least square algorithm based on the data produced by the economy itself. If the rational expectation equilibrium is learnable, then, the learning dynamics eventually tend toward, and eventually coincide with, the rational expectations equilibrium. Learnability is an obviously desired feature of monetary policy.

We apply E-stability results outlined in Evans and Honkapohja (2001, section 10.2.1). Agents are assumed to have identical beliefs and to forecast using variables that appear in the minimal state variable (MSV) solution of the system under rational expectations. The form of agents' perceived law of motion (PLM) coincides with the system MSV solution. Given our model, thus, the PLM will not contain any constant term. Agents are assumed to know just the autocorrelation of the shocks but they have to estimate the remaining parameters. Each period, as additional data become available, they re-estimate the coefficients of their model. We then ask whether agents are able to learn the MSV equilibrium of the system (see Appendix A2 for details).

### 2.2.2 Transparency versus opacity

In defining opacity (OP) and transparency (TR) of monetary policy, we follow closely the work of Preston (2005) and Eusepi and Preston (2010). We assume that the central bank is perfectly credible: the public believes and fully incorporates central bank's announcements. Agents are uncertain about the economy ( $\hat{\pi}$ ,  $\hat{y}$  and  $\hat{b}$ ) and about the path of nominal interest rates ( $\hat{R}$ ). Communication by the central bank simplifies agents' problem in that it gives them information on how the monetary authority sets interest rates, that is, on the monetary policy strategy. Therefore: (i) under OP, the private sector has to make learning about the economy and about monetary policy; under TR, it needs to forecast just the economy but not the path of nominal interest rates, since the central bank announces its precise reaction function.

In case of TR, we incorporate the reaction function directly in the aggregate demand equation and the agents' problem boils down to forecast inflation, output and debt. This,



as we will show, should be of help in anchoring expectations by aligning agents' beliefs with central bank's monetary policy strategy.

### 3 Results

This section presents the main results of the paper. We first consider the standard case of a zero inflation target (i.e., zero inflation steady state), for which analytical results are presented. We then move to the more general and realistic case of a positive inflation target for which some analytical conditions and numerical simulations are provided.

We follow Leeper's (1991) definition of active/passive monetary and fiscal policy: a policy authority has an "active" behaviour when it pursues its objective unconstrained by the actions of the other authority; instead, if the authority is constrained, its behaviour is "passive". Table 1 shows the possible policy mixes that can take place in our model for different values of the parameters in the policy functions. Note that active fiscal policy corresponds to two different areas depending on the value of  $\gamma$ , respectively  $\gamma < (1/\beta - 1)$  (AF<sub>down</sub> region) and  $\gamma > (1/\beta + 1)$  (AF<sup>up</sup> region). Previous studies concentrate only on the AF<sub>down</sub> region, where the additional tax revenue generated by a small increase in the level of debt is less than the increase in interest rates payments. The other region is disregarded, as the values of  $\gamma$  appear non reasonable. For  $\gamma > (1/\beta + 1)$ , in fact, a shock raising debt would make taxes increase even beyond the amount required to pay off the debt (including interest rates). In what follows, as an intellectual curiosity, we show results even for this second area.

Table 1: Monetary/fiscal policy mixes for different values of  $\phi_\pi$  and  $\gamma$

	$0 < \phi_\pi < 1$	$\phi_\pi > 1$
$\gamma > \frac{1}{\beta} + 1$	PM/AF <sup>up</sup>	AM/AF <sup>up</sup>
$\frac{1}{\beta} - 1 < \gamma < \frac{1}{\beta} + 1$	PM/PF	AM/PF
$\gamma < \frac{1}{\beta} - 1$	PM/AF <sub>down</sub>	AM/AF <sub>down</sub>

#### 3.1 Determinacy under zero-inflation target

Consider equations (2)-(4). After assuming zero trend inflation ( $\bar{\pi} = 1$ ), one obtains  $\eta_{\bar{\pi}} = \xi_{\bar{\pi}} = 0$ , thus both the auxiliary variable and the measure of relative price dispersion

become irrelevant for inflation dynamics and the three equations collapse into

$$\hat{\pi}_t = \beta E_{t-1}^* \hat{\pi}_{t+1} + \kappa E_{t-1}^* \hat{y}_t, \quad (8)$$

where  $\kappa = \lambda(1 + \sigma_n)$ . Therefore, the model reduces to a three-equation dynamic system in the variables  $\hat{y}_t$ ,  $\hat{\pi}_t$ , and  $\hat{b}_t$ . The Blanchard-Kahn (1980) conditions for determinacy of the REE depend, beside the Taylor principle, on the fiscal policy implemented by the government. The following two propositions report the binding condition for determinacy corresponding to the cases of passive and active fiscal policy (proofs can be found in Appendix A3).

**Proposition 3.1** *In case of passive fiscal policy, i.e.  $|1/\beta - \gamma| < 1$ , there is determinacy if:*

$$\phi_\pi > 1, \quad (9)$$

*i.e. the Taylor principle must hold and an active monetary policy is needed.*

**Proposition 3.2** *In case of active fiscal policy, i.e.  $|1/\beta - \gamma| > 1$ , determinacy holds if*

$$\phi_\pi < 1.$$

*That is, monetary policy must be passive.*

Summarizing, the determinacy properties of our model trace the original findings obtained by Leeper (1991) in his flexible price model. The equilibrium is determinate under both AM/PF and PM/AF, indeterminate under PM/PF and explosive under AM/AF.

## 3.2 Determinacy under positive inflation

We now study how a positive trend inflation modifies the determinacy regions. In order to compare our results with the existing literature, we discuss the flexible price and sticky price cases separately.

### 3.2.1 Flexible prices

Following the pivotal article by Leeper (1991) that adopts a representative agent, flexible price, endowment economy, we check what happens to determinacy when trend inflation

rises in absence of price rigidity ( $\alpha = 0$ ). Figure 1 illustrates the determinacy regions for different values of the parameters of the monetary ( $\phi_\pi$ ) and fiscal ( $\gamma$ ) policy rules.<sup>5</sup> The white area indicates determinacy, the dark grey area instability, the light grey area explosiveness. It is important to note that the areas are identical to the zero inflation case and do not change as the inflation target increases. We can therefore conclude that:

**Result 1.1** *Under flexible prices trend inflation does not affect determinacy.*

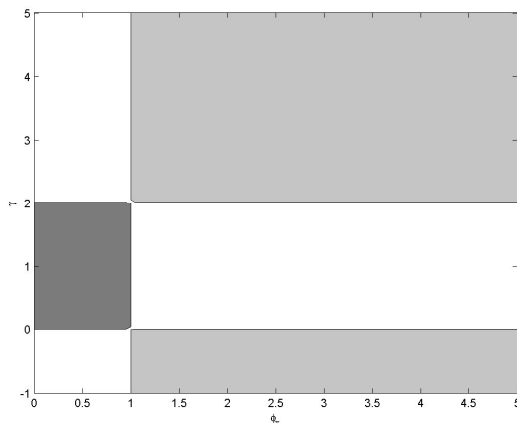


Figure 1. Determinacy for every trend inflation

Indeterminacy: dark grey area; instability: light grey area; determinacy: white area.

### 3.2.2 Sticky prices

We now return to our sticky price model with trend inflation (proofs can be found in Appendix A4). In order to get analytical conditions for determinacy, throughout this section we set  $\sigma_n = 0$  (i.e. indivisible labor) and  $\varepsilon = 0$  (no price indexation).

**Proposition 3.3** *Determinacy of the REE under positive trend inflation and passive fiscal policy obtains if and only if*

$$\phi_\pi > \max(1, z(\bar{\pi})).$$

$z(\bar{\pi})$  is the largest root of a second degree inequality for  $\phi_\pi$  (equation 26 in the Appendix). We adopt a numerical approach to compute the two roots of such equation for different values of  $\bar{\pi}$ : we find both roots to be real and of opposite sign, so we label the largest root  $z(\bar{\pi})$ . Therefore the binding condition for determinacy is  $\phi_\pi > \max(1, z(\bar{\pi}))$ .

<sup>5</sup>In the numerical simulation of the paper, unless otherwise stated, we use rather standard parameter values reported in Table 2.

The function  $z(\bar{\pi})$  is shown in Figure 2 and is computed by fixing all parameters other than  $\bar{\pi}$  to their benchmark calibrated values. Importantly, our simulations show that  $\partial z(\bar{\pi})/\partial \bar{\pi} > 0$  and that  $z(\bar{\pi}) > 1$  when annualized trend inflation increases above 2.3%.

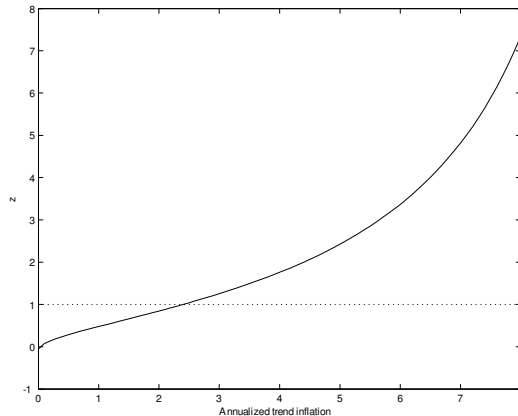


Figure 2. Determinacy for every trend inflation

**Proposition 3.4** *Under active fiscal policy, determinacy is obtained if and only if*

$$\phi_{\pi} < 1.$$

To investigate the relevance of these analytical results we revert to numerical simulations. Figure 3 illustrates the determinacy region in the plane  $(\phi_{\pi}, \gamma)$  for different levels of annualised trend inflation under a contemporaneous rule with  $\alpha = 0.75$  and  $\sigma_n = 1$ .<sup>6</sup>

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<sup>6</sup>For the sake of brevity, we go straight to the more interesting case  $\sigma_n = 1$ ; however, the case  $\sigma_n = 0$  gives comparable results and it is available from the authors.

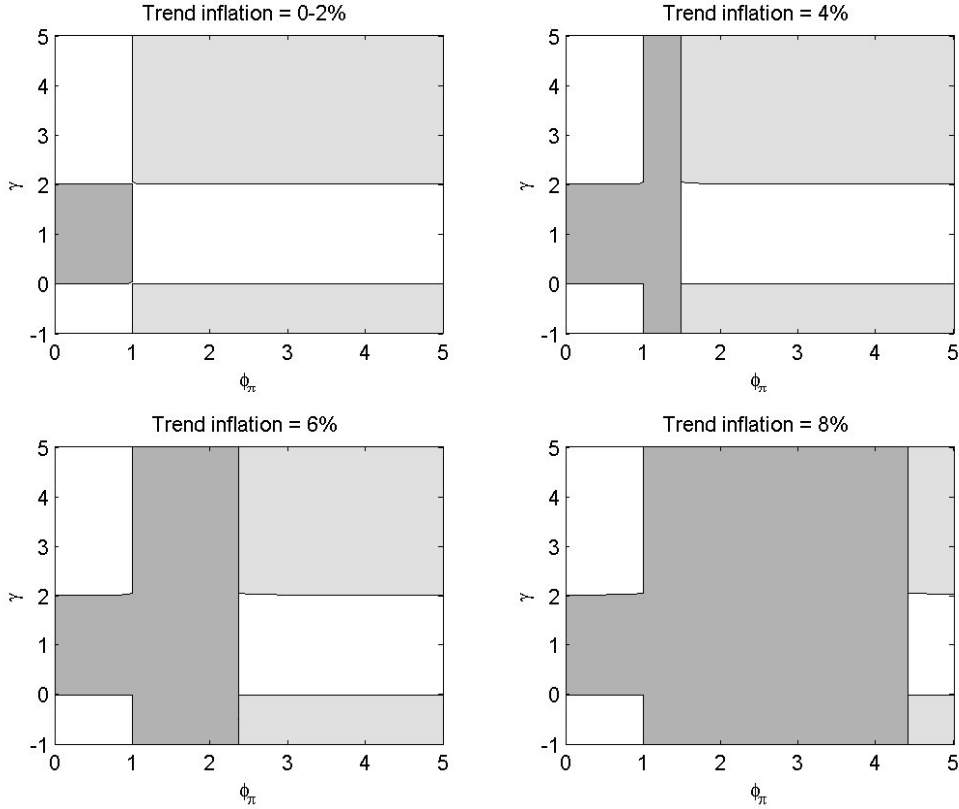


Figure 3. Determinacy under positive trend inflation

Indeterminacy: dark grey area; instability: light grey area; determinacy: white area.

**Result 1.2** *Trend inflation does not affect determinacy in the PM case (that is in the  $\phi_\pi < 1$  region).*<sup>7</sup>

In particular, when a PM/AF regime is in place, there are two determinate areas: one characterized by low taxes, the other by high taxes. Following an increase in inflation expectations, output and inflation, stimulated by a decline in real interest rates, increase. However, the increase in inflation changes the value of real debt and, thanks to wealth effect, drives inflation and thus the economy back to the steady state. Trend inflation does not change these determinacy conditions. Even the PM/PF area remains indeterminate as trend inflation increases.

**Result 1.3** *Trend inflation affects determinacy for AM cases.*

As can be gauged from the figure,  $z(\bar{\pi})$  corresponds to a vertical line to the right of  $\phi_\pi = 1$  for inflation targets greater than 2%. In particular, with trend inflation higher

<sup>7</sup>Rossi (2009) gets the same result employing a model with distortionary taxation and taxes levied on wages.

than that value, in order to have the AM/PF zone determinate, the central bank has to react more to inflation (i.e. increase  $\phi_\pi$ ).

Under zero trend inflation, in the AM/PF case, the equilibrium is determinate: the active central bank responds to, say, an increase in inflation expectations by increasing the real interest rate and this restrains aggregate demand and lowers inflation. If monetary policy is called “active” when the Taylor principle is satisfied ( $\phi_\pi > 1$ ) then we have that, as trend inflation rises, active monetary policy does not guarantee determinacy *per se* (even in the presence of passive fiscal policy). We instead need monetary policy to be hawkish ( $\phi_\pi \gg 1$ ). As trend inflation increases, in fact, the output weight in the Phillips curve declines. Hence, in order to get the desired effect on inflation, the real rate has to increase by a greater extent ( $\phi_\pi \gg 1$ ). The explosive AM/AF area shrinks, as trend inflation increases, together with the determinacy area in the AM/PF case.

From a central bank’s perspective, as for determinacy is concerned, the level of inflation that returns the larger area for determinacy in the active monetary case would be around 0-2%.

### 3.3 Learning under trend inflation

The figures below report the values of the coefficients of monetary and fiscal policy that return E-stability (white area) under opacity (Figure 4) and transparency (Figure 5) for different values of trend inflation.

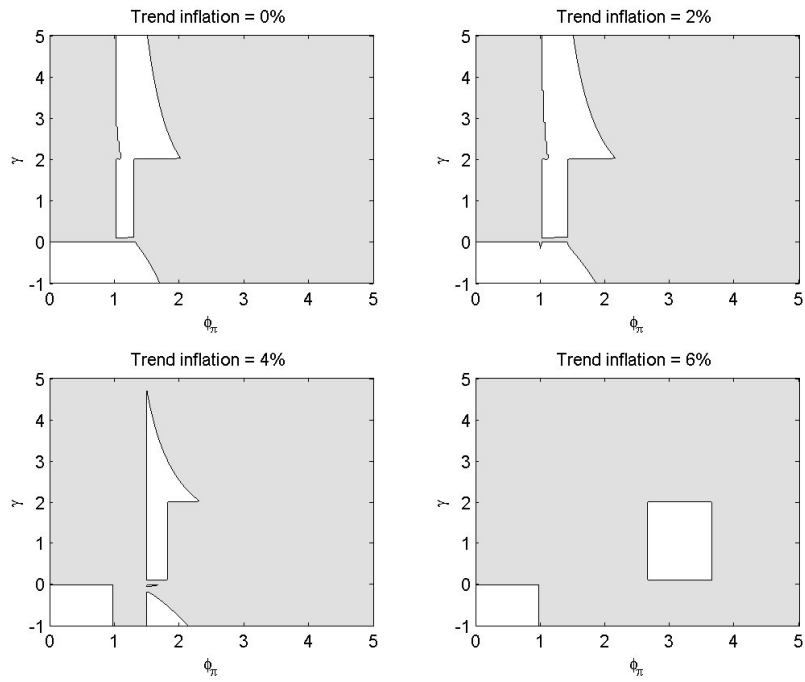


Figure 4. Learning under monetary policy opacity

White area: E-stability; grey area: E-instability.

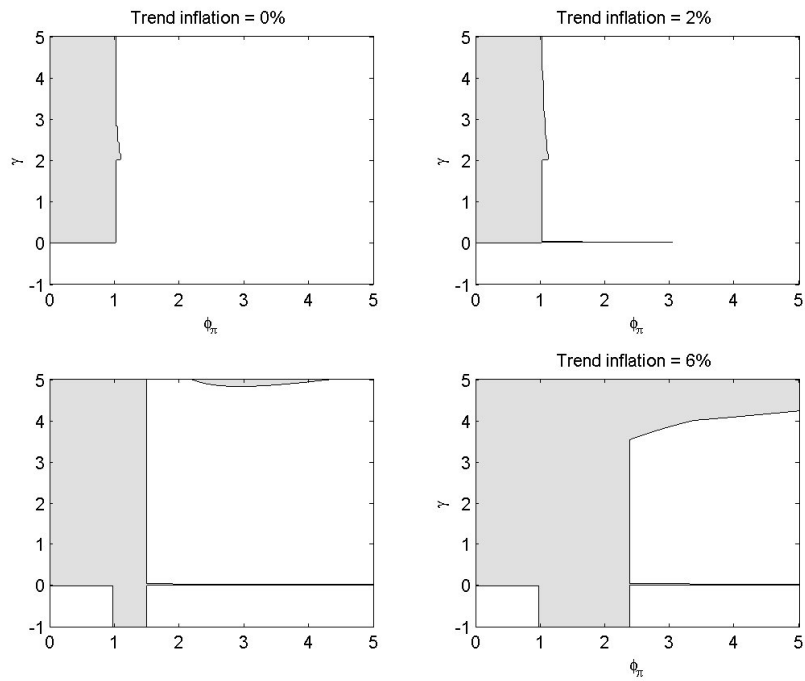


Figure 5. Learning under monetary policy transparency

White area: E-stability; grey area: E-instability

The case with zero trend inflation and monetary policy transparency returns results similar to those derived by Evans and Honkapohja (2007) employing the (flexible price) model by Leeper (1991): in particular, determinacy implies E-stability.<sup>8</sup>

At a first glance, E-stability results are not tightly linked to the determinacy ones. Different trend inflation can have the same determinacy zones but different E-stability (compare the cases with trend inflation equal to 0 and 2%).

The AM/PF regime, if determinate, is always E-stable under TR, not under OP. Expectational stability under OP requires a not too aggressive response by the central bank ( $\phi_\pi$  near one). The area with PM/AF<sup>up</sup> is always determinate but never E-stable (neither under TR nor under OP); the area PM/AF<sub>down</sub> is always determinate and always E-stable under both TR and OP. If a regime is indeterminate (see the PM/PF case) it is never E-stable; if it is explosive (see AM/AF) can instead be E-stable. In case of TR with AM/AF the explosive solution can be learnable. Therefore the monetary and fiscal authority need not to coordinate their policies if they want to implement a learnable equilibrium. In particular, in case of OP, the regime AM/AF<sup>up</sup> is E-stable just for low values of  $\phi_\pi$  but E-stability shrinks and gradually disappears as trend inflation rises. The same regime is E-stable under TR but, again, E-stability shrinks as trend inflation rises becoming more unstable for higher  $\gamma$ . AM/AF<sub>down</sub> (except near  $\gamma = 0$ ) is E-stable with TR, not with OP (except for  $\phi_\pi$  near one).

### 3.4 Impulse response functions

Figures 6 and 7 report impulse response of output, inflation and real debt to, respectively, monetary and fiscal shocks employing the usual calibration (see Table 2) on our model. Each figure is divided in three columns: one for the AM/PF case, one for the PM/AF<sup>up</sup>, the last for PM/AF<sub>down</sub>.<sup>9</sup> Furthermore, every single impulse response is depicted for different values of trend inflation. Again, our aim is to check if higher values of trend inflation change the responses to shocks when different policy mixes are in place.

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<sup>8</sup>Evans and Honkapohja (2007) do not consider the upper area of active fiscal policy.

<sup>9</sup>We restrict attention to the two determinate regimes since the double passive and the double active regimes imply, respectively, indeterminacy and no solution.



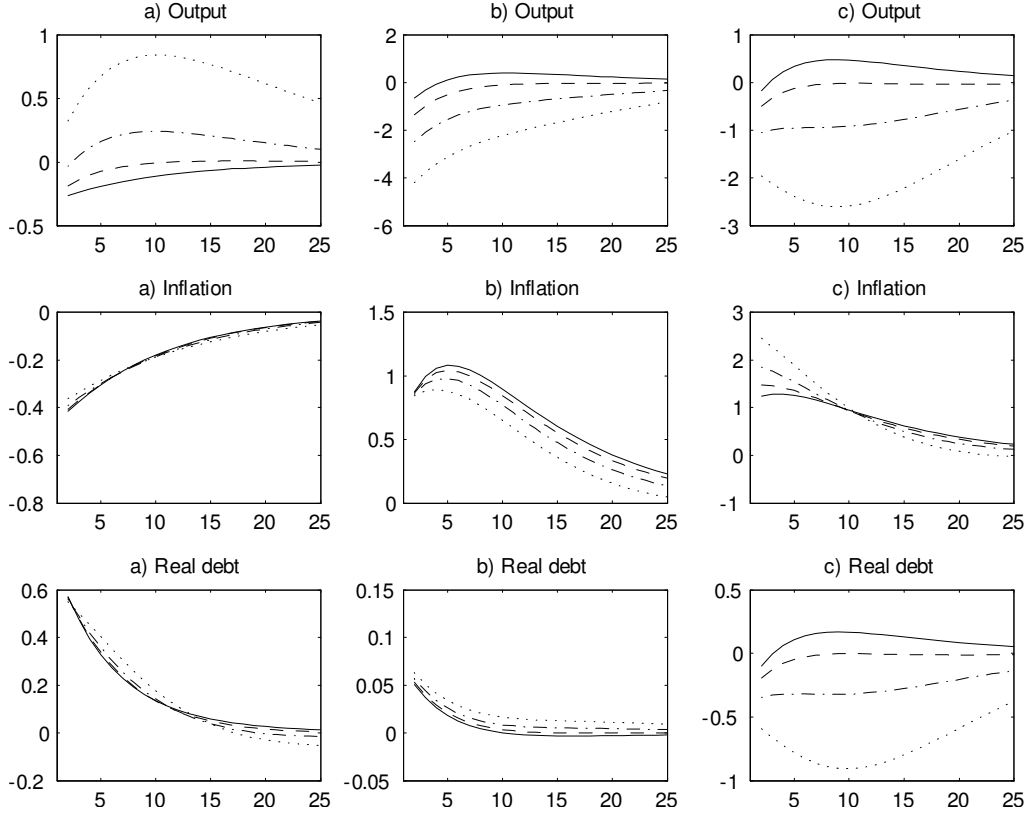


Figure 6. Impulse response to a contractionary, one standard deviation monetary policy shock

a) AM/PF ( $\phi_\pi = 3, \gamma = 0.2$ ); b) PM/AF<sup>up</sup> ( $\phi_\pi = 0.5, \gamma = 3$ ); c) PM/AF<sub>down</sub> ( $\phi_\pi = 0.5, \gamma = 0$ )

Trend inflation: solid line=0%, dashed=2%, dash-dotted=4%, dotted=6%.

Figure 6 shows the responses to a monetary policy tightening. In the AM/PF regime with low levels of trend inflation, both inflation and output decrease on impact, but the contractionary effect on both these variables weakens as trend inflation increases. Moreover, for high levels of trend inflation, not only output reacts less but it even increases on impact causing an *output puzzle*: this is related to the sign change in the Phillips curve slope described by Ascari and Ropele (2009). As already noted by Ascari and Ropele (2007), persistence of the impulse response for both output and inflation increases with trend inflation. In the PM/AF<sub>down</sub> case, with zero trend inflation, inflation and output rise. An interest rate increase raises the value of government debt to cover the larger interest rate expenses. Agents, in this non-Ricardian setup, perceive the higher debt as net wealth since they do not expect it to be backed up by future taxes hence increase spending and this, in turn, pushes up the price level causing a *price puzzle*. The

higher trend inflation, the more the price puzzle grows while output (through lower real interest rates) and real debt decrease. Conversely, in the PM/AF<sup>up</sup> zone, the (high) increase in the tax rate causes a negative wealth effect that leads output and inflation to decrease on impact. Lower inflation counteracts the effect of higher taxes making real debt increase. The higher trend inflation, the more output (through lower real interest rates) and inflation decrease bringing about a real debt increase.

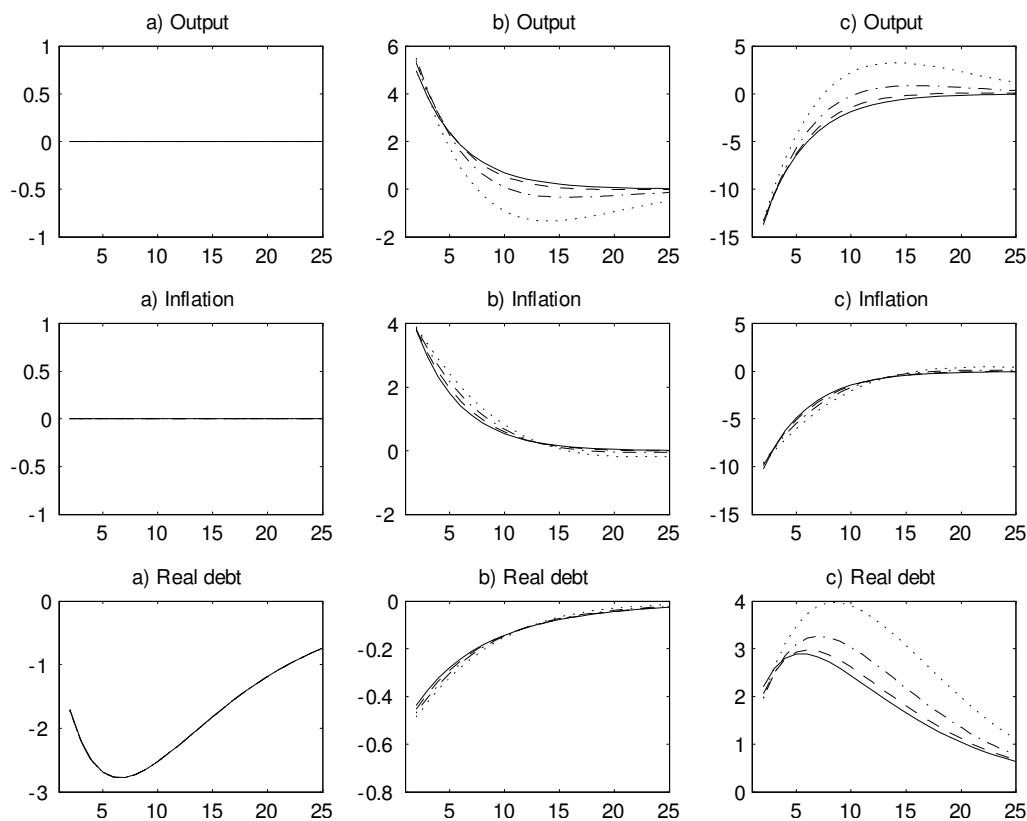


Figure 7. Impulse response to a contractionary, one standard deviation fiscal policy shock

a) AM/PF ( $\phi_\pi = 3$ ,  $\gamma = 0.2$ ); b) PM/AF<sup>up</sup> ( $\phi_\pi = 0.5$ ,  $\gamma = 3$ ); c) PM/AF<sub>down</sub> ( $\phi_\pi = 0.5$ ,  $\gamma = 0$ )

Trend inflation: solid line=0%, dashed=2%, dash-dotted=4%, dotted=6%.

A fiscal shock (Figure 7) does not affect output and inflation under a AM/PF regime since the monetarist solution holds hence their dynamics do not depend on the dynamics of fiscal variables. Higher taxes in the PM/AF<sub>down</sub> regime cause a negative wealth effect that makes output and inflation decrease. The reduction in prices makes real debt increase. On the other hand, in the PM/AF<sup>up</sup> regime the initial decrease in debt carried by the shock makes taxes fall by a great extent leading to an increase in output and

inflation that, in turn, makes real debt fall. Higher trend inflation tempers the effects on output while, in the PM/AF<sub>down</sub> case, it exacerbates the debt increase.

### 3.5 Robustness

In this section we investigate the robustness of our results along different dimensions.<sup>10</sup>

**Policy Rule.** First, we investigate if and how results change when a different policy rule is considered. A forward looking interest rate rule under flexible prices confirm, even under trend inflation, the results by Branch et al. (2008), according to whom determinacy holds provided fiscal policy is active. With a backward looking interest rate rule, instead, as in Branch et al. (2008), determinacy obtains provided the policy mix is PM/AF and this is true for every level of trend inflation. Then Result 1.1 still holds. *“As a corollary, we find that for both forward- and backward-looking rules, determinacy implies that the unique rational expectations equilibrium (REE) is non-Ricardian.”* Turning to the sticky price model, employing a forward looking rule, irrespective of fiscal policy behavior, the REE is never determinate under AM as long as trend inflation is positive while Result 1.2 still holds. As for E-stability, while under zero trend inflation a forward looking rule returns about the same results as the benchmark contemporaneous case, for positive trend inflation, under both TR and OP, the only E-stable regime is the PM/AF<sub>down</sub> one. While determinacy results under price rigidity with a backward looking policy rule are similar to the benchmark case, results concerning learning differ. There is always E-stability under TR as for the benchmark case but, as inflation increases, the double active mix is always E-stable. Results under OP differ from those under TR just for the incomplete learnability of the AM/AF<sup>up</sup> case.

**Model structure.** First, we check how results change for different degrees of price rigidity. We find the degree of price stickiness to be a crucial structural parameter both for determinacy and E-stability. While under flexible prices trend inflation does not affect determinacy, the more rigidity increases (higher  $\alpha$ ) the more, as trend inflation rises, results differ from the zero inflation case. Furthermore, for  $\alpha = 0.91$  the AM/PF mix is never determinate for trend inflation larger than 2%. However, with less rigid prices ( $\alpha = 0.35$ ),<sup>11</sup> we get for determinacy the same results as with flexible prices. What

<sup>10</sup>Details on the results, including figures, are omitted for brevity but are available from the authors upon requests.

<sup>11</sup>The calibration values for high ( $\alpha = 0.91$ ) and low ( $\alpha = 0.35$ ) degree of price rigidity are taken,

is more, transparency is needed to stabilize expectations in countries with flexible prices since it always sharply improves E-stability with respect to opacity. On the contrary, with high rigidity, TR does not improve so much under positive trend inflation relative to OP and, with trend inflation greater than 2%, non-Ricardian policies are less E-stable (and the more so under TR).

Secondly, we examine the effects of a higher debt/GDP ratio (in the benchmark case fixed at  $\bar{b} = 0.4$ ). We find that higher levels of  $\bar{b}$  do not affect determinacy, slightly change E-stability results under OP but make the E-stability area under AM/AF in case of TR shrink considerably.

Third, decreasing the value of the elasticity of substitution ( $\theta$ ) weakens the effects of trend inflation on determinacy and on E-stability results. Increasing the intertemporal elasticity of labour supply ( $\sigma_n$ ) reduces the E-stability area under both OP and TR and this effect worsens with trend inflation.

Finally, we examine the effects of including price indexation. It is well-known that indexation counteracts the effects of trend inflation. We find that this is true both for determinacy and E-stability.

## 4 Conclusions

This paper proves that a higher inflation target unanchors expectations under active monetary regimes but not under passive monetary ones. When a central bank, for example, follows a Taylor rule in the Ricardian regime, the higher the inflation target, the smaller the determinacy and the E-stability regions. Moreover, the higher the inflation target, the more the policy should be hawkish with respect to inflation to stabilize expectations. This is not true in a non-Ricardian context, where determinacy and E-stability are not sensible to changes in trend inflation. Transparency helps anchoring expectations - returning a E-stability region wider under transparency than under opacity - for all inflation targets under active monetary policy. The AM/PF regime, if determinate, is always E-stable under TR; the PM/AF<sub>down</sub> is always determinate and E-stable. A double active policy can be learnable under transparency. Therefore, with the help of transparency, the monetary and the fiscal authority could coordinate to a lesser extent

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respectively, from the estimates by Christiano et al. (2005) for the United States and by Smets and Wouters (2003) for the Euro area.

their policies to get E-stability under learning.

Finally, the more flexible are prices, the more transparency is valuable. In a low price rigidity country, say the United States, adhering to the Taylor principle is a sufficient condition for equilibrium determinacy under the AM/PF regime, irrespective of the level of trend inflation. Still, the central bank must be transparent to stabilize expectations. On the contrary, in a high price rigidity economy, say the Euro-zone, to have determinacy under AM/PF mix, we find that the inflation target cannot be larger than 2% but the central bank needs not to be transparent to stabilize expectations. Furthermore, high rigidity makes non-Ricardian policies less E-stable.

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Table 2: Parameters description

Parameter	Description
$\beta = 0.99$	Intertemporal discount factor
$\sigma_n = 1$	Inverse intertemporal elasticity of substitution in labour supply
$\theta = 11$	Dixit-Stiglitz elasticity of substitution
$\alpha = 0.75$	Calvo probability not to optimize prices
$\bar{\pi}$	Central bank's inflation target (or trend inflation)
$\phi_\pi$	Inflation parameter in the Taylor rule
$\gamma$	Fiscal rule parameter
$\bar{b} = 0.4$	Debt/gdp
NKPC coefficients definition	
$\lambda_{\bar{\pi}}$	$(1 - \alpha\bar{\pi}^{\theta-1}) (1 - \alpha\beta\bar{\pi}^\theta) (\alpha\bar{\pi}^{\theta-1})^{-1}$
$\eta_{\bar{\pi}}$	$\beta (\bar{\pi} - 1) (1 - \alpha\bar{\pi}^{\theta-1})$
$\xi_{\bar{\pi}}$	$\theta\alpha\bar{\pi}^{\theta-1} (\bar{\pi} - 1) (1 - \alpha\bar{\pi}^{\theta-1})^{-1}$

## A Appendix

### A.1 The Model

The model is based on Ascari and Ropele (2009), augmented with fiscal policy.

**Households.** Households live forever and their expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \chi \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right), \quad (10)$$

where  $\beta \in (0, 1)$  is the subjective rate of time preference,  $E_0$  is the expectation operator conditional on time  $t = 0$  information,  $C$  is consumption,  $N$  is labour,  $\chi$  and  $\sigma_n$  are parameters. The period budget constraint is given by:

$$P_t C_t + B_t \leq P_t w_t N_t + (1 + i_{t-1}) B_{t-1} + F_t \quad (11)$$

where  $P_t$  is the price of the final good,  $B_t$  represents holding of bonds offering a one-period nominal return  $i_t$ ,  $w_t$  is the real wage, and  $F_t$  are firms' profits rebated to house-



holds. The households maximize (10) subject to the sequence of budget constraints (11), yielding the following first order conditions:

$$w_t = \chi N_t^{\sigma_n} C_t, \quad (12)$$

$$\frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} \frac{(1+i_t) P_t}{P_{t+1}} \right]. \quad (13)$$

**Final Good Producers.** In each period, a final good  $Y_t$  is produced by perfectly competitive firms, using a continuum of intermediate inputs  $Y_{i,t}$  indexed by  $i \in [0, 1]$  and a standard CES production function  $Y_t = \left[ \int_0^1 Y_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}$ , with  $\theta > 1$ . The final good producer demand schedule for intermediate good quantities is:  $Y_{i,t} : Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$ . The aggregate price index:  $P_t = \left[ \int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}$ .

**Intermediate Goods Producers.**  $Y_{i,t}$  are produced by a continuum of firms indexed by  $i \in [0, 1]$ . The production function for intermediate input firms is:  $Y_{i,t} = N_{i,t}$ . Intermediate goods producers sets prices according to the usual Calvo mechanism. In each period there is a fixed probability  $1 - \alpha$  that a firm can re-optimize its nominal price, i.e.,  $P_{i,t}^*$ . With probability  $\alpha$ , instead, the firm may either keep its nominal price unchanged. The first order condition of the problem is:

$$\frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left( \frac{C_t}{C_{t+j}} \right) \left( \frac{P_t}{P_{t+j}} \right)^{-\theta} Y_{t+j} w_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left( \frac{C_t}{C_{t+j}} \right) \left( \frac{P_t}{P_{t+j}} \right)^{1-\theta} Y_{t+j}}. \quad (14)$$

The aggregate price level evolves as

$$P_t = \left[ \alpha (P_{t-1})^{1-\theta} + (1 - \alpha) (P_{i,t}^*)^{1-\theta} \right]^{1/(1-\theta)}. \quad (15)$$

As shown by Ascari and Ropele (2009), we will assume that for given parameter values of  $0 \leq \alpha < 1$ ,  $0 < \beta < 1$ ,  $0 \leq \varepsilon \leq 1$  and  $\theta > 1$ , the positive level of trend inflation fulfils the restrictions:  $1 \leq \bar{\pi} < \left( \frac{1}{\alpha} \right)^{\frac{1}{\theta-1}}$  and  $1 \leq \bar{\pi} < \left( \frac{1}{\alpha\beta} \right)^{\frac{1}{\theta}}$ , to ensure existence of the steady state of the model (see Ascari and Ropele, 2009 for details).

**Relative price dispersion.** At the level of intermediate firms, it holds true that  $(P_{i,t}/P_t)^{-\theta} Y_t = N_{i,t}$ . Aggregating this expression for all the firms  $i$  yields  $Y_t s_t = N_t$ , where  $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\theta} di$  and  $N_t \equiv \int_0^1 N_{i,t} di$ . The variable  $s_t$  measures the relative price dispersion across intermediate firms.  $s_t$  is bounded below at one and it represents

the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism.  $s_t$  evolves as

$$s_t = (1 - \alpha) \left( \frac{P_{i,t}^*}{P_t} \right)^{-\theta} + \alpha (\Pi_t)^\theta s_{t-1}, \quad (16)$$

where  $\Pi_t = P_t/P_{t-1}$  denotes the gross inflation rate. The variable  $s_t$  directly affects the real wage via the labour supply equation (12):  $w_t = \chi Y_t^{\sigma_n} s_t^{\sigma_n} C_t$ .

**Government budget constraint.** The flow budget constraint is given by

$$b_t = \frac{R_{t-1} b_{t-1}}{\Pi_t} + G_t - \tau_t,$$

where  $b_t$  is the value of real debt ( $b_t = B_t/P_t$ ). After setting government spending  $G_t$  to zero for simplicity, straightforward log-linearization lead to equation (7).

**Market clearing conditions.** The market clearing conditions in the goods and labour markets are:  $Y_t = C_t$ ;  $Y_{i,t}^s = Y_{i,t}^D = (P_{i,t}/P_t)^{-\theta} Y_t, \forall i$ ; and  $N_t = \int_0^1 N_{i,t} di$ .

## A.2 Learning

We write our problem as:

$$\begin{aligned} Y_t &= \beta_0 E_{t-1} Y_t + \beta_1 E_{t-1} Y_{t+1} + \delta Y_{t-1} + k w_t, \\ w_t &= \varphi w_{t-1} + e_t, \end{aligned} \quad (17)$$

where  $Y_t$  is the vector of endogenous variables and  $w_t$  are the shocks (exogenous) and  $e_t$  white noise. According to EH's notation the minimal state variable (MSV) solution is written as:

$$Y_t = a Y_{t-1} + c w_{t-1} + k e_t, \quad (18)$$

where the matrices  $a, c$  are to be determined.<sup>12</sup>

Following Evans and Honkapohja (2001), we assume that agents' perceived law of motion (PLM) coincides with the system's MSV solution. In this case, even the PLM will not contain any constant term.<sup>13</sup> Agents are assumed to know just the autocorrelation

<sup>12</sup>Since our model does not include constant terms, the MSV solution will not contain any constant.

<sup>13</sup>Using a PLM with a constant term, our main conclusions do not change. Results are available from

of the shocks but they have to estimate remaining parameters. To justify this choice we apply to the now usual argument of Bullard and Mitra (2002):

If the equilibrium can not be learned even under these very favourable conditions, then one might be quite pessimistic about the possibility of observing such an equilibrium in an actual economy.

Once the MSV solution is determined one can apply the E-stability criterion in Proposition 10.1 of Evans and Honkapohja (2001). An MSV solution  $\bar{a}, \bar{c}$  to (17) is E-stable if:

- (i) all the eigenvalues of  $DT_a(\bar{a})$  have real parts less than 1.
- (ii) all the eigenvalues of  $DT_c(\bar{a}, \bar{c})$  have real parts less than 1.

Assuming none of the eigenvalues has real part equal to 1, the solution is not E-stable if any of conditions (i), (ii) do not hold. In other words, we have stabilising expectations or E-stability when expectations converge to REE.

In order to determine the matrices  $DT_a(a)$  and  $DT_c(a, c)$  one has to proceed with the following steps. Starting from the PLM one can compute the following expectations:

$$E_{t-1}Y_t = aY_{t-1} + cw_{t-1} \quad (19)$$

and

$$E_{t-1}Y_{t+1} = aE_{t-1}Y_t + cE_{t-1}w_t = a^2Y_{t-1} + (ac + c\varphi)w_{t-1}. \quad (20)$$

Substituting these computed expectations into model (17) one obtains the actual law of motion (ALM):

$$Y_t = (\beta_1 a^2 + \beta_0 a + \delta)Y_{t-1} + (\beta_0 c + \beta_1 ac + \beta_1 c\varphi + \kappa\varphi)w_{t-1} + ke_t \quad (21)$$

The mapping from the PLM to the ALM takes the form

$$T(b, c) = (\beta_1 a^2 + \beta_0 a + \delta, \beta_0 c + \beta_1 ac + \beta_1 c\varphi + \kappa\varphi). \quad (22)$$

Expectational stability is then determined by the matrix differential equation

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the authors upon request.

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c). \quad (23)$$

The fixed points of equation (23) give us the MSV solution  $(\bar{a}, \bar{c})$ . The derivatives of the T-map are:<sup>14</sup>

$$DT_a(a) = a' \otimes \beta_1 + I \otimes (\beta_0 + \beta_1 a) \quad (24)$$

$$DT_c(a, c) = \varphi' \otimes \beta_1 + I \otimes (\beta_0 + \beta_1 a), \quad (25)$$

where  $I$  denotes an identity matrix of conformable size.

### A.3 Determinacy conditions under zero trend inflation

In absence of trend inflation, the model can be reduced to a three equation system

$$\begin{aligned} \hat{y}_t &= E_{t-1}^* \hat{y}_{t+1} - \phi_\pi E_{t-1}^* \hat{\pi}_t + E_{t-1}^* \hat{\pi}_{t+1}, \\ \hat{\pi}_t &= \beta E_{t-1}^* \hat{\pi}_{t+1} + \kappa E_{t-1}^* \hat{y}_t, \\ \bar{b}\beta^{-1} \hat{\pi}_t + \hat{b}_t &= (\beta^{-1} - \gamma) \hat{b}_{t-1} + \bar{b}\beta^{-1} \phi_\pi E_{t-2}^* \hat{\pi}_{t-1} + \bar{b}\beta^{-1} \varepsilon_{m,t-1} - \varepsilon_{\tau,t}, \end{aligned}$$

where  $\kappa = \lambda(1 + \sigma_n)$ . Sims's (2001) method offers a convenient framework for solving a model where expectations with different information sets appear. If we assume for simplicity that the policy shocks are uncorrelated, the model can be written recursively

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<sup>14</sup>See Evans and Honkapohja (2001, p.231-232) for details.

as

$$\begin{bmatrix} 1 & 0 & 0 & \phi_\pi & -1 & -1 & 0 \\ 0 & 1 & -\kappa & 0 & 0 & -\beta & 0 \\ 0 & \frac{\bar{b}}{\beta} & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ E_{t-1}y_t \\ E_{t-1}\pi_t \\ E_{t-1}y_{t+1} \\ E_{t-1}\pi_{t+1} \\ b_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\bar{b}}{\beta}\phi_\pi & 0 & 0 & \frac{1}{\beta} - \gamma \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ E_{t-2}y_{t-1} \\ E_{t-2}\pi_{t-1} \\ E_{t-2}y_t \\ E_{t-2}\pi_t \\ b_{t-1} \end{bmatrix} \\
+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & \frac{\bar{b}}{\beta} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{m,t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \\ \zeta_{3,t-1} \\ \zeta_{4,t-1} \end{bmatrix},$$

where the  $\zeta_i$ 's are zero-mean expectation errors. The matrix on the left hand side can be inverted to obtain

$$\begin{aligned}
\begin{bmatrix} y_t \\ \pi_t \\ E_{t-1}y_t \\ E_{t-1}\pi_t \\ E_{t-1}y_{t+1} \\ E_{t-1}\pi_{t+1} \\ b_t \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{\kappa}{\beta} + 1 & \phi_\pi - \frac{1}{\beta} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 0 & 0 & \frac{\bar{b}}{\beta}\phi_\pi & 0 & -\frac{\bar{b}}{\beta} & \frac{1}{\beta} - \gamma \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ E_{t-2}y_{t-1} \\ E_{t-2}\pi_{t-1} \\ E_{t-2}y_t \\ E_{t-2}\pi_t \\ b_{t-1} \end{bmatrix} \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & \frac{\bar{b}}{\beta} \end{bmatrix} \begin{bmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{m,t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -\frac{1}{\beta} & \frac{1}{\beta}\kappa + 1 & \phi_\pi - \frac{1}{\beta} \\ 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \\ 0 & -\frac{\bar{b}}{\beta} & 0 & -\frac{\bar{b}}{\beta} \end{bmatrix} \begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \\ \zeta_{3,t-1} \\ \zeta_{4,t-1} \end{bmatrix}.
\end{aligned}$$

These equations are block-recursive, so the dynamic properties depend on the sub-system

$$\begin{aligned}
\begin{bmatrix} E_{t-1}\pi_t \\ E_{t-1}y_{t+1} \\ E_{t-1}\pi_{t+1} \\ b_t \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \frac{\kappa}{\beta} + 1 & \phi_\pi - \frac{1}{\beta} & 0 \\ 0 & -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{\bar{b}}{\beta}\phi_\pi & 0 & -\frac{\bar{b}}{\beta} & \frac{1}{\beta} - \gamma \end{bmatrix} \begin{bmatrix} E_{t-2}\pi_{t-1} \\ E_{t-2}y_t \\ E_{t-2}\pi_t \\ b_{t-1} \end{bmatrix} \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & \frac{\bar{b}}{\beta} \end{bmatrix} \begin{bmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{m,t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & -\frac{1}{\beta} & \frac{1}{\beta}\kappa + 1 & \phi_\pi - \frac{1}{\beta} \\ 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \\ 0 & -\frac{\bar{b}}{\beta} & 0 & -\frac{\bar{b}}{\beta} \end{bmatrix} \begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \\ \zeta_{3,t-1} \\ \zeta_{4,t-1} \end{bmatrix}
\end{aligned}$$

$$Y_t = AY_{t-1} + B\varepsilon_t + C\zeta_t.$$

The three non-zero eigenvalues of matrix  $A$  are given by  $1/\beta - \gamma$  and by the two eigenvalues of submatrix

$$\check{A} = \begin{bmatrix} \frac{\kappa}{\beta} + 1 & \phi_\pi - \frac{1}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Since  $\hat{b}_t$  is a state-variable, the Blanchard-Kahn (1980) conditions for determinacy require two eigenvalues of  $A$  to lie outside the unit circle. The conditions can be expressed in term of the parameters that represent the stance of monetary and fiscal policies.

Before discussing the proofs of Proposition 1 and 2, it is worth observing that

$$\det(\check{A}) = \frac{1}{\beta}(\kappa\phi_\pi + 1), \quad \text{tr}(\check{A}) = \frac{1}{\beta} + \frac{\kappa}{\beta} + 1.$$

**Proof of Proposition 1 (passive fiscal policy)** The fiscal policy is passive when  $|1/\beta - \gamma| < 1$ . In this case, there exists a unique stable rational expectation equilibrium if both eigenvalues of  $\check{A}$  lie outside the unit circle, which is true if and only if<sup>15</sup>

$$|\text{tr}(\check{A})| < |1 + \det(\check{A})| \quad \text{and} \quad |\det(\check{A})| > 1.$$

It is straightforward to obtain the two conditions

$$\begin{aligned} \phi_\pi &> 1, \\ \phi_\pi &> \frac{\beta - 1}{\kappa}. \end{aligned}$$

As  $\beta < 1$ , the binding condition for determinacy is  $\phi_\pi > 1$ , i.e. the central bank must satisfy the Taylor principle.

**Proof of Proposition 2 (active fiscal policy)** The fiscal policy is passive when  $|1/\beta - \gamma| > 1$ . Determinacy is then achieved if  $\check{A}$  has one eigenvalue inside and one outside the unit circle, which is the case if and only if

$$|\text{tr}(\check{A})| > |1 + \det(\check{A})|.$$

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<sup>15</sup>See La Salle (1986).

Again, this inequality is satisfied for

$$\phi_\pi < 1,$$

i.e. the central bank must not satisfy the Taylor principle.

#### A.4 Determinacy conditions under positive trend inflation

We limit our attention to the simplified case where  $\sigma_n = 0$ . Accordingly, the price dispersion does not appear in the Phillips curve and the system reduces to a set of four equations for the variables  $\hat{y}_t$ ,  $\hat{\pi}_t$ ,  $\hat{\phi}_t$  and  $\hat{b}_t$ . By handling the system as shown in Section A.3, we obtain that the four eigenvalues that control the determinacy properties are given by  $1/\beta - \gamma$  and by the three eigenvalues of matrix

$$\tilde{A} = \begin{bmatrix} \frac{\lambda_\pi}{\bar{\pi}\beta} + 1 & \phi_\pi - \frac{1}{\bar{\pi}\beta} & \frac{1}{\bar{\pi}^\theta \alpha \beta^2} \eta \bar{\pi} \\ -\frac{\lambda_\pi}{\bar{\pi}\beta} & \frac{1}{\bar{\pi}\beta} & -\frac{1}{\bar{\pi}^\theta \alpha \beta^2} \eta \bar{\pi} \\ \frac{\lambda_\pi}{\bar{\pi}\beta} (\theta - 1) & \frac{1}{\bar{\pi}\beta} (1 - \theta) & \frac{1}{\bar{\pi}^\theta \alpha \beta^2} (\bar{\pi}\beta - \eta \bar{\pi} + \theta \eta \bar{\pi}) \end{bmatrix}.$$

**Passive fiscal policy** Again, when the fiscal policy is passive  $1/\beta - \gamma$  is inside the unit circle and the presence of three jump variables requires all eigenvalues of  $\tilde{A}$  to lie outside the unit circle. In other words, the system is determinate if  $\tilde{A}^{-1}$  is a stable matrix. Theorem 1 of Brooks (2004) states the necessary and sufficient conditions for checking the stability of a matrix in the three-variable case. In particular, stability requires

$$\begin{aligned} |\det(\tilde{A}^{-1})| &< 1, \\ |\det(\tilde{A}^{-1}) + \text{tr}(\tilde{A}^{-1})| &< M(\tilde{A}^{-1}) + 1, \\ \det(\tilde{A}^{-1})^2 - \text{tr}(\tilde{A}^{-1}) \det(\tilde{A}^{-1}) + M(\tilde{A}^{-1}) &< 1, \end{aligned}$$



where  $M(\cdot)$  indicates the sum of the principal minors. We obtain that

$$\begin{aligned}\det(\tilde{A}^{-1}) &= \frac{\bar{\pi}^\theta \alpha \beta^2}{1 + \lambda_{\bar{\pi}} \phi_\pi}, \\ tr(\tilde{A}^{-1}) &= \frac{1 + \lambda_{\bar{\pi}} - \eta_{\bar{\pi}} + \bar{\pi} \beta + \theta \eta_{\bar{\pi}}}{1 + \lambda_{\bar{\pi}} \phi_\pi} + \bar{\pi}^{\theta-1} \alpha \beta, \\ M(\tilde{A}^{-1}) &= \frac{\pi \beta + (\theta - 1) \eta_\pi + \pi^\theta \alpha \beta^2 + \pi^{\theta-1} \alpha \lambda_\pi \beta + \pi^{\theta-1} \alpha \beta}{1 + \lambda_\pi \phi_\pi}.\end{aligned}$$

The first condition gives

$$\phi_\pi > \frac{\bar{\pi}^\theta \alpha \beta^2 - 1}{\lambda_{\bar{\pi}}}.$$

We choose to restrict the analysis to relative low values of trend inflation (i.e.  $\alpha \bar{\pi}^\theta < 1$ ) so that coefficients  $\lambda_{\bar{\pi}}, \eta_{\bar{\pi}}$  and  $\xi_{\bar{\pi}}$  are all positive. If we do so, this restriction is always satisfied when  $\phi_\pi$  takes positive values. The second condition returns the Taylor principle  $\phi_\pi > 1$ . The third condition is more involved and turns out to be a quadratic inequality for  $(1 + \lambda_{\bar{\pi}} \phi_\pi)^{-1}$ :

$$\begin{aligned}& \frac{\bar{\pi}^\theta \alpha \beta^2 [1 + \lambda_{\bar{\pi}} + (\theta - 1) \eta_{\bar{\pi}} + \bar{\pi} \beta - \bar{\pi}^\theta \alpha \beta^2]}{(1 + \lambda_{\bar{\pi}} \phi_\pi)^2} \\ & - \frac{[(\theta - 1) \eta_{\bar{\pi}} + \bar{\pi} \beta + (1 + \lambda_{\bar{\pi}}) \bar{\pi}^{\theta-1} \alpha \beta + \bar{\pi}^\theta \alpha \beta^2 - \bar{\pi}^{2\theta-1} \alpha^2 \beta^3]}{1 + \lambda_{\bar{\pi}} \phi_\pi} + 1 > 0.\end{aligned}\tag{26}$$

If we focus only on  $\phi_\pi > 0$  and fix all parameters other than  $\bar{\pi}$  to their calibrated values (see Table 2), the condition is satisfied for  $\phi_\pi > z(\bar{\pi})$ , where  $z(\bar{\pi})$  is an increasing function of  $\bar{\pi}$ . In conclusion, when the fiscal authority is passive determinacy can be obtained if  $\phi_\pi > \max(1, z(\bar{\pi}))$ .

**Active fiscal policy** If the fiscal policy is active, determinacy is obtained when two eigenvalues of  $\tilde{A}$  are explosive and the third one is stable. Woodford (2003, appendix C, p. 675) derives the necessary and sufficient conditions for two roots to lie outside and one root within the unit circle. The conditions are stated in term of the determinant,

the trace, and the sum of principal minors of  $\tilde{A}$ , whose values are the following:

$$\begin{aligned}\det(\tilde{A}) &= \frac{1 + \lambda_{\bar{\pi}}\phi_{\pi}}{\bar{\pi}^{\theta}\alpha\beta^2}, \\ \text{tr}(\tilde{A}) &= \frac{1 + \lambda_{\bar{\pi}}}{\bar{\pi}\beta} + \frac{\bar{\pi}\beta + (\theta - 1)\eta_{\bar{\pi}}}{\bar{\pi}^{\theta}\alpha\beta^2} + 1, \\ M(\tilde{A}) &= \frac{1 + \lambda_{\bar{\pi}}\phi_{\pi}}{\bar{\pi}\beta} + \frac{1 + \lambda_{\bar{\pi}} + (\theta - 1)\eta_{\bar{\pi}} + \bar{\pi}\beta}{\bar{\pi}^{\theta}\alpha\beta^2}.\end{aligned}$$

In particular, one of the following three cases must hold.

**Case 1** Two restrictions should be satisfied simultaneously:

$$\begin{aligned}1 - \text{tr}(\tilde{A}) + M(\tilde{A}) - \det(\tilde{A}) &< 0, \\ -1 - \text{tr}(\tilde{A}) - M(\tilde{A}) - \det(\tilde{A}) &> 0.\end{aligned}$$

It is immediate to verify that the second condition is never satisfied, hence this case does not hold.

**Case 2** In the second case three conditions are required:

$$\begin{aligned}1 - \text{tr}(\tilde{A}) + M(\tilde{A}) - \det(\tilde{A}) &> 0, \\ -1 - \text{tr}(\tilde{A}) - M(\tilde{A}) - \det(\tilde{A}) &< 0, \\ \det(\tilde{A})^2 - \det(\tilde{A})\text{tr}(\tilde{A}) + M(\tilde{A}) - 1 &> 0.\end{aligned}$$

While the second condition is always true, the first condition gives

$$\frac{(\phi_{\pi} - 1)(1 + \bar{\pi}^{\theta-1}\alpha\beta)}{\bar{\pi}^{\theta-1}\alpha\beta} > 0,$$

which is satisfied for  $\phi_{\pi} < 1$ . The third condition can be shown, after some work, to be identical to the inequality (26), which is true for  $\phi_{\pi} > z(\bar{\pi})$ . Therefore this case holds for  $z(\bar{\pi}) < \phi_{\pi} < 1$ .

**Case 3** In the last case four conditions are required:

$$\begin{aligned}
1 - \text{tr}(\tilde{A}) + M(\tilde{A}) - \det(\tilde{A}) &> 0, \\
-1 - \text{tr}(\tilde{A}) - M(\tilde{A}) - \det(\tilde{A}) &< 0, \\
\det(\tilde{A})^2 - \det(\tilde{A}) \text{tr}(\tilde{A}) + M(\tilde{A}) - 1 &< 0, \\
|\text{tr}(\tilde{A})| &> 3.
\end{aligned}$$

The first two conditions have just been discussed and give  $\phi_\pi < 1$ . The second condition is again identical to inequality (26) but with opposite sign, so we have  $\phi_\pi < z(\bar{\pi})$ . The last condition is satisfied for our benchmark calibration, so this case holds for  $\phi_\pi < \min(z(\bar{\pi}), 1)$ .

In conclusion, for low values of trend inflation  $z(\bar{\pi}) < 1$  and the union of the second and third case will give determinacy for  $\phi_\pi < 1$ . Similarly, if  $z(\bar{\pi}) > 1$  the second case never applies and the third case gives again  $\phi_\pi < 1$ . Therefore, when the fiscal policy is active determinacy can be always achieved for  $\phi_\pi < 1$ , disregarding trend inflation.