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Staggered Price Setting, Bertrand Competition and Optimal Monetary Policy

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Staggered Price Setting, Bertrand Competition and Optimal Monetary Policy

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Abstract

We reconsider the New-Keynesian model with staggered price setting when each market is characterized by a small number of firms competing in prices *à la* Bertrand rather than a continuum of isolated monopolists. Price adjusters change their prices *less* when there are more firms that do not adjust, creating a natural and strong form of real rigidity. In a DSGE model with Calvo pricing and Bertrand competition, we obtain a modified New-Keynesian Phillips Curve with a lower slope. This reduces the level of nominal rigidities needed to obtain the estimated response of inflation to real marginal costs and to generate high reactions of output to monetary shocks. As a consequence, the determinacy region enlarges and the optimal monetary rule under cost push shocks, obtained through the linear quadratic approach, becomes less aggressive. Notably, the welfare gains from commitment decrease in more concentrated markets in reaction to inflationary shocks.

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The core idea of Keynesian economics is that in the presence of nominal price rigidities aggregate demand shocks have an impact on the real variables at least in the short run. Following Mankiw (1985), consider a restaurant with monopoly power on its customers: if there are some "small menu costs" such a restaurant would not adjust prices for a while. If this is the case for other isolated monopolists in other sectors, monetary shocks induce small price adjustments and large effects on the aggregate production levels and the other real variables. In this work we show how the impact of nominal rigidities is magnified when firms interact strategically *à la* Bertrand in the choice of prices rather than being independent monopolists.

To follow up on the metaphor of Mankiw (1985), a restaurant is rarely a monopolist, but is likely to compete with another restaurant across the street or a small number of similar restaurants in the neighborhood. Before changing prices, such a restaurant would consider what the rivals are doing. If none of them is going to revise the menu, the incentives to do it are limited. If all of the other restaurants are increasing their prices, there are high incentives to increase prices, though less than the rivals to gain customers. How much the restaurant should adjust depends on how many restaurants are adjusting their prices and how substitutable are their meals: a sushi bar may increase prices even if a fast food on the other side of the street does not, but a pizza restaurant may not want to increase prices if another pizza restaurant across the street has not revised its menu. Paradoxically, if products are (almost) homogenous, as long as there is a single firm that does not increase its prices, none of the others will be able to increase prices without losing (almost) the entire demand. As a consequence, small nominal rigidities induce additional (endogenous) rigidities and large changes in quantities when the economy is characterized by high inter-sectoral substitutability in concentrated markets. Arguably, most local markets for traditional goods and services, which represent a big portion of our economies (restaurants in a neighborhood are just an example), do involve a small number of competitors interacting strategically.[1]

The role of nominal rigidities under monopolistic competition with CES preferences and an exogenous number of firms was formalized in general equilibrium by Blanchard and Kiyotaki (1987) and then crystallized in the New-Keynesian DSGE models assuming price staggering *à la* Calvo (1983), where each firm has the chance of adjusting its price with a fixed probability in each period.[2] When this is the case, firms re-optimize taking into account future changes in their marginal cost, which delivers the link between real and nominal variables. However, under monopolistic competition between a large number of firms, price adjusters take as given the current and future price levels and the number of firms, ignoring

[1] Nevertheless, one should not think that global high-tech markets are that different in this dimension: a well known result of industrial organization theory and empirics (Sutton, 1991) is that also global markets tend to be highly concentrated because of a process of escalation of R&D costs.

[2] See Yun (1996), King and Wolman (1996) and Woodford (2003).

any strategic interaction with the competitors. This is unrealistic when local markets include a small number of firms producing highly substitutable goods: for instance, in the spatial model of monopolistic competition of Salop (1979), each firm is competing with its two immediate neighbors, and even in the Dixit-Stiglitz framework strategic interactions become relevant when the number of firms is finite and small. Following this insight, we consider a small number of direct competitors producing imperfectly substitute goods in each different market.

Other works, at least since Ball and Romer (1990), have already stressed the role of strategic complementarities between firms' prices as a source of real rigidities (see Nakamura and Steinsson, 2013, for a survey),[3] but (quite surprisingly) we are not aware of a formalization of price staggering with Bertrand competition as the natural source of strategic complementarities. A first approach to real rigidities, due to Basu (1995), relies on the fact that each firm employs all the other goods as intermediate inputs, and a second approach, adopted by Woodford (2003) or Altig *et al.* (2011), relies on firm-specific inputs: in both cases marginal costs depend on firms' own relative prices, so as to generate optimal prices increasing in the price index. A third approach, advanced by Kimball (1995), relies on a demand elasticity that is increasing in the relative price, generating again strategic complementarity between prices. Recent applications of this approach by Dotsey and King (2005), Eichenbaum and Fisher (2007), Levin *et al.* (2007) and Sbordone (2007) have been based on a generalization of the Dixit-Stiglitz aggregator to obtain elasticities increasing in prices and in the number of goods, but ignoring strategic interactions between price-setters.[4] To compare this line of research with ours and introduce our key idea, in Section I we propose a simple partial equilibrium model of Bertrand competition microfounded with the general class of indirectly separable preferences (Bertoletti and Etro, 2013), which includes the CES case and also cases of variable elasticity of demand. Within such toy model we show how price adjusters change less their prices when a) the fraction of non-adjusters is higher, b) the market is more concentrated or c) demand becomes more elastic. Then, we move to a DSGE model restricting our attention on the traditional CES case to isolate the role of Bertrand competition under price staggering.

In Section II we derive a modified New Keynesian Phillips Curve (NKPC) under Bertrand competition and Calvo pricing characterized by a lower slope. Such slope decreases with the inter-sectoral elasticity of substitution between goods and with the level of concentration of the markets, up to a third of the slope of the standard NKPC with monopolistic competition.

[3] Blanchard and Kyotaki (1987) and others have also stressed the potential role of generic strategic complementarities in generating multiple equilibria, but multiple equilibria are not the focus of our work.

[4] Bergin and Feenstra (2000) have replaced CES preferences with translog preferences, that are homothetic and deliver an elasticity of demand increasing in a (finite) number of goods. Nevertheless, they focus on different issues and, again, thy neglect the role of strategic interactions.

Thanks to this, our model contributes to reconcile the micro-evidence of frequent price adjustments (Bils and Klenow, 2004; Nakamura and Steinsson, 2008) with the macroeconomic data indicating that inflation is rather inertial (see Altig *et al.*, 2011). Moreover, we provide a natural foundation (from an industrial organization perspective) for the shape of the long run Phillips curve: an increase in the number of competitors, for instance due to globalization, increases the reactivity of inflation to marginal cost shocks.

In Section III we then introduce monetary policy with a standard Taylor rule. The lower slope of the Phillips curve implies that the determinacy region enlarges as the number of firms or the elasticity of substitution between goods decrease. We also derive the welfare-based loss function for our context applying the linear-quadratic approach of Rotemberg and Woodford (1997) and we characterize the optimal monetary policy in front of cost push shocks: the optimal reaction of the interest rate rule to inflationary shocks becomes less aggressive as the number of firms or the elasticity of substitution between goods decrease. Finally, we examine the welfare gains from a commitment to monetary policy and find that they decrease in more concentrated markets in reaction to inflationary shocks.

The recent research on dynamic entry has been mostly focused on standard monopolistic competition (Bilbiie *et al.,* 2008, 2012, 2014). Strategic interactions and Bertrand competition have been explicitly introduced in a flexible price model by Etro and Colciago (2010),[5] and the first applications in the New Keynesian framework have been developed by Faia (2012) to analyze the Ramsey problem of choosing the optimal state contingent inflation tax rates and Lewis and Poilly (2012) for estimation purposes.[6] However, all these models neglect price staggering and adopt a price adjustment cost *à la* Rotemberg (1982), which implies that all firms (rather than a fraction of them) adjust prices simultaneously and identically in each period.[7] Introducing time-dependent staggered pricing *à la* Calvo we obtain a different form of real rigidity associated with the substitutability between goods: if few firms in a market do not change prices after a cost shock, the price adjustment of the others are smaller the more substitutable are the goods.[8] The extension of our framework to endogenous entry is a promising avenue to reproduce realistic reactions to supply and demand shocks (see Etro and Rossi, 2013).

[5] See also Colciago and Etro (2010), Colciago and Rossi (2011) and Minniti and Turino (2013).

[6] See also Cecioni (2010) for an empirical assessment of the Bertrand model and Benigno and Faia (2010) on open economy issues.

[7] See also Auray *et al.* (2012). Cavallari (2013) has adopted Calvo pricing but focusing on monopolistic competition and ignoring strategic interactions.

[8] Contrary to this, Rotemberg pricing delivers larger adjustments when substitutability increases, both under monopolistic and Bertrand competition. Notice that the empirical analysis of Lewis and Poilly (2012) has found a small competition effect in their model with Rotember pricing, but this is not surprising since, besides various differences between setups, they have focused on a relatively high number of firms and low inter-sectoral substitutability.

In what follows we first develop the basic idea in a one sector model in Section 1. We then introduce the latter in the simplest New-Keynesian DSGE model to compare monopolistic and Bertrand competition with Calvo pricing in Section 2. In Section 3 we revisit the theory of optimal monetary policy applying the linear quadratic approach to our framework. We conclude in Section 4, leaving technical details to the Appendix.

1 An Example

To build the intuition for our main result we first analyze Bertrand competition in a isolated market in partial equilibrium. To emphasize the different roles of variable elasticity of demand or substitution between goods (Kimball, 1995) and strategic interactions in magnifying nominal rigidities, we propose a generalization of the Dixit-Stiglitz microfoundation of demand based on the class of indirectly additive preferences (see Bertoletti and Etro, 2013). Consider a representative agent with nominal expenditure (income) I and the following indirect utility over a finite number of goods $n \in [2, \infty)$:

$$V = \sum_{j=1}^{n} v\left(\frac{p(j)}{I}\right) \tag{1}$$

where $v > 0$, $v' < 0$ and $v'' > 0$ and $p(j)$ is the price of good j. Notice that we assume separability of the indirect utility[9] and we exploit its homogeneity of degree zero, which excludes money illusion on the consumers' side (doubling prices and income leaves utility unchanged). When the subutility is isoelastic, as with $v(p) = p^{1-\theta}$ with $\theta > 1$, we are in the traditional case of CES preferences, where the indirect utility is $V = I(\sum_{j=1}^{n} p(j)^{1-\theta})^{1/(1-\theta)}$ and θ is the constant elasticity of substitution. Beyond this particular case, our preferences generate a variable elasticity of substitution between goods.

The direct demand can be derived through the Roy identity $C(i) = -V_{p(i)}/V_I$ as:

$$C(i) = \frac{v'\left(\frac{p(i)}{I}\right) I}{\sum_{j=1}^{n} v'\left(\frac{p(j)}{I}\right) p(j)} \tag{2}$$

which delivers profits $d(i) = [p(i) - MC]\, C(i)$ for firm i, where MC is the nominal marginal cost of production.

Under Bertrand competition each firm i chooses its own price $p(i)$ to maximize profits,

[9] Dixit and Stiglitz (1977) assumed separability of the direct utility, and Kimball (1995) assumed implicit separability of the direct utility. A similar (but more complex) analysis could be based on non-separable homothetic utilities $V = I \cdot V(p_1, p_2, .., p_n)$, which provides the general class of consumption aggregators needed with infinitely living agents to insure two-stage budgeting.

with the FOC:

$$\frac{v'\left(\frac{p(i)}{I}\right)}{\sum_{j=1}^{n} v'\left(\cdot\right) p(j)} - \frac{\left[p(i) - MC\right] v''\left(\frac{p(i)}{I}\right)}{\sum_{j=1}^{n} v'\left(\cdot\right) p(j)} = \frac{\left[p(i) - MC\right]\left[v'\left(\frac{p(i)}{I}\right) + p(i) v''\left(\frac{p(i)}{I}\right)\right]}{\left[\sum_{j=1}^{n} v'\left(\cdot\right) p(j)\right]^2} \tag{3}$$

The last term is the effect of the price choice of the firm on the marginal utility of income (or the standard price index under CES preferences). Under Bertrand competition this pushes prices up due to strategic complementarities. Let us define $\theta(p/I) \equiv -v''(p/I)(p/I)/v'(p/I) > 1$ as the (approximate) elasticity of demand with respect to the price, which here is also the elasticity of substitution between symmetric goods. Such elasticity is constant only in the traditional CES case, otherwise it depends on the price-income ratio.

Consider *flexible prices* for all firms. Imposing symmetry between the n firms, the equilibrium price must satisfy:

$$p = \frac{\theta(p/I) + (n-1)^{-1}}{\theta(p/I) - 1} MC \tag{4}$$

Remarkably, the markup is variable in both the elasticity $\theta(p/I)$ and the number of firms n. In the (probably) more realistic case in which demand becomes more rigid with higher income and lower prices, that is when $\theta'(p/I) > 0$, the equilibrium markup is positively related to income: this provides a demand-side rationale for procyclical markups (see Bertoletti and Etro, 2013). On the other side, an increase in the number of firms n, which may be associated with a boom, reduces the markup because it intensifies competition: this provides a supply-side rationale for countercyclical markups (Etro and Colciago, 2010).

Now, let us introduce *sticky prices à la* Calvo (1983). Assume that a fraction λ of the n firms cannot adjust the nominal price, maintaining the pre-determined level P_{-1}, while the fraction $1 - \lambda$ can reoptimize. Employing this in (3) it is easy to derive that the symmetric Bertrand equilibrium is now characterized by new nominal prices reset as:

$$\begin{aligned} p &= \frac{\theta(p/I) + \left[n\Gamma(\lambda, p, P_{-1}) - 1\right]^{-1}}{\theta(p/I) - 1} MC \\ \text{with } \Gamma(\lambda, p, P_{-1}) &= \lambda \frac{v'(P_{-1}/I) P_{-1}}{v'(p/I) p} + 1 - \lambda \end{aligned} \tag{5}$$

This emphasizes our new mechanism of amplification of price rigidities, due to the strategic interactions and related to the term $\Gamma(\lambda, p, P_{-1})$. The presence of firms that do not adjust their prices leads also the optimizing firms to adjust less their own prices. More formally, as long as n is finite, we have $\partial p/\partial \Gamma < 0$ and $\partial \Gamma/\partial \lambda \gtrless 0$ if $p \gtrless P_{-1}$, therefore price adjustments decrease with λ.[10] Moreover, one can verify how the level of competition in the market affects this mechanism. When n increases the amplification mechanism becomes less important: for

[10] Notice that $v'(p/I)p < 0$ is increasing in the price under our assumption $\theta(p/I) > 1$. Notice that we also have $\partial\Gamma/\partial P_{-1} < 0$, which confirms strategic complementarity ($\partial p/\partial P_{-1} > 0$).

$n \to \infty$ we return to the standard monopolistic competition in which each firm acts independently and adjusts its prices in the same way for any λ. It is in concentrated markets that strategic interactions amplify the nominal rigidities: for instance, this is the case when markets are typically characterized by duopolies or restricted oligopolies.[11] Finally, the contribution of the variable elasticity of demand to nominal rigidities can be complementary to our mechanism when we assume (as in Kimball, 1995) that $\theta'(p/I) > 0$ and we keep income fixed. In such case there is an additional tendency to make smaller adjustments in front of a price increase: this is due both to the increase in the demand elasticity $\theta(p/I)$ (the Kimball effect) and to the increase in $\Gamma(\lambda, p, P_{-1})$ under Bertrand competition (our effect).[12]

To isolate the new role of strategic interactions, in the rest of the paper we will focus on the traditional case of CES preferences. In such case, the only source of amplification of price stickiness is the strategic one, and we have:

$$\Gamma(\lambda, p, P_{-1}) = \lambda \left(\frac{p}{P_{-1}} \right)^{\theta - 1} + 1 - \lambda \tag{6}$$

Since $\partial \Gamma / \partial \theta > 0$ if $p \gtrless P_{-1}$, we can clearly determine the additional impact that an increase of substitutability between goods exerts on pricing in concentrated markets with nominal rigidities: firms adjust prices less when they produce more similar goods. This mechanism is more relevant when the number of firms is low: in other words, more concentrated markets (low n) producing more substitutable goods (high θ) deliver a given price adjustment for a smaller amount of nominal rigidity λ.[13]

2 The Model

In this section we introduce Bertrand competition in a dynamic general equilibrium model with price staggering. Consider a representative household with utility:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \log C_{\iota t} d\iota - \frac{\upsilon L_t^{1+\phi}}{1+\phi} \right\} \quad \upsilon, \phi \geq 0 \tag{7}$$

where $\beta \in (0, 1)$ is the discount factor, L_t is labor supply and $E_0[\cdot]$ is the expectation operator. We restrict our analysis to the traditional case of CES preferences, so that $C_{\iota t}$ is

[11] While it is natural to think of a small number of competitors in local markets (i.e. restaurants in a neighborhood), one of the leading results in the theory of endogenous market structures is that also global markets tend to be highly concentrated because of a process of escalation of advertising and R&D costs (Sutton, 1991).

[12] However, notice that, if an expansionary shock increased spending, the variable elasticity of demand would generate a counteracting effect which makes demand more rigid and leading to larger price adjustments (associated with procyclical markups).

[13] For the purpose of realism, notice that identical results emerge if n is interpreted the number of multiproduct firms, each one producing an arbitrary large number of goods and choosing all their prices to maximize profits (on multiproduct firms in related models see Minniti and Turino, 2013).

the Dixit-Stiglitz consumption index for a continuum of sectors $\iota \in [0,1]$:

$$C_{\iota t} = \left[\sum_{j=1}^{n} C_{\iota t}(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{8}$$

In each sector, there is a fixed number $n \in [2, \infty)$ of producers of differentiated varieties and the price index is $P_{\iota t} = [\sum_{j=1}^{n} p_{\iota t}(j)^{1-\theta}]^{1/(1-\theta)}$. As in Etro and Colciago (2010) substitutability between goods is low across sectors (namely unitary given the log utility) but high within sectors, and each sector is highly concentrated. The household maximizes (7) choosing how much to work and how much to consume in each period and sector, under the following budget constraint in nominal terms:

$$B_t + \int_0^1 P_{\iota t} C_{\iota t} d\iota = (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t \tag{9}$$

where B_t are nominal risk-free bonds, purchased at time t and maturing at time $t+1$. We defined i_t as the nominal interest rate on the risk-free bonds agreed at time t, while W_t is the nominal wage. Since households own the firms, they receive an additional income coming from nominal profits, that are entirely distributed in form of dividends, D_t. In each period t the optimality condition for consumption across sectors requires the same expenditure $I_t = P_{\iota t} C_{\iota t}$ for any ι. Symmetry between sectors will allow us to consider a representative sector with consumption C_t and (average) price index P_t.

Given the nominal wage W_t, the optimal labor supply function can be derived as:

$$L_t = \left(\frac{W_t}{\upsilon P_t C_t} \right)^{1/\phi} \tag{10}$$

Each firm i produces a good with a linear production function. Labor is the only input, and output of firm i is:

$$y_t(i) = A_t l_t(i) \tag{11}$$

where A_t is total factor productivity at time t, and $l_t(i)$ is total labor employed by firm i. This implies that the real marginal costs in aggregate terms are $mc_t = w_t/A_t$, where we define the real wage as $w_t = W_t/P_t$. The Euler equation is:

$$(P_t C_t)^{-1} = \beta E_t \left[(1 + i_t) (P_{t+1} C_{t+1})^{-1} \right] \tag{12}$$

Loglinearizing the above equations around the zero inflation steady state, we have:

$$\begin{aligned} \hat{w}_t &= \hat{C}_t + \phi \hat{L}_t = \hat{A}_t + \widehat{mc}_t && (13) \\ \hat{C}_t &= \hat{A}_t + \hat{L}_t && (14) \\ \hat{C}_t &= \hat{C}^e_{t+1} - \left(\hat{\imath}_t - \pi^e_{t+1} \right) && (15) \end{aligned}$$

where $\hat{\imath}_t \equiv \log i_t - \log(1/\beta)$ is the log-deviation of the nominal interest rate from its steady state value, $\pi^e_{t+1} \equiv P_{t+1}/P_t - 1$ is expected inflation and $\hat{A}_t$ is the log-deviation of productivity, which will evolve exogenously following a standard AR(1) process $\hat{A}_t = \rho_a \hat{A}_{t-1} + \varepsilon_{a,t}$ where $\rho_a \in [0, 1)$ and $\varepsilon_{a,t}$ is a white noise.

In each period, a fraction λ of the firms across all sectors cannot adjust the nominal price and maintains its pre-determined price, and a fraction $1-\lambda$ can reoptimize the nominal price at the new level p_t, which maximizes the discounted value of future profits. Applying the law of large numbers, the average price index across all sectors in the economy reads as:

$$P_t = \left[\lambda {P_{t-1}}^{1-\theta} + (1-\lambda) n p_t^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{16}$$

whose steady state version implies $P^{1-\theta} = np^{1-\theta}$. The usual log-linearization around a zero inflation steady state is

$$\widehat{P}_t = \lambda \widehat{P}_{t-1} + (1-\lambda)\widehat{p}_t \tag{17}$$

which provides the average inflation rate $\pi_t = (1-\lambda)(\hat{p}_t - \hat{P}_{t-1})$. In every period t, each optimizing firm i chooses the new price p_t to maximize the expected profits until the next adjustment, taking into account the probability that there will be a new adjustment λ:

$$\max_{p_t} \sum_{k=0}^{\infty} \lambda^k \left\{ Q_{t+k} \left(p_t - \frac{W_{t+k}}{A_{t+k}} \right) \left(\frac{p_t}{P_{t+k}} \right)^{-\theta} C_{t+k} \right\} \tag{18}$$

where $Q_{t+k} = \beta^k I_t / I_{t+k}$ is the stochastic discount factor (from the Euler equation), with $I_t = P_t C_t$ being total expenditure in period t, and we used the fact that demand at time $t+k$ is $C_{t+k}(i) = (p_t/P_{t+k})^{-\theta} C_{t+k}$. The problem can be simplified to:

$$\max_{p_t} \sum_{k=0}^{\infty} (\lambda\beta)^k \frac{\left(p_t - \frac{W_{t+k}}{A_{t+k}}\right) p_t^{-\theta} I_t}{\sum_{j=1}^{n} p_{t+k}(j)^{1-\theta}} \tag{19}$$

where the prices of the $n-1$ intersectoral competitors in the initial period and in all the future periods are taken as given. The FOC of this problem, after simplifying and imposing symmetry of all the adjusted prices, reduces to:[14]

$$p_t \sum_{k=0}^{\infty} \frac{(\lambda\beta)^k}{\sum_{j=1}^{n} p_{t+k}(j)^{1-\theta}} - \theta \sum_{k=0}^{\infty} \frac{(\lambda\beta)^k \left(p_t - \frac{W_{t+k}}{A_{t+k}}\right)}{\sum_{j=1}^{n} p_{t+k}(j)^{1-\theta}} = (1-\theta)\, p_t^{1-\theta} \sum_{k=0}^{\infty} \frac{(\lambda\beta)^k \left(p_t - \frac{W_{t+k}}{A_{t+k}}\right)}{\left(\sum_{j=1}^{n} p_{t+k}(j)^{1-\theta}\right)^2} \tag{20}$$

[14] The FOC can be rearranged as:

$$\left(\frac{p_t}{P_t}\right) F_t = K_t + \left(\frac{p_t}{P_t}\right)^{2-\theta} G_t - \left(\frac{p_t}{P_t}\right)^{1-\theta} H_t$$

where $F_t \equiv \sum_{k=0}^{\infty} (\lambda\beta)^k (P_{t+k}/P_t)^{\theta-1}$, $K_t \equiv \frac{\theta}{\theta-1} \sum_{k=0}^{\infty} (\lambda\beta)^k mc_{t+k} (P_{t+k}/P_t)^{\theta}$ which correspond to the terms emerging under monopolistic competition (see Benigno and Woodford, 2005), $G_t \equiv \sum_{k=0}^{\infty} (\lambda\beta)^k (P_{t+k}/P_t)^{2(\theta-1)}$ and $H_t \equiv \sum_{k=0}^{\infty} (\lambda\beta)^k mc_{t+k} (P_{t+k}/P_t)^{2\theta-1}$. A closed form solution for p_t/P_t is not available as it was for monopolistic competition.

The steady state implies:

$$\left[1-\theta\left(1-\frac{MC}{p}\right)-(1-\theta)\,(p/P)^{1-\theta}\left(1-\frac{MC}{p}\right)\right]=0 \tag{21}$$

and $(p/P)^{1-\theta}=1/n$, therefore we can express the long run Lerner index as:

$$\frac{p-MC}{p}=\frac{n}{\theta n+1-\theta} \tag{22}$$

Loglinearizing the FOC we obtain:

$$a_0\hat{p}_t\sum_{k=0}^{\infty}(\lambda\beta)^k+a_1\sum_{k=0}^{\infty}(\lambda\beta)^k\,\widehat{P}_{t+k}+a_2\sum_{k=0}^{\infty}(\lambda\beta)^k\,\widehat{MC}_{t+k}=0 \tag{23}$$

where, using the steady state conditions we obtain the following coefficients:

$$a_0 = -\frac{\theta-1}{n}\left[n+\theta-2-\frac{(\theta-1)^2\,(n-1)}{\theta n+1-\theta}\right] \tag{24}$$

$$a_1 = \frac{(\theta-1)^2}{\theta n+1-\theta} \tag{25}$$

$$a_2 = \frac{(\theta-1)\,(n-1)}{n} \tag{26}$$

To rewrite the expression above in terms of the real marginal cost we add and subtract $a_2\sum_{k=0}^{\infty}(\lambda\beta)^k\,\widehat{P}_{t+k}$, so that we define $\widehat{mc}_{t+k}=\widehat{MC}_{t+k}-\widehat{P}_{t+k}$ and (23) becomes:

$$a_0\hat{p}_t\sum_{k=0}^{\infty}\gamma^k+(a_1+a_2)\sum_{k=0}^{\infty}(\lambda\beta)^k\,\widehat{P}_{t+k}+a_2\sum_{k=0}^{\infty}(\lambda\beta)^k\,\widehat{mc}_{t+k}=0 \tag{27}$$

One can verify that $a_1+a_2=-a_0$. Therefore we can solve for $\hat{p}_t$ in recursive form as:

$$\hat{p}_t=[1-\lambda\beta(1-\delta)]\,\widehat{P}_t-[1-\lambda\beta(1-\delta)]\,\frac{a_2}{a_0}\widehat{mc}_t+\lambda\beta(1-\delta)\hat{p}_{t+1} \tag{28}$$

In order to substitute for $\hat{p}_t$ and $\hat{p}_{t+1}$, consider the log-linearization of the price index (17). Solving it for $\hat{p}_t$ we have:

$$\hat{p}_t=\frac{\widehat{P}_t-\lambda\widehat{P}_{t-1}}{1-\lambda}\quad\text{and}\quad\hat{p}_{t+1}=\frac{\widehat{P}_{t+1}-\lambda\widehat{P}_t}{1-\lambda} \tag{29}$$

Substituting both into (28) we have:

$$\frac{\widehat{P}_t-\lambda\widehat{P}_{t-1}}{1-\lambda}=[1-\lambda\beta(1-\delta)]\left(\widehat{P}_t-\frac{a_2}{a_0}\widehat{mc}_t\right)+\lambda\beta(1-\delta)\left(\frac{\widehat{P}_{t+1}-\lambda\widehat{P}_t}{1-\lambda}\right) \tag{30}$$

Multiplying each side by $1-\lambda$, adding to both sides $\lambda\widehat{P}_t$ and simplifying, we finally reach:

$$\lambda\left(\widehat{P}_t-\widehat{P}_{t-1}\right)=\lambda\beta(1-\delta)\left(\widehat{P}_{t+1}-\widehat{P}_t\right)-(1-\lambda)\,[1-\lambda\beta(1-\delta)]\,\frac{a_2}{a_0}\widehat{mc}_t \tag{31}$$

Replacing $\pi_t=\widehat{P}_t-\widehat{P}_{t-1}$ and $\pi^e_{t+1}=\widehat{P}_{t+1}-\widehat{P}_t$, dividing by λ, and using $a_2/a_0=-(n-1)\,[\theta\,(n-1)+1]\,/\,[\theta-1+\theta n\,(n-1)]$ we finally obtain our main result:

Proposition 1. *Under Bertrand competition and Calvo pricing the inflation rate satisfies:*

$$\pi_t = \beta\pi^e_{t+1} + \frac{(1-\lambda)(1-\lambda\beta)(n-1)[\theta(n-1)+1]}{\lambda[\theta-1+\theta n(n-1)]}\widehat{mc}_t \tag{32}$$

where $\widehat{mc}_t$ is the change in the real marginal cost from the steady state level.

Before analyzing the properties of the inflation dynamics, we rewrite (32) as a modified NKPC. It is easy to show that as in the basic NK model, the flexible-price efficient output, in log-deviation from its steady state, is $\widehat{y}^*_t = \widehat{A}_t$. Defining the output gap around a deterministic flexible price equilibrium with zero inflation as $x_t = \hat{C}_t - \widehat{A}_t$, and using $\widehat{mc}_t = (1+\phi)\hat{L}_t = (1+\phi)x_t$ from (13)-(15), we finally have the modified NKPC, that we augment with a cost-plus shock:

$$\pi_t = \beta\pi^e_{t+1} + \kappa(\theta,n)x_t + \xi_t \tag{33}$$

where $\xi_t = \rho_\xi\xi_{t-1} + \varepsilon_{\xi,t}$ is an exogenous AR(1) shock with $\rho_\xi \in [0,1)$ and $\varepsilon_{\xi,t}$ white noise. The coefficient of the output gap is:

$$\kappa(\theta,n) \equiv \frac{\bar{\kappa}(n-1)[\theta(n-1)+1](1+\phi)}{\theta-1+\theta n(n-1)} \tag{34}$$

where $\bar{\kappa} = (1-\lambda)(1-\lambda\beta)/\lambda$.

First of all, notice that the modified NKPC collapses to the standard one with monopolistic competition in two cases: for $n \to \infty$, since we are back to the case in which each firm is negligible in its own market, and when $\theta \to 1$, since firms tend to produce independent goods and strategic interactions disappear even between few firms. In all the other cases, the slope of the NKPC is smaller than the standard one and, at most, it becomes a third of it, i.e. $\kappa(\infty,2) = \bar{\kappa}/3$. This reduces drastically the impact of the output gap on inflation, a result which is desirable from an empirical point of view since, as known in the literature, the basic NKPC implies an excessive reaction of inflation to changes in real marginal costs. Usual estimates from macrodata for the coefficient of the NKPC on the marginal cost range between 0.03 and 0.05 (see Levin *et al.*, 2007). Recently, Altig *et al.* (2011) found a coefficient of 0.014. Since in the standard Calvo model the value of λ implies price adjustments on average every $1/(1-\lambda)$ quarters, assuming $\beta = 0.99$, this last coefficient implies $\lambda = 0.9$ under monopolistic competition and therefore price adjustments on average every 30 months, much more than what appears to be reasonable.

To illustrate the effect of our form of strategic complementarity on the NKPC we adopt the graph used by Woodford (2005). Figures 1-3 show the relationship between λ and the elasticity of inflation on real marginal costs, that is $\kappa(\theta,n)/(1+\phi)$ for different values of n (2,3,5,10) and two alternative values of θ, that is different values of goods' elasticity of substitutions, respectively $\theta = 6$ (low substitutability) and $\theta = 30$ (high substitutability). The red line and the blue line report the estimated value of the NKPC coefficient of real marginal costs, $\kappa/(1+\phi)$, found by Altig *et al.* (2011) et al and Levin *et al.* (2007) respectively. The green line represents the value of the coefficient under monopolistic competition.

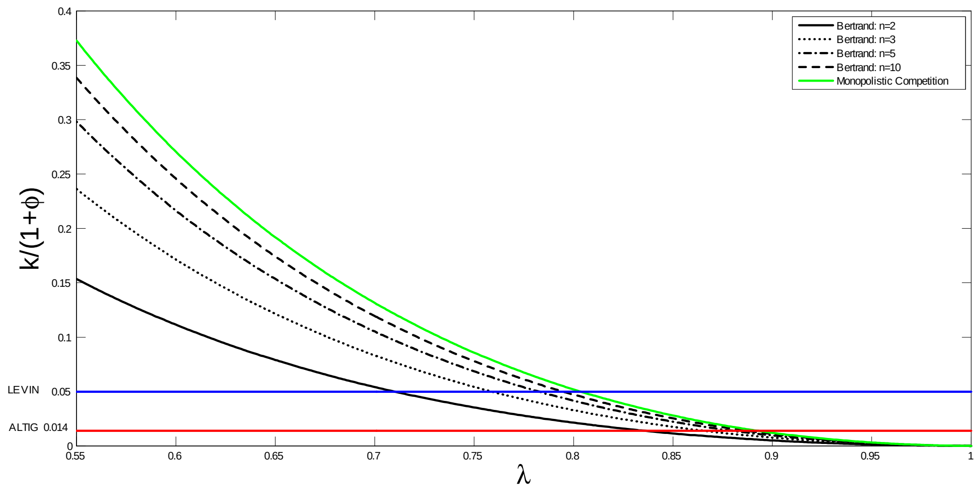

Figure 1: Slope of the Phillips Curve and Nominal Rigidities. Low substitutability ($\theta = 6$)

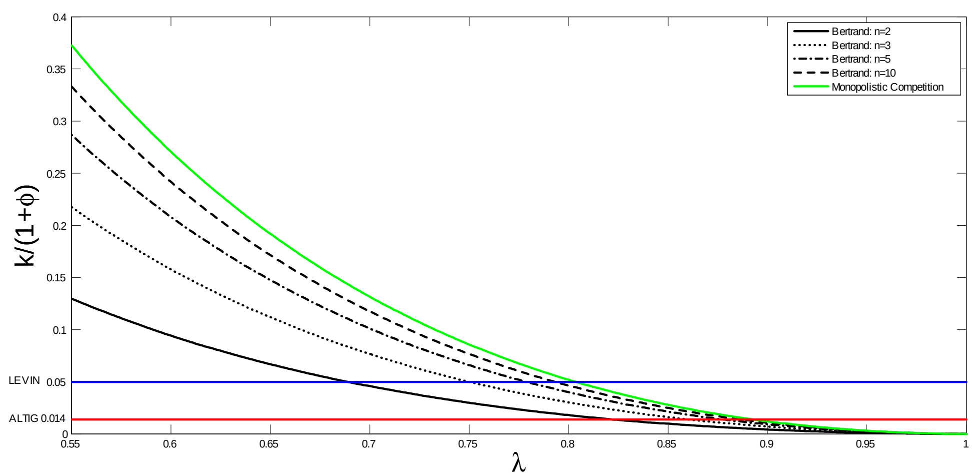

Figure 2: Slope of the Phillips Curve and Nominal Rigidities. High substitutability ($\theta = 30$)

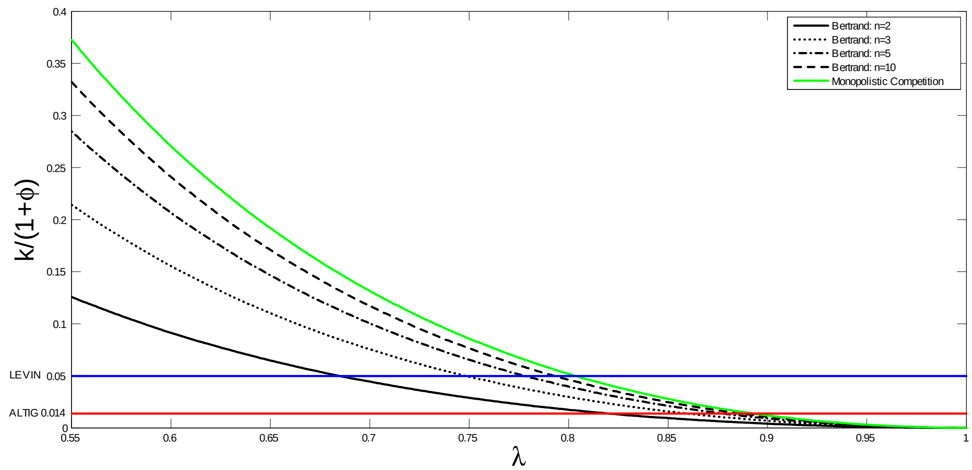

Figure 3: Slope of the Phillips Curve and Nominal Rigidities. Almost perfect substitutabilty ($\theta = 100$)

The introduction of strategic interactions reduces the implied degree of nominal frictions up to $\lambda = 0.82$, for $n = 2$, $\theta = 6$ to obtain the coefficient estimated by Altig *et al.* (2011). Notice that, a value of $\lambda = 0.82$ implies price adjustments on average every 5.8 quarters: thus, strategic interaction almost halves the average period of price adjustment, being more in accordance with the micro-evidence: Bils and Klenow (2004) find that half of prices last 5.5 months excluding sales, but Nakamura and Steinsson (2008) increase this estimate to 12-13 months excluding both sales and product substitutions.[15]

Importantly, for the value estimated by Levin et al (2007), i.e. for $\kappa/(1+\phi) = 0.05$, the corresponding value of λ falls to 0.71 for $\theta = 6$ and almost to $\lambda = 0.67$ for $\theta = 30$, and becomes slightly lower for a value of θ implying that goods are almost homogeneous (see Figure 3). Notice that, ceteris paribus, the corresponding value of λ is 0.82, under monopolistic competition. Thus, other things equal, Bertrand competition strongly reduces the value of λ, implying that firms adjust prices more often, thus being more in line with the micro-evidence.

To look at things from a different perspective, the standard model with monopolistic competition with $\lambda = 0.67$ (price adjustments every 3 quarters) generates a coefficient on the

[15] Notice that Smets and Wouters (2007) have estimated their model for the U.S. economy by replacing the Dixit-Stiglitz aggregator with the Kimball aggregator. The latter implies that the price elasticity of demand becomes increasing in the firm's price. Therefore, prices in the model become more rigid and respond by smaller amounts to shocks, for a given frequency of price changes λ. Using their macro-model, they obtain a much smaller estimate of the Calvo parameter, about $\lambda = 0.67$, which implies that price contracts last 3 quarters on average, more in accordance with US microeconomic evidence (see Maćkowiak and Smets, 2008). Our setup suggests an alternative way to reconcile the micro with the macro-evidence. Needless to say, the different sources of real rigidity can be complementary.

marginal cost change equal to 0.166, but this does not fit with the mentioned macroevidence on the small reaction of inflation to changes in marginal costs. Instead, Bertrand competition generates a smaller coefficient, at least 0.055, which is much closer to the macroevidence.[16]

In our model, the degree of concentration of markets (inversely related to n) and the substitutability between goods within sectors (increasing in θ) do affect the impact of the output gap on inflation. In particular, we can derive the following comparative statics on the slope of the Phillips curve:

Proposition 2. *Under Bertrand competition and Calvo pricing the NKPC becomes flatter when the elasticity of substitution among goods increases or the number of firms decreases.*

To verify the first result notice that:

$$\frac{d\kappa(\theta,n)}{d\theta} = \frac{-n^2\kappa(\theta,n)}{\left[\theta-1+\theta n\left(n-1\right)\right]\left[\theta\left(n-1\right)+1\right]} < 0 \tag{35}$$

Contrary to what happens in the baseline model with monopolistic competition, the slope of the NKPC depends on the substitutability between goods. This is quite important since most firms compete mainly with few rivals whose products are close substitutes. And in this case high values of the demand elasticity are associated with smaller price adjustments. When θ increases, firms become less prone to change prices, because their demand is more sensitive to price differentials. As a consequence, monetary shocks have smaller effects on the inflation and therefore larger effects on the real economy. The limit behavior for (almost) homogenous goods is

$$\lim_{\theta\to\infty}\kappa(\theta,n) = \frac{\bar{\kappa}\left(n-1\right)^2}{1+n\left(n-1\right)} \tag{36}$$

which represents the lower bound of the slope of the Phillips curve. Moreover, we have:

$$\frac{d\kappa(\theta,n)}{dn} = \frac{\left(\theta-1\right)\left[\theta\left(n^2-1\right)+1\right]\kappa(\theta,n)}{\left(n-1\right)\left[\theta-1+\theta n\left(n-1\right)\right]\left[\theta\left(n-1\right)+1\right]} > 0 \tag{37}$$

A change in real marginal costs implies a smaller reaction of current inflation in more concentrated markets because firms tend to adjust less their prices. This is going to amplify the real impact of monetary shocks and downplay the impact of technology shocks. Another implication, is that entry of firms increases the long run slope of the Phillips curve: this is in line with the idea that globalization and deregulation increase the pass-through of shocks on prices (see Benigno and Faia, 2010).

[16] Our model implies an even better fit to euro-area data, where as emphasized by many authors the median consumer price lasts about 3.7 quarters months (Maćkowiak and Smets, 2008).

3 Monetary Policy

To close the model we adopt a standard Taylor rule:

$$\hat{\imath}_t = \gamma_\pi \pi_t + \gamma_x x_t + \mu_t \tag{38}$$

where $\gamma_\pi \in [0, \infty)$ and $\gamma_x \in [0, \infty)$ with at least one coefficient different from zero, and where μ_t is a stationary policy shock following an AR(1) process $\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t}$ with $\rho_\mu \in [0, 1)$ and $\varepsilon_{\mu,t}$ white noise.

3.1 Determinacy and the Taylor Principle

To assess the determinacy of the equilibrium, we rewrite the log-linearized Euler equation (15) as a forward looking IS curve in terms of the output gap. In the absence of technology shocks this reads as:

$$x_t = x_{t+1}^e - \hat{\imath}_t + \pi_{t+1}^e \tag{39}$$

and we can substitute the Taylor rule (38) into it. This and the NKPC (33) form a $2x2$ system, that we rewrite in the following matrix format $\mathbf{X}_t = \mathbf{A}\mathbf{X}_{t+1}^e + \mathbf{B}\mathbf{u}_t$, where vector $\mathbf{X}_t$ includes the inflation rate π_t and the output gap x_t, while $\mathbf{u}_t$ is a complete vector of all the shocks. Determinacy is obtained if the standard Blanchard and Kahn (1980) conditions are satisfied. The model is isomorphic to the standard New Keynesian model, therefore the necessary and sufficient conditions for determinacy require:

$$\gamma_\pi + \frac{1-\beta}{\kappa(\theta, n)} \gamma_x > 1 \tag{40}$$

Since $d\left[(1-\beta)/\kappa(\theta,n)\right]/dn < 0$ and $d\left[(1-\beta)/\kappa(\theta,n)\right]/d\theta > 0$ from Proposition 2, we can conclude with:

Proposition 3. *Under Bertrand competition and Calvo pricing the determinacy region enlarges in the parameter space (γ_π, γ_x) as the number of firms decreases or the elasticity of substitution increases.*

In the basic New Keynesian model a Central Bank must react to an increase in inflation by increasing the nominal interest rate enough to increase the real one and reduce current consumption, so that the negative output gap brings inflation under control. In an economy with concentrated markets (low n) where few firms produce extremely substitutable goods (high θ), firms are less prone to change prices because they can lose a large portion of the customers in their market, therefore the monetary authority can use a less aggressive policy rule to avoid self-fulfilling inflation. Remarkably, the elasticity of substitution is irrelevant for determinacy in the standard NKPC under monopolistic competition by independent firms, but it becomes relevant in concentrated markets.

3.2 Welfare based loss function

In the absence of cost-push shocks, our model inherits the standard property of the New Keynesian model: as long as an appropriate labor subsidy restores the efficiency of the long-run equilibrium, by eliminating the wedge between the marginal rate of substitution (of consumption and labor) and the marginal product of labor, the short run efficient equilibrium is obtained with zero inflation rate (which is the same in terms of consumer price or producer price inflation). In other words, the so called *divine coincidence* holds and there is no trade-off between stabilizing the output gap and the inflation rate.[17] Solving the Ramsey problem in the absence of subsidies, one can also confirm the standard result for which zero inflation remains optimal in the steady state (for a proof in case of monopolistic competition see Schmitt-Grohe and Uribe, 2011). This justifies our approximations around a zero inflation steady state.

In the presence of an exogenous cost-push shocks, however, the stabilizing role of monetary policy is affected by the number of firms and the elasticity of substitution among goods. To verify this, we characterize the optimal monetary policy in the presence of a cost-push shock to the NKPC. We follow the linear-quadratic approach of Rotemberg and Woodford (1997) and derive the second order Taylor approximation of utility around the steady state (see the Appendix). The next characterization of the loss function is valid around the efficient steady state in the presence of the optimal subsidy, or in case of small distortions:

Proposition 4. *The welfare-based loss function is:*

$$\mathcal{L}_t = \sum_{k=0}^{\infty} \beta^k \left\{ \vartheta \left[\frac{x_{t+k}^2}{2} + \Phi\left(\theta, n, \tau\right) x_{t+k} \right] + \frac{\pi_{t+k}^2}{2} \right\} + O\left(\|\xi\|^3 \right) + t.i.p. \tag{41}$$

where $\vartheta = \bar{\kappa}(1+\phi)/\theta$*, the coefficient on the linear term is* $\Phi\left(\theta, n, \tau\right) \equiv \left[n - \tau\left(\theta - 1\right)\left(n - 1\right)\right] / \left(\theta n - \theta + 1\right)$*, and* $O(\|\xi\|^3)$ *and t.i.p. contains terms of order higher than second and independent from the policy.*

Remarkably, Bertrand competition affects only the linear term compared to the standard result, because of the distortions introduced by strategic interactions. The labor subsidy that restores efficiency can be easily derived as:

$$\tau^* = \frac{n}{\theta(n-1)+1} \tag{42}$$

which insures $\Phi\left(\theta, n, \tau^*\right) = 0$. Accordingly, the coefficient of the linear term $\Phi\left(\theta, n, \tau\right)$ becomes negligible with a subsidy close to the optimal one or simply when the elasticity of substitution is high, because this reduces the steady state distortion (eliminating it for $\theta \to \infty$). As mentioned, the approximation of the welfare-based loss function is valid in this case.

[17] See Blanchard and Galì (2007) for a detailed discussion on the divine coincidence and on the possibility to endogenize the trade-off.

3.3 Optimal monetary policy around a distorted steady state

If the Central Bank cannot credibly commit in advance to a sequence of future policy actions, the optimal monetary policy under discretion can be derived generalizing the result in Clarida *et al.* (1999). The problem of optimal monetary policy under discretion is:

$$\begin{aligned} &\min_{\{x_t,\pi_t\}} \vartheta\frac{x_t^2}{2} + \vartheta\Phi\left(\theta,n,\tau\right)x_t + \frac{\pi_t^2}{2} + \mathcal{L}_{t+1} \\ &s.t.: \quad \pi_t = \kappa(\theta,n)x_t + f_t \end{aligned} \tag{43}$$

where f_t and $\mathcal{L}_{t+1}$ are taken as given. The FOC is:

$$x_t = \left[\Phi\left(\theta,n,\tau\right) - \frac{\kappa(\theta,n)}{\vartheta}\right]\pi_t \tag{44}$$

and provides:

$$\pi_t = \frac{\vartheta\Phi\left(\theta,n,\tau\right)\kappa(\theta,n)}{\kappa(\theta,n)^2 + \vartheta\left(1-\beta\right)} + \vartheta\Psi\xi_t \tag{45}$$

where $\Psi = \left[\kappa(\theta,n)^2 + \vartheta(1-\beta\rho_\xi)\right]^{-1}$. The optimal output gap in response to a cost-push shock is:

$$x_t = \frac{\vartheta\Phi\left(\theta,n,\tau\right)\left(1-\beta\right)}{\kappa(\theta,n)^2 + \vartheta\left(1-\beta\right)} - \kappa(\theta,n)\Psi\xi_t \tag{46}$$

Thus, the presence of a distorted steady state does not affect the response of the output gap and inflation to shocks under the optimal policy. It only affects the average inflation and output gap around which the economy fluctuates. In particular, as long as $\Phi\left(\theta,n,\tau\right) > 0$ the optimal discretionary policy requires a positive average inflation giving rise to the classical *inflation bias à la* Barro-Gordon. This occurs because the steady state level of output is below the efficient one, and the Central Bank has an incentive to push output above its natural steady state level. Such incentive increases with the degree of inefficiency of the steady state. Further notice that $\Phi\left(\theta,n,\tau\right)$ decreases in both n and θ, but the average inflation is ambiguously affected by changes in $\kappa(\theta,n)$, therefore we cannot sign the impact of changes in concentration and substitutability on the average inflation bias.

Substituting for x_t and x_{t+1}^e in $x_t = x_{t+1}^e - \hat{\imath}_t + \pi_{t+1}^e$ and simplifying yields:

$$-\frac{\kappa}{\vartheta}\pi_t = -\frac{\kappa}{\vartheta}\pi_{t+1}^e - \hat{\imath}_t + \pi_{t+1}^e \tag{47}$$

Since $\pi_{t+1}^e = \rho_\xi\pi_t$ and

$$\pi_{t+1}^e = \frac{\vartheta\Phi\kappa}{\kappa^2 + \vartheta\left(1-\beta\right)} + \vartheta\Psi\xi_{t+1}^e = \frac{\vartheta\Phi\kappa}{\kappa^2 + \vartheta\left(1-\beta\right)} + \vartheta\Psi\rho_\xi\xi_t \tag{48}$$

we have

$$\begin{aligned} \rho_\xi\pi_t &= \frac{\rho\vartheta\Phi\kappa}{\kappa^2 + \vartheta\left(1-\beta\right)} + \vartheta\Psi\rho_\xi\xi_t \\ &= \pi_{t+1}^e - \frac{\vartheta\Phi\kappa\left(1-\rho_\xi\right)}{\kappa^2 + \vartheta\left(1-\beta\right)} \end{aligned} \tag{49}$$

Solving for π_t and replacing above provides:

$$-\frac{\kappa}{\vartheta\rho_\xi}\pi^e_{t+1}+\frac{\kappa^2}{\vartheta\rho_\xi}\frac{\vartheta\Phi\left(1-\rho_\xi\right)}{\kappa^2+\vartheta\left(1-\beta\right)}=-\frac{\kappa}{\vartheta}\pi^e_{t+1}-\widehat{i}_t+\pi^e_{t+1} \tag{50}$$

which can be solved for $\widehat{i}_t$ to obtain the optimal interest rate rule:

$$\widehat{i}_t=\left[1+\frac{\kappa(\theta,n)(1-\rho_\xi)}{\vartheta\rho_\xi}\right]\pi^e_{t+1}-\Upsilon\left(\theta,n,\tau,\rho_\xi\right)$$

where the last term represents the downward distortion of the steady state interest rate relative to the efficient level:

$$\Upsilon\left(\theta,n,\tau,\rho_\xi\right)\equiv\frac{\Phi\left(\theta,n,\tau\right)\left(1-\rho_\xi\right)\kappa(\theta,n)^2}{\rho_\xi\left[\kappa(\theta,n)+\vartheta\left(1-\beta\right)\right]}$$

One can verify that this bias is reduced when the elasticity of substitution θ increases (and approximately eliminated when goods become close substitutes).

Using $\pi^e_{t+1}=\rho_\xi\pi_t$ and replacing $\kappa(\theta,n)/\vartheta$, we can also rewrite the rule as:

$$\widehat{i}_t=\left[1+\left(1-\rho_\xi\right)\left(\frac{\theta\left(n-1\right)\left[\theta\left(n-1\right)+1\right]}{\theta-1+\theta n\left(n-1\right)}-1\right)\right]\pi_t-\Upsilon\left(\theta,n,\tau,\rho_\xi\right) \tag{51}$$

This is a Taylor rule of the kind (38) with $\gamma_\pi>1$, which insures determinacy, and $\gamma_x=0$. The optimal coefficient on the inflation rate is much smaller compared to the case of monopolistic competition.[18] Therefore, Bertrand competition with a small number of firms in each sector requires a less aggressive monetary policy compared to monopolistic competition. Moreover, straightforward comparative statics provides:

Proposition 5. *Under Bertrand competition and Calvo pricing the optimal monetary rule requires a less aggressive reaction to inflationary shocks, with a coefficient increasing in the number of firms and in the elasticity of substitution between goods.*

One may notice that higher substitutability induces smaller price adjustments, which would allow for a less aggressive policy. However, when goods become more substitutable, the dispersion of consumption creates a smaller welfare loss, which asks for less output stabilization and more inflation stabilization, and therefore a more aggressive policy. This second effect is prevailing on the first one.

3.4 Discretion vs Commitment

We will now focus our analysis to the case in which the optimal subsidy is available and the classic inflation bias problem disappears ($\Phi\left(\theta,n,\tau^*\right)=0$). Nevertheless, the optimal

[18] For instance, assume uncorrelated shocks ($\rho_\xi=0$) with three firms per sector on average. Under monopolistic competition the optimal coefficient is $\gamma_\pi=\theta$. However, when $\theta=3$ the optimal coefficient under Bertrand competition becomes $\gamma_\pi=2.1$, and when $\theta=30$ the optimal coefficient under Bertrand competition is $\gamma_\pi=17.5$.

discretionary policy above can be improved by committing to a different rule, which creates a welfare loss. In order to evaluate the welfare implications of monetary policy, we compute the (per period) unconditional welfare-loss around the efficient steady state under discretion. Combining $\pi_t = \vartheta\Psi\xi_t$ with $x_t = -\kappa(\theta, n)\Psi\xi_t$ we obtain $x_t = -\vartheta\Psi\xi_t$, where $\Psi = \left[\kappa(\theta, n)^2 + \vartheta(1-\beta\rho_\xi)\right]^{-1}$. If we substitute this and $\pi_t = \vartheta\Psi\xi_t$ in the loss-function we can express the unconditional welfare-loss under discretion as:

$$l_D = \frac{1}{2}\left[(\vartheta\Psi)^2 + \vartheta\left(\kappa(\theta, n)\Psi\right)^2\right] Var\left(\xi_t\right). \tag{52}$$

We now asses the welfare gains that the Central Bank may obtain by committing to a state-contingent rule of the kind studied by Clarida *et al.* (1999), with $x_t^c = -\omega\xi_t$ where ω is a feedback parameter to choose optimally. This implies the same qualitative rule as before with $\vartheta\left(1-\beta\rho_\xi\right)$ replacing ϑ. The optimal Taylor rule becomes more aggressive compared to the one under discretion. This allows us to compute the (per period) unconditional welfare-loss, which is now given by:

$$l_C = \frac{1}{2}\left[\left(\vartheta\left(1-\beta\rho_\xi\right)\Psi^c\right)^2 + \vartheta\left(\kappa(\theta, n)\Psi^c\right)^2\right] Var(\xi_t) \tag{53}$$

where $\Psi^c = [\kappa(\theta, n)^2 + \vartheta\left(1-\beta\rho_\xi\right)^2]^{-1}$. As a measure of the welfare gains implied by commitment we take the ratio between the unconditional loss under discretion l_D and the one under commitment l_C. A ratio greater than one means that, as expected, the commitment rule implies a *welfare gain,* due to a more stabilizing effect on inflation compared to the discretionary rule. Figure 4 plots the welfare gains from commitment for different values of n and θ.

As shown in Figure 4 the welfare gains from commitment decrease as n decreases and θ increases (we calibrated parameters with $\phi = 1/4$, $\beta = 0.99$, $\lambda = 0.67$ and $\rho_\xi = 0.9$). The gains from commitment start to decrease substantially with less than eight firms. Importantly, notice that the welfare gains are almost halved moving from twenty to four firms, as the ratio between the two losses passes from a value of 2.8 to almost 1.4. Thus, the gains from commitment decrease by a large amount when markets become highly concentrated. Instead, the effect of the elasticity of substitution appears to be less important, with small reductions of the welfare gains obtained when θ reaches high levels. To sum up, under Bertrand competition a lower number of firms or a higher elasticity of substitution go in the same direction of stabilizing inflation and thus of reducing the gains from a better stabilizing rule, therefore discretion creates a lower inflation bias, reducing the advantages of commitment.

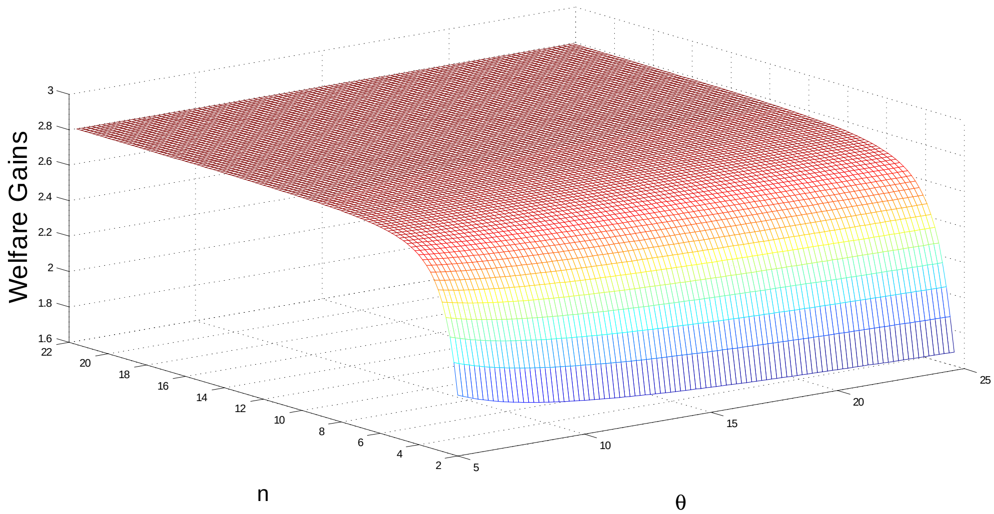

Figure 4: Welfare Gains from Commitment to a rule.

4 Conclusion

We have reconsidered the New-Keynesian framework under time-dependent staggered pricing *à la* Calvo in markets characterized by a small number of competitors engaged in Bertrand competition. Such a description of the relevant form of competition can be quite realistic from an industrial organization point of view. Most local markets for traditional goods and services (as the classic restaurants in a neighborhood) do involve a small number of competitors and represent a big portion of our economies. However, also many global markets tend to be highly concentrated because of a process of escalation of R&D costs. In these conditions, strategic interactions cannot be ignored and create important real rigidities that affect the propagation of shocks. Price adjusters do change their prices less when there are more firms that do not adjust: this strengthens the impact of nominal rigidities, which is at the heart of New-Keynesian economics. Indeed, we have shown that Bertrand competition reduces the level of nominal rigidities required to obtain the estimated response of inflation to real marginal costs, thus contributing to reconcile the macro with the micro-evidence. Furthermore, it implies a lower level of nominal rigidities to generate high reactions of output to monetary shocks. As a consequence, the determinacy region enlarges and the optimal monetary rule in response to a cost push shocks becomes less aggressive. Finally, we found that Bertrand competition reduces the inflationary bias usually characterizing a discretionary rule. This implies that the welfare gains from commitment decrease in more concentrated markets in reaction to inflationary shocks.

A number of extensions are left for future research. Price indexation at the sector level can affect the size of the real rigidities as long as firms take in consideration the impact of their price choice on the future indexation. One may also follow the inspiration of models with variable elasticity of substitution and endogenize changes in such elasticity. In general, as long as the elasticity is procyclical (countercyclical), an expansionary shock tends to reduce (increase) inflation over time. More interesting would be to adopt different kinds of preferences, such as the translog preferences (see Bilbiie *et al.*, 2008, 2014) or other homothetic preferences. The mechanism of amplification of the shocks presented here depends crucially on the elasticity of substitution between goods, which is indeed constant in the CES case but dependent on the number of goods with any other preferences.

Last, but not least, the natural extension of our framework to endogenous entry is a promising avenue to reproduce realistic reactions to supply and demand shocks and investigate optimal monetary policy. On one side, dynamic entry introduces an additional mechanism of propagation of the shocks, and establishes a link between inflation and the process of business creation, but it does not qualitatively affect the source of real rigidity developed here with a fixed number of firms. On the other side, endogenous entry creates intertemporal links that do not allow one to easily apply the linear-quadratic approach to monetary policy issues. Further progress can be obtained examining Ramsey-optimal allocations as in Faia (2012), Bilbiie *et. al.* (2014) and, under Calvo pricing, in Etro and Rossi (2013).

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Appendix: Derivation of the Welfare-Based Loss Function

Second order approximation of welfare around the efficient steady state

The efficient steady state is reached with a labor subsidy τ that eliminates the wedge between marginal rate of substitution (of consumption and labor) and the marginal product of labor:

$$\frac{\upsilon L^{\phi}}{C^{-1}} = (1+\tau)\, w = \frac{(1+\tau)\,(\theta-1)\,(n-1)}{\theta n - \theta + 1} A n^{\frac{1}{\theta-1}} \tag{54}$$

where $An^{\frac{1}{\theta-1}}$ is the marginal product of labor. This means that the optimal subsidy must be:

$$\tau^{*} = \frac{n}{(\theta-1)\,(n-1)} \tag{55}$$

To find the efficient output level, let us consider the labor market equilibrium equation. In log-deviations it implies $\widehat{mc}_t + \widehat{A}_t = \widehat{C}_t + \phi \widehat{L}_t$. From the resource constraint and the production function we have $\widehat{y}_t = \widehat{C}_t$ and $\widehat{L}_t = \widehat{y}_t - \widehat{A}_t$. Substituting into the labor market equilibrium yields $\widehat{mc}_t + \widehat{A}_t = \widehat{y}_t + \phi(\widehat{y}_t - \widehat{A}_t)$. Now, notice that under the flexible-price efficient equilibrium we must have marginal cost pricing: $\widehat{mc}_t = 0$. Imposing this we find $\widehat{A}_t = \widehat{y}_t + \phi(\widehat{y}_t - \widehat{A}_t)$. Solving for output we reach an expression for the behavior of the flexible-price efficient output, $\widehat{y}_t^{*} = \widehat{A}_t$, and we can define the output gap as:

$$x_t \equiv \hat{y}_t - y_t^{*} = \hat{y}_t - \widehat{A}_t$$

Let us take the second order Taylor expansion of the sub-utility function $U(C_t, L_t) = \log C_t - (1+\phi)^{-1} \upsilon L_t^{1+\phi}$ around the steady state ignoring terms of order higher than the second, $O\left(\|\xi\|^3\right)$. We will often use the second order approximations of relative deviations $\frac{Z_t-Z}{Z}\approx\hat{z}_t+\frac{1}{2}\hat{z}_t^2$ and $\left(\frac{Z_t-Z}{Z}\right)^2\approx\hat{z}_t^2$, where $\hat{z}_t=\log(Z_t/Z)$ is the log deviation of a variable Z_t from the efficient steady state value Z. This leads to:

$$\begin{aligned}
U\left(C_t,L_t\right)-U(C,L) &\approx U_C C\left(\frac{C_t-C}{C}\right)+\frac{U_{CC}C^2}{2}\left(\frac{C_t-C}{C}\right)^2+U_L L\left(\frac{L_t-L}{L}\right)+\frac{U_{LL}L^2}{2}\left(\frac{L_t-L}{L}\right)^2 \\
&= U_C C\left(\hat{y}_t+\frac{1}{2}\hat{y}_t^2\right)+\frac{1}{2}U_{CC}C^2\hat{y}_t^2+U_L L\left(\hat{l}_t+\frac{1}{2}\hat{l}_t^2\right)+\frac{1}{2}U_{LL}L^2\hat{l}_t^2 \\
&= \hat{y}_t+\frac{1}{2}\hat{y}_t^2+\frac{1}{2}\frac{U_{CC}C}{U_C}\hat{y}_t^2-\hat{l}_t-\frac{1}{2}\hat{l}_t^2-\frac{1}{2}\frac{U_{LL}L}{U_L}\hat{l}_t^2 \\
&= \hat{y}_t-\hat{l}_t-\frac{1+\phi}{2}\hat{l}_t^2
\end{aligned} \tag{56}$$

where the first line is the Taylor expansion, the second line uses second order approximations and the fact that $y_t=C_t$, the third line uses the efficiency condition for the steady state $U_C C=-U_L L=1$ under our functional form, and the fourth line uses $U_{CC}C/U_C=-1$ and $U_{LL}L/U_L=\phi$ under our functional form.

From the production function $C_t=A_tL_t=y_t\Delta_t$, where $\Delta_t\equiv\sum_{j=1}^{n}\left[p_t(j)/P_t\right]^{-\theta}$ represents price dispersion (though it is not an index larger than unity as with a continuum of goods in the unit interval). Since $\hat{l}_t=\hat{y}_t-\widehat{A}_t+\hat{\Delta}_t$ and $\hat{l}_t^2=\left(\hat{y}_t-\widehat{A}_t\right)^2=x_t^2$, we obtain:

$$U\left(C_t,L_t\right)-U(C,L)=-\hat{\Delta}_t-\frac{1}{2}\left(1+\phi\right)x_t^2+O\left(\|\xi\|^3\right)+t.i.p. \tag{57}$$

where $t.i.p.$ contains all terms independent from the policy.

We can now follow Woodford (2003) to show that for any number of goods n, we have $\hat{\Delta}_t=(\theta/2)\,Var_j\left[\log p_t\left(j\right)\right]$, where $Var_j\left[\log p_t\left(j\right)\right]$ is the cross-sectional variance of the log-prices. Define $s_t\left(i\right)\equiv p_t\left(i\right)/P_t$. Notice that in steady state $s_t\left(i\right)\to n^{1/(\theta-1)}$, $\Delta_t\to n^{1/(1-\theta)}$ and $\sum_{j=1}^{n}s_t(j)^{1-\theta}\to 1$. Then, up to second order we have:

$$\begin{aligned}
s_t\left(i\right)^{1-\theta} &= \frac{1}{n}+\left(1-\theta\right)n^{\theta-1}\left[\hat{s}_t\left(i\right)+\frac{1}{2}\hat{s}_t\left(i\right)^2\right]-\theta\left(1-\theta\right)\frac{n^{\theta-1}}{2}\hat{s}_t\left(i\right)^2+O\left(\|\xi\|^3\right)= \\
&= \frac{1}{n}+\left(1-\theta\right)n^{\theta-1}\hat{s}_t\left(i\right)+\frac{\left(1-\theta\right)^2n^{\theta-1}}{2}\hat{s}_t\left(i\right)^2+O\left(\|\xi\|^3\right)
\end{aligned} \tag{58}$$

Summing over all firms we have:

$$\sum\nolimits_{j=1}^{n}\hat{s}_t\left(j\right)\approx\frac{\theta-1}{2}\sum\nolimits_{j=1}^{n}\hat{s}_t\left(j\right)^2 \tag{59}$$

Up to second order we also have:

$$\begin{aligned}
s_t\left(i\right)^{-\theta} &= n^{\frac{\theta}{1-\theta}}-\theta n^{\frac{\theta}{1-\theta}}\left[\hat{s}_t\left(i\right)+\frac{1}{2}\hat{s}_t\left(i\right)^2\right]+\theta\left(\theta+1\right)\frac{n^{\frac{\theta}{1-\theta}}}{2}\hat{s}_t\left(i\right)^2+O\left(\|\xi\|^3\right)= \\
&= n^{\frac{\theta}{1-\theta}}-\theta n^{\frac{\theta}{1-\theta}}\hat{s}_t\left(i\right)+\theta^2\frac{n^{\frac{\theta}{1-\theta}}}{2}\hat{s}_t\left(i\right)^2+O\left(\|\xi\|^3\right)
\end{aligned} \tag{60}$$

where we used the previous relation. Summing over all firms we obtain:

$$\begin{aligned}
\Delta_t &\approx \sum\nolimits_{j=1}^{n} s_t(j)^{-\theta} = n^{\frac{1}{1-\theta}} - \theta n^{\frac{\theta}{1-\theta}} \sum\nolimits_{j=1}^{n} \hat{s}_t(j) + \theta^2 \frac{n^{\frac{\theta}{1-\theta}}}{2} \sum\nolimits_{j=1}^{n} \hat{s}_t(j)^2 = \\
&= n^{\frac{1}{1-\theta}} \left(1 + \frac{\theta}{2} \frac{\sum_{j=1}^{n} \hat{s}_t(j)^2}{n} \right) = n^{\frac{1}{1-\theta}} \left(1 + \frac{\theta}{2} Var_j \left[\log \frac{p_t(j)}{P_t} \right] \right) = \\
&= n^{\frac{1}{1-\theta}} \left(1 + \frac{\theta}{2} Var_j \left[\log p_t(j) \right] \right) \qquad (61)
\end{aligned}$$

Accordingly, we reach:

$$\hat{\Delta}_t = \log \frac{\Delta_t}{\Delta} = \log \left(1 + \frac{\theta}{2} Var_j \left[p_t(j) \right] \right) \approx \frac{\theta}{2} Var_j \left[\log p_t(j) \right] \qquad (62)$$

To relate this to the inflation rate, let us define $P_t^e = \frac{1}{n} \sum_{j=1}^{n} \log p_t(j)$, so that:

$$\begin{aligned}
\pi_t &\approx P_t^e - P_{t-1}^e = \frac{1}{n} \sum\nolimits_{j=1}^{n} \left[\log p_t(j) - P_{t-1}^e \right] = \\
&= \frac{\lambda}{n} \sum\nolimits_{j=1}^{n} \left[\log p_{t-1}(j) - P_{t-1}^e \right] + (1-\lambda) \left(\log p_t - P_{t-1}^e \right) = \\
&= 0 + (1-\lambda) \left(\log p_t - P_{t-1}^e \right) \qquad (63)
\end{aligned}$$

where the first step is an approximation, the second holds by definition, the third employs the adjusted price p_t for a fraction $1-\lambda$ of firms at time t, and the last exploits the lack of adjustment for the remaining fraction λ. We then obtain:

$$\begin{aligned}
Var_j \left[\log p_t(j) \right] &= Var_j \left[\log p_t(j) - P_{t-1}^e \right] = \\
&= \frac{1}{n} \sum\nolimits_{j=1}^{n} \left[\log p_t(j) - P_{t-1}^e \right]^2 - \left[\frac{1}{n} \sum\nolimits_{j=1}^{n} \left[\log p_t(j) - P_{t-1}^e \right] \right]^2 = \\
&= \lambda Var_j \left[\log p_{t-1}(j) \right] + (1-\lambda) \left(\log p_t - P_{t-1}^e \right)^2 - \left(P_t^e - P_{t-1}^e \right)^2 = \\
&\approx \lambda Var_j \left[\log p_{t-1}(j) \right] + \frac{\lambda}{1-\lambda} \pi_t^2 \qquad (64)
\end{aligned}$$

where the first line derives from the properties of cross-sectional variance, the second by its decomposition, the third by using the definition of variance in the previous period for the firms that do not adjust prices, and the fourth from the previous approximation of the inflation rate. Iterating forward one gets:

$$\sum_{t=0}^{\infty} \beta^t Var_j \left[\log p_t(j) \right] = \sum_{t=0}^{\infty} \beta^t \frac{\lambda \pi_t^2}{(1-\lambda)(1-\lambda\beta)} \qquad (65)$$

We finally combine all these results to confirm that, for any number of firms n, we obtain the standard approximation of the intertemporal welfare function at time t and the associated welfare loss $\mathcal{L}_t \equiv U(C, L)/(1-\beta) - \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, L_{t+k})$, or:

$$\mathcal{L}_t = \sum_{k=0}^{\infty} \beta^k \frac{1}{2} \left[\frac{\lambda\theta}{(1-\lambda)(1-\lambda\beta)} \pi_{t+k}^2 + (1+\phi) x_{t+k}^2 \right] + O\left(\|\xi\|^3 \right) + t.i.p. \qquad (66)$$

Second order approximation of welfare around the distorted steady state

We now derive a second order approximation of the utility function around the distorted steady state. In particular, as in Galì (2008, Ch. 5), we define the steady state distortion as the wedge between the marginal rate of substitution between consumption and labor and the marginal product of labor, both evaluated at the steady state. Formally, the steady state distortion Φ satisfies:

$$\frac{\upsilon L^{\phi}}{C^{-1}} = \left(1 - \Phi\left(\theta, n, \tau\right)\right) w = \frac{\left(1+\tau\right)\left(\theta-1\right)\left(n-1\right)}{\theta n - \theta + 1} A n^{\frac{1}{\theta-1}}$$

with:

$$\Phi\left(\theta, n, \tau\right) \equiv \frac{n - \tau\left(\theta-1\right)\left(n-1\right)}{\theta n - \theta + 1} \tag{67}$$

which becomes negligible when the elasticity of substitution and the number of firms are large enough or the subsidy is close to the optimal one.

The flexible-price efficient output is still given by $\widehat{y}_t^* = \widehat{A}_t$. The second order Taylor expansion of $U(C_t, L_t)$ around the distorted steady state, ignoring terms of order higher than the second, $O(\|\xi\|^3)$, is now:

$$\begin{aligned}
U\left(C_t, L_t\right) - U(C, L) &= \hat{y}_t + \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\frac{U_{CC}C}{U_C}\hat{y}_t^2 - \left(1 - \Phi\left(\theta, n, \tau\right)\right)\left(\hat{l}_t + \frac{1}{2}\hat{l}_t^2 + \frac{1}{2}\frac{U_{LL}L}{U_L}\hat{l}_t^2\right) = \\
&= \hat{y}_t - \hat{l}_t + \Phi\left(\theta, n, \tau\right)\hat{l}_t - \frac{1+\phi}{2}\hat{l}_t^2 \\
&= \hat{y}_t - \hat{y}_t + \Phi\left(\theta, n, \tau\right)\hat{y}_t - \frac{1+\phi}{2}\left(\hat{y}_t - \widehat{A}_t\right)^2 \\
&= \Phi\left(\theta, n, \tau\right)\hat{y}_t - \frac{1+\phi}{2}\hat{y}_t^2 - \frac{1+\phi}{2}\hat{y}_t\widehat{A}_t + t.i.p \\
&= -\hat{\Delta}_t - \frac{1}{2}\left(1+\phi\right)x_t^2 + \Phi\left(\theta, n, \tau\right)x_t + t.i.p.
\end{aligned} \tag{68}$$

where we used the fact that $\Phi\left(\theta, n, \tau\right)\left(1+\phi\right)\hat{l}_t^2/2$ is a term of third order and we defined the output gap as $x_t = \hat{y}_t - y_t^* = \hat{y}_t - \widehat{A}_t$. Expliciting $\hat{\Delta}_t$ as before, we finally get:

$$\mathcal{L}_t = \sum_{k=0}^{\infty}\beta^k \frac{1}{2}\left[\frac{\lambda\theta}{\left(1-\lambda\right)\left(1-\lambda\beta\right)}\pi_{t+k}^2 + 2\left(1+\phi\right)x_{t+k}^2 + \Phi\left(\theta, n, \tau\right)x_{t+k}\right] + t.i.p. + O\left(\|\xi\|^3\right) \tag{69}$$

which can be rewritten as in Proposition 4.