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**Conditional graphical models for
systemic risk measurement**

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Conditional graphical models for systemic risk measurement

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Abstract

Financial network models are a useful tool to model interconnectedness and systemic risks in financial systems. They are essentially descriptive, and based on highly correlated networks. In this paper we embed them in a stochastic framework, aimed at a more parsimonious and more realistic representation. First we introduce Gaussian graphical models in the field of systemic risk modelling, thus estimating the adjacency matrix of a network in a robust and coherent way. Second, we propose a conditional graphical model that can usefully decompose correlations between financial institutions into correlations between countries and correlations between institutions, within countries. While the former may be further explained by macroeconomic variables, the latter may be further explained by idiosyncratic balance sheet indicators. We have applied our proposed methods to the largest European banks, with the aim of identifying central institutions, more subject to contagion or, conversely, whose failure could result in further distress or breakdowns in the whole system. Our results show that, in the transmission of the perceived default risk, there is a strong country effect, that reflects the weakness and the strength of the underlying economies. In addition, each country reveals specific idiosyncratic factors, with communalities among similar countries.

Keywords: Conditional independence, network models, financial risk management

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1. Introduction

In the latest financial crisis, started in 2007, the core capital of banks has become insufficient to cover impairment losses arising from loans and security portfolios. Consequently, several banks have been strengthened their capital base or reduced their asset exposure. Other banks have been bailed out by state aids or have defaulted. To reduce the risk of similar crises in the future and to enhance the resilience of the banking sector, a new regulatory framework, the so-called Basel III package, has been proposed, implying more stringent capital requirements for financial institutions [6]. The effectiveness of the new regulatory framework to prevent banking default and financial crisis is an open problem, particularly as regulations themselves are still in progress, and may thus benefit from the results of research findings in the field.

Research studies on bank failures can be classified in two main streams: financial market models and scoring models.

Financial market models originate from the seminal paper of Merton [28], in which the market value of a bank's assets, typically modeled as a diffusion process, is insufficient to meet its liabilities. Due to its practical limitations, Merton's model has been evolved into the reduced form [34], leading to a widespread diffusion of the resulting model, and the related implementation in Basel II credit portfolio models. In order to implement market models, diffusion process parameters and, therefore, bank default probabilities can be obtained on the basis of share price data that can be collected almost in real time from financial markets. Market data are relatively easy to collect, are public, and are quite objective. On the other hand, they may not reflect the true fundamentals of the underlying financial institutions, and may lead to a biased estimation of the probability of failure. Indeed, the recent paper by [19] and [16] show that market models may be good in very short-term predictions, but not in medium and long-term ones, where the importance of fundamental financial data emerge.

Scoring models are typically based on financial fundamentals, taken from
30 publicly available balance sheets. Their diffusion has followed the seminal paper
by Altman [3], and has induced the production of scoring models for banks them-
selves: noticeable examples are [32], [33], [11]. The development of the Basel
regulation (www.bis.org) and the recent financial crisis have further boosted the
literature on scoring models for banking failure predictions. Recent examples
35 include [4], [13] and [22]. Scoring models have been extended in different ways:
interesting developments include the incorporation of macroeconomic compo-
nents (see e.g. [23], [27], [20] and [21]) and the explicit consideration of the
credit portfolio, as in the Symbol model of [14], that allows stress tests of bank-
ing asset quality and capital, as emphasized in the recent paper by [17]. The
40 problem with scoring models is that they are mostly based on balance sheet
data, which have, differently from the market, a low frequency of update (an-
nual or, at best, quarterly) and do depend on subjective management choices.
They may thus be good to predict defaults (especially in the medium term) but
not in the assessment of systemic risks, which occur very dynamically and with
45 short notice.

In this contribution we consider a novel mixed approach that uses both mar-
ket and balance sheet data. In addition, our focus will not be on the prediction
of single defaults but rather on how they are correlated with each other, in a
systemic perspective. The research literature on systemic risk is very recent,
50 and follows closely the developments of the recent financial crisis. A compre-
hensive review is provided in [9] who also provide a historical comparison of
different crisis. Specific measures of systemic risk have been proposed, in par-
ticular, by [2], [1], [8], [18], [7] and, from a different perspective, [30]. All of
these approaches are built on financial market price information, on the basis of
55 which they lead to the estimation of appropriate quantiles of the estimated loss
probability distribution of a financial institution, conditional on a crash event
in the financial market. These literature developments have led and are still
contributing to the identification of the Systemically Important Financial Insti-
tutions (SIFIs), at the global and regional level. They however do not address

60 the issue of how risks are transmitted between different institutions.

Trying to address this aspect of systemic risks, researchers have recently proposed financial network models. In particular, [7] propose several econometric measures of connectedness based on principal component analysis and Granger-causality networks. They find that hedge funds, banks and insurance companies
65 have become highly interrelated over the past decade, likely increasing the level of systemic risk through a complex and time-varying network of relationships. [10] and [5] follow similar approaches. Here we aim to statistically learn financial networks from the available data and, to achieve this aim, we propose embedding network models into multivariate graphical models. Graphical models are
70 based on the idea that interactions among random variables in a system can be represented in the form of graphs, whose nodes represent the variables and whose edges show their interactions. For an introduction to graphical models see, for example, [29, 26, 36, 35, 15].

In particular, graphical models can be employed to accurately estimate the
75 adjacency matrix, aimed at measuring interconnectedness between different financial institutions and, in particular, to assess central ones that may be the most contagion or the strongest source of contagion [7]. Network models use the correlation matrix estimated from the data to derive the adjacency matrix. Although useful, this approach takes into account only the marginal effect
80 of a variable on another, without looking at the indirect effect of other variables. Graphical models, instead, focus on the partial correlation matrix, that is obtained by measuring the direct correlation between two variables. A partial correlation coefficient can express the change in the expected value of a variable, caused by a unitary change of another variable, when the remaining variables
85 are held constant. In so doing, the effect of a bank on another is split into a direct effect (estimated by the partial correlation) and an indirect effect (what is left in the marginal correlation). Here we follow this approach and derive the adjacency matrix, the main input of a financial network model, not from the correlation matrix but, rather, from the partial correlation matrix obtained
90 from the application of graphical models to the available data.

The methodological contribution of this paper is a novel graphical model, that allows correlations between financial institutions to be decomposed into a country effect plus a bank-specific effect. This similarly to what is assumed for the asset returns in CAPM models [31].

95 The applied contribution of this paper is in the understanding of whether and how a distress probability is transmitted between different banks, that belong to different countries, with different regulatory systems. A very interesting case study, in this respect, is the Eurozone, where the European Central Bank is about to assume the supervision of the largest banks (with total assets greater
100 than 30bn euro) in each of the member states. Thus, eventually, the euro banking market will evolve into a single market but, at the time being, it is still fragmented. It thus becomes timely and rather interesting to study the degree of convergence towards a European banking union, looking at the comovements between share returns of the banks in the European area, and at their depen-
105 dence on both macroeconomic and balance sheet variables. In this respect, we remark that, to our knowledge, this is the first paper that models systemic risks using jointly market and balance sheet data.

The rest of the paper is organized as follows. In Section 2, we introduce the proposed methodology based on conditional graphical models. In Section 3
110 we describe the empirical results obtained with the application of our proposed model to data that concern the largest European banks. Finally, Section 4 contains some concluding remarks.

2. Methodology

In this section we first review graphical Gaussian models and, then, present
115 our methodological proposal.

2.1. Graphical gaussian models

Let $g = (V, E)$ be an undirected graph, with vertex set $V = \{1, \dots, n\}$, and edge set $E = V \times V$, a binary matrix, with elements e_{ij} , that describes whether

pairs of vertices are (symmetrically) linked between each other ($e_{ij} = 1$), or not ($e_{ij} = 0$). If the vertices V of the graph g are put in correspondence with a vector of random variables $X = X_1, \dots, X_n$, the edge set E induces conditional independence on X via the so-called Markov properties [25]. More precisely, the pairwise Markov property determined by the graph g states that, for all $1 \leq i < j \leq n$,

$$e_{ij} = 0 \iff X_i \perp X_j | X_{V \setminus \{i,j\}};$$

that is, the absence of an edge between vertices i and j is equivalent to independence between the random variables X_i and X_j , conditionally on all other variables $X_{V \setminus \{i,j\}}$.

120 Here we are concerned with quantitative random variables and, therefore, the graphical model we assume is a graphical Gaussian model, specified as follows.

Let $X = (X_1, \dots, X_n) \in R^n$ be a random vector distributed according to a multivariate normal distribution $\mathcal{N}(\mu, \Sigma)$. In this paper, without loss of generality, we will assume that the data are generated by a stationary process, and, 125 therefore, $\mu = 0$. In addition, we will assume throughout that the covariance matrix Σ is non singular.

[36] proved that the pairwise Markov property implies that the following equivalence holds, for graphical gaussian models:

$$X_i \perp X_j | X_{V \setminus \{i,j\}} \iff \rho_{ijV} = 0,$$

where

$$\rho_{ijV} = \frac{-\sigma^{ij}}{\sqrt{\sigma^{ii}\sigma^{jj}}}$$

130 denotes the ij -th partial correlation, that is, the correlation between X_i and X_j conditionally on the remaining variables $X_{V \setminus \{i,j\}}$.

Therefore, given an undirected graph $g = (V, E)$, a graphical Gaussian model can be defined as the family of all N -variate normal distributions $\mathcal{N}(0, \Sigma_g)$ that satisfy the constraints induced by a graph g on the variance-covariance matrix, 135 in terms of zero partial correlations.

Statistical inference for graphical models can be of two kinds: quantitative learning, which means that, given a graphical structure, with associated Markov

properties, data is employed to estimate the unknown parameters of the model; and structural learning, which means that the graphical structure itself is estimated on the basis of the data.

Here we focus on structural learning, as our aim is to infer from the data the network model that best describes the interrelationships between the financial institutions we consider. To achieve this aim, we now recall the expression of the likelihood of a graphical Gaussian model, on which structural learning will be based.

For a given graph g , consider a sample X of size n from $P = \mathcal{N}(0, \Sigma_g)$, and let S be the corresponding observed variance-covariance matrix. For a subset of vertices $A \subset N$, let Σ_A denote the variance-covariance matrix of the variables in X_A , and define with S_A the corresponding observed submatrix.

When the graph g is decomposable the likelihood of a graphical gaussian model specified by P nicely decomposes as follows (see e.g. [12]):

$$p(x|\Sigma, g) = \frac{\prod_{C \in \mathcal{C}} p(x_C|\Sigma_C)}{\prod_{S \in \mathcal{S}} p(x_S|\Sigma_S)},$$

where \mathcal{C} and \mathcal{S} respectively denote the set of cliques and separators of the graph G , and:

$$P(x_C|\Sigma_C) = (2\pi)^{-\frac{n|C|}{2}} |\Sigma_C|^{-n/2} \exp[-1/2 \text{tr} (S_C (\Sigma_C)^{-1})]$$

and similarly for $P(x_S|\Sigma_S)$.

Note that the likelihood depends on the parameter Σ and on g , which indicates the set of cliques and separators that factorise the likelihood, and determine which submatrices of Σ to consider.

Structural learning can be achieved replacing Σ with its maximum likelihood estimator, the observed sample variance covariance matrix S (constrained by g) and comparing all possible graphical models in terms of their resulting maximised likelihood. AIC and BIC criteria can enforce such choice adding penalty terms to the maximum likelihood, that depend on the number or parameters of the model (AIC) or on the number of observations (BIC).

2.2. Proposal

165 In high dimensional settings, such as those occurring in systemic risk modelling, there is a high chance that model selection algorithms used for structural learning get trapped, spending much time to learn local optimum structures which might not be optimal at a global level. In addition, it may be difficult to extract interpretable information from a learned structure that is large and
 170 shows many interrelationships. A possible solution to the above problems is to add more structure to graphical models, and this is what we propose. To ease understanding, and without loss of generality, we will from now on refer the notation to the specific systemic risk problem we face.

Assume that $R_{i,j,t}$ is a random variable representing the return for the i -th
 175 bank in the j -th country at time t and that \bar{R}_{jt} is the mean return of all banks present in country j , at time t . The joint distribution of all bank returns can then be factorised, using the country mean variables, as follows:

$$P(R_{11t}, \dots, R_{IJt}) = P(R_{11t}, \dots, R_{IJt} | \bar{R}_{1t}, \dots, \bar{R}_{Jt}) * p(\bar{R}_{1t}, \dots, \bar{R}_{Jt}).$$

Our main modelling assumption concerns the conditional distribution of bank returns, $P(R_{11t}, \dots, R_{IJt} | \bar{R}_{1t}, \dots, \bar{R}_{Jt})$. For any given time point we assume that:
 180

$$\begin{aligned} & P\left\{ \underbrace{(R_{11t}, \dots, R_{I1t})}_{\text{Country1}}, \underbrace{(R_{12t}, \dots, R_{I2t})}_{\text{Country2}}, \underbrace{(R_{1Jt}, \dots, R_{IJt})}_{\text{CountryJ}} \mid \bar{R}_{1t}, \dots, \bar{R}_{Jt} \right\} = \\ & = P(R_{11t}, \dots, R_{I1t} | \bar{R}_{1t}) * P(R_{12t}, \dots, R_{I2t} | \bar{R}_{2t}) * \dots * P(R_{1Jt}, \dots, R_{IJt} | \bar{R}_{Jt}). \end{aligned}$$

that is, returns of banks from different countries are assumed to be independent, conditionally on the knowledge of their country means. From an economic viewpoint, this assumption relies on the fact that, at least in Europe, countries
 185 and banks are strongly intertwined and, as the recent history has shown, the main source of market volatility of bank share prices is the state of the economy of the country where they operate.

According to the previous assumption, we can thus proceed decomposing the dependence between bank returns into a dependence between country means and a dependence between banks returns, within each country. Our second
190 assumption is that both dependence structures can be described by a graphical Gaussian model, as follows.

First, we assume that the vector of all bank returns $R_{i,j,t}$, conditionally on the country means, is distributed as a graphical Gaussian model with mean
195 $\mu_{j,t}$, the country mean. Indeed the (observed) country mean return can be subtracted out, leading to the extra returns $Y_{i,j,t} = R_{i,j,t} - \bar{R}_{j,t}$ be distributed as follows:

$$Y_t = \begin{pmatrix} Y_{1,1,t} \\ Y_{2,1,t} \\ \cdot \\ \cdot \\ Y_{I,1,t} \\ Y_{1,2,t} \\ Y_{2,2,t} \\ \cdot \\ \cdot \\ Y_{I,2,t} \\ \dots \\ Y_{1,J,t} \\ Y_{2,J,t} \\ Y_{I,J,t} \end{pmatrix} \sim N(0, \Sigma_b), t = 1, \dots, n,$$

where Σ_b indicates the (non singular) variance-covariance matrix between bank extra returns. According to the pairwise Markov property, the following
200 then holds, for any pair of banks (i, i') :

$$e_{ii'} = 0 \iff \rho_{ii'} = 0 :$$

a missing edge between two banks is equivalent to a zero partial correlation between the corresponding extra-returns.

Note that conditional independence between banks in different countries implies that Σ_b is a block-diagonal matrix, with each block describing the variance-covariance matrix between bank extra returns in each country:

$$\Sigma_b = \begin{pmatrix} \Sigma^{11} & 0 & 0 & 0 \\ & \Sigma^{22} & 0 & 0 \\ & & \dots & \\ & & & \Sigma^{JJ} \end{pmatrix}$$

Further constraints on the matrix Σ_b are to be estimated from the data, in a graphical model selection procedure, specific for each country, in accordance with the pairwise Markov property.

Second, we assume that the vector of all country mean returns, \bar{R}_{jt} , is also distributed as a graphical Gaussian model, with mean μ_t , the overall mean. The overall mean return can be subtracted out, leading to the extra returns $Z_{j,t} = \bar{R}_{jt} - \bar{R}_t$ distributed as follows:

$$Z_t = \begin{pmatrix} Z_{1,t} \\ Z_{2,t} \\ \dots \\ Z_{J,t} \end{pmatrix} \sim N(0, \Sigma_c), t = 1, \dots, n,$$

where Σ_c indicate the (non singular) variance-covariance matrix between country mean extra returns. According to the pairwise Markov property the following then holds, for any pair of countries j, j' :

$$e_{jj'} = 0 \iff \rho_{jj'} = 0.$$

a missing edge between two countries means is equivalent to a zero partial correlation between the corresponding extra-returns.

Further constraints on the matrix Σ_c are to be estimated from the data, in a graphical model selection procedure based on the country mean returns.

220 Note that the model we have specified is made up of two components: a graphical model between country mean returns and, conditionally on the mean returns of each country, a graphical model between bank returns of that country.

We now further extend our proposed model to take into account covariates that may explain the returns and their correlations. We will consider two types
 225 of covariates: macroeconomic ones, that may affect returns at the country level, and microeconomic ones, that may affect returns at the bank level. This will allow us "to mix" market data with bank specific balance sheet data, and with macroeconomic data as well. We remark that, to our knowledge, this is the first paper that models systemic risk using more than one data source.

230 Covariates can be introduced in our model as further conditioning variables. For example, we may condition country mean returns on macroeconomic variables, and check whether the correlations between countries are due to common macroeconomic cycles. Or we may condition bank returns on microeconomic balance sheet ratios and check whether the correlations between banks in a
 235 country are due to common bank management policies.

Mathematically, the introduction of covariates can be done maintaining the hierarchical structure seen before, specifying a linear regression model of bank returns on the covariates, both at the country and at the bank level and, then, model the regression residuals with a graphical Gaussian model.

240 More formally, we first assume that:

$$Y_t \sim N(X\beta, \Sigma_{b'}), t = 1, \dots, n,$$

where $\Sigma_{b'}$ is such that, for any pair of banks (i, i') :

$$e_{ii'} = 0 \iff \rho_{ii'} = 0,$$

and X is a data matrix containing balance sheet micro economic explanatory variables with β the corresponding vector of regression coefficients.

Second, we assume that:

$$Z_t \sim N(W\Gamma, \Sigma_{c'}), t = 1, \dots, n,$$

where $\Sigma_{c'}$ is such that, for any pair of countries (j, j') :

$$e_{jj''} = 0 \iff \rho_{jj'V} = 0,$$

245 and W is a data matrix containing macro-economic explanatory variables with Γ the corresponding vector of regression coefficients.

From an interpretational viewpoint, it is important to compare the selected graphical models, before and after the introduction of covariates. At the country level, it may happen that: i) c contains more partial correlations than c' :
250 this means that the correlation between the macroeconomic variables of two countries explains that between the mean returns; ii) c contains less partial correlations than c' : this means that financial markets adds relationships that are not coherent with macroeconomic fundamentals; they may be due, for example, to strategies of portfolio diversification of the investors.

255 The same kind of reasoning can be carried out at the bank-specific idiosyncratic level. In this case if b' contains more edges than b it can be interpreted as a sign that bank returns reflect common management strategies of banks (rather than specific behaviours of the markets). Conversely, if b contains less edges than b' there may be additional market strategies, not consistent with
260 fundamentals (for example, due to portfolio diversification, or to speculative behaviours).

3. Application

In this section we apply our proposed model to the estimation of the systemic risk of European banks. Europe is an interesting test case as banking
265 systems, that differ among many different countries, are progressing towards the integration in a single Banking Union.

We consider only large banks, whose total assets are greater than 30*b* euros, and are therefore included in the European Central Bank comprehensive assessment review, for the Eurozone countries. To obtain a more complete representation of European risks, we also consider countries in the European Union,
270 not belonging to the Eurozone, as well as Switzerland and Norway. We consider only publicly listed banks, for which market data are available. In the case of a banking group with more entities that satisfy the above criteria, we consider only the controlling entity. The complete list of the 61 considered financial institutions is in Table 1, with the corresponding ticker code acronyms. Table 1
275 contains, besides bank names and their codes, their prevalent country, and their Total Assets from the last available balance sheet (in thousands of euro), at 4Q 2013.

For each bank, we have collected data on their reported (quarterly) balance
280 sheet, as well as on their (daily) market performance, from Bankscope, for the period 2009-2013. Concerning balance sheets, as they are published in the quarter that follows their reference period, the corresponding data has been shifted by one quarter, to make it consistent with the period in which they are published. In more detail, from the balance sheet of each bank we have extracted
285 the ratio indicators suggested in the C.A.M.E.L. approach extensively employed in scoring models (see for example [4]), for a total of 9 candidate explanatory variables: Leverage (equity/total assets) and Tier1 (capital/risk weighted assets), that measure Capital adequacy; Coverage (loan reserves/gross loans) and Impairments (loan losses/gross losses), that measure Asset quality; Assets (total assets), that measure Management; NIM (net interest margin) and ROAA
290 (return/average assets), that measure Earnings; Liquid (liquid assets/total assets) and Loans (loans/assets) that measure Liquidity. We found some missing data, especially for ratios which are less common; we have decided to impute them using a regression based approach, when the missing variable is strongly
295 correlated with another (as it is the case for Coverage and Impairments), or a moving average approach, in other cases.

Market performance has instead been measured, consistently with balance

sheet data, taking the average quarterly share prices for each bank. Average prices have than been converted into returns. Let P_t be the average price of a bank in a quarter t . Its return can then be defined as: $R_t = \log(P_t/P_{t-1})$, where t is a quarter and $t - 1$ the quarter that precedes it.

Besides the above bank-specific data, we have extracted from Eurostat the main macroeconomic information from the countries to which the considered banks belong to, for the same period under consideration.

We now present the main results from the application of our conditional graphical gaussian models. We first search the graphical gaussian model that best fits the interdependences between the mean returns of the 20 considered countries.

Figure 1 shows the graphical network model between countries that is selected by the stepwise graphical model selection procedure implemented in the software R, packages gRbase and gRim.

Figure 1 about here

Figure 1 shows that the largest and strongest economies of Europe (DE, FR, GB) are related, and form an isolated clique. Southern and/or weaker economies (ES, PT, IE, AT, HU) are also related, with IT acting as a central agent of contagion. Other economies, typically smaller, are (weakly) connected to the latter, with a position that depends on the performance of their banks. In terms of systemic risk, Figure 1 shows that, besides IT, the most contagious/subject to contagion countries are NL (with its multinational bank, ING) and GR (with its troubled economy).

The links in Figure 1 may indeed be due to correlations in the macroeconomic cycles. This can be checked looking at the selected graphical models on the residuals from the regression of country mean returns on macroeconomic variables, reported in Figure 2.

Figure 2 about here

The structure of Figure 2 is similar to that of Figure 1: this indicates that the relationships among country mean returns are not due only to macroeconomic factors. We however underline two important differences. On one hand, CH

gets linked to the strongest economies, making them connected with the others,
330 and more central: this, presumably, on the basis of market portfolio strategies.
On the other hand, FI is removed from the group of weaker economies, revealing
that the link with IT that appears in Figure 1 may be due to a similar economic
cycle, but it is not supported by the market.

According to Figure 2 the most contagious/subject to contagion economies
335 are: IT , CH, GB, PL and IE: weaker economies and financial hubs, as one
would expect. For the sake of completeness, we report the results from the
regression of country mean returns on macroeconomic variables, on which the
graph of residuals in Figure 2 is based, in Table 2.

Table 2 about here

340 Table 2 shows that, while the GDP affects positively the mean returns, the
Unemployment rate and the Inflation rate are negatively related, at the overall
European level. The result for Unemployment and GDP indicates that down-
turn periods are associated to lower returns. The result for the inflation rate
indicates that in the considered deflationary period, the rate is negatively related
345 with the returns. These results are consistent with the economic literature.

Having estimated the country-to-country connections, we now look in detail
at the interdependencies between banks, within countries. Figures 3 through 6
present the selected graphical models for four selected Eurozone countries, two
from northern Europe (France and Germany) and two from southern Europe
350 (Spain and Italy).

Figures 3 through 6 about here.

Figure 3 shows that, in the case of Germany, the selected graph contains
three independent cliques: one that contains the large commercial banks (CBK
and DBK), and one that contains the smaller ones (ARL and WUW), with the
355 specialized bank IKB on its own, with the lowest systemic risk.

Figure 4 shows that, in France, all banks are pairwise connected, apart from
KN, which has three connections, and therefore, appears the most risk systemic.

Figure 5 shows that in Spain there is one central clique, composed of the
largest two banks, BBVA and SAN, connected with the medium sized banks

360 BKT and POP. SAB and CABK are instead disconnected. Therefore, in Spain,
the largest banks are also the most risk systemic.

Figure 6 shows the more complex case of Italy. The most connected bank
appears to be PMI, whereas the large bank UCG and the investment bank MB,
together with CVAL, are disconnected. Besides PMI, the banks that are more
365 risk systemic are the large sized ISP and the medium size BPSO.

We now investigate whether the previously found connections can be ex-
plained by a similar management strategy, as described by balance sheet ratios
or are, instead, due to other latent variables, such as market portfolio strategies.
To achieve this aim, we consider the graphical model selected on the basis of the
370 residuals from the regression model of bank extra returns on the balance sheet
ratios. Table 3 presents, in a combined form, the results of the regressions for
the four considered countries.

Table 3 about here.

From Table 3 note that the significant variables differ considerably among
375 countries, with a degree of similarity present between the two southern countries,
ES and IT and, on the other hand, between the two northern countries, DE and
FR.

In more detail, for both ES and IT, Asset quality has an important negative
sign on the returns (as expected), with an important difference between the two
380 countries: while in Italy what matters is the Coverage of credit losses, in Spain
both Impairments and Coverage are significant: higher Impairments are related
with lower returns, and higher coverages partly correct this effect, increasing
returns. Besides asset quality, the only other variable that significantly affect
the returns, in Italy, is Tier1, with a positive effect, as expected. In Spain,
385 instead, further significant variables are Liquid and NIM. While the former has
the expected positive sign, the sign of the net interest margin is instead coun-
terintuitive. One possible way to interpret it is in "reversing" the causal chain:
banks with more difficulties, reflected by a lower return, have been prompted
to improve, increasing the interest margin. None of the remaining variables are
390 significant in the two southern countries.

The structure of the estimated linear regression model for France and Germany is different from the previous one. For both countries, the significant ratios concern Asset quality, Management and Liquidity, with neither Capital adequacy nor Earnings being significant. Asset quality acts through the
395 Impairments variable, significant and negative for both countries (as expected); Management through the Assets variable, significant and positive for both countries (as expected); Liquidity through the Loans variable (for Germany) and the Liquid variable for France. None of the remaining variables are significant in the two northern countries.

400 To summarise, the findings in Table 3 emphasize the importance of the country effect, as similar countries have a similar structure of significant explanatory variables.

As done for the country mean returns we now evaluate whether the introduction of balance sheet variables changes the dependence structure between
405 the banks described in Figure 3 through 6. Figures 7 through 10 present the selected graphical model for the residuals of the same four countries: Germany, France, Spain and Italy.

Figures 7 through 10 about here.

Comparing Figures 7-10 with Figures 3-6 note that some new links are added,
410 and others are deleted.

In more detail: the graph of Germany and that of France are unchanged, revealing that the relationship between bank returns are not determined univocally by balance sheet ratios but, rather, by other factors, such as portfolio diversification strategies, based on the economical strength of the underlying
415 country. In the case of Spain, the only change concerns the position of POP which is now connected to SAN rather than BBVA. This means that SAN becomes the most central bank, from a systemic risk viewpoint. In Italy, the differences concern the positions of BPE which is now connected to ISP, rather than BMPS. This means that, when balance sheet data is taken into account,
420 BMPS becomes less contagious, and ISP more contagious. This is good news, given the difficult situation of BMPS at the moment and conversely the good

state of ISP.

Note, once, more, the difference between northern and southern countries: the estimated systemic risk network is affected by microeconomic factors only
425 for the latter, weaker economically and, therefore, more fragile from a banking point of view.

4. Conclusions

Financial network models are a useful tool to model interconnectedness and systemic risks in financial systems. Such models are essentially descriptive,
430 and based on highly correlated networks. This paper provides a stochastic framework for financial network models, based on graphical Gaussian models. In addition, it proposes a conditional graphical model that can usefully decompose dependencies between financial institutions into correlations between countries and correlations between institutions, within countries. A model that can be
435 extended to include explanatory covariates, both at the microeconomic and at the macroeconomic level.

We have applied our proposed methods to the largest European banks, with the aim of identifying central institutions, more subject to contagion or, conversely, whose failure could result in further distress or breakdowns in the whole
440 system. Our methods can use market prices, CAMEL based balance sheet ratios and macroeconomic variables, and it is therefore the first paper that considers jointly more than one source of data in the estimation of systemic risks.

From an interpretational viewpoint the paper shows that, in the transmission of the perceived default risk, there is a strong country effect, that reflects the
445 weakness and the strength of the underlying economies. Besides the country effect, the most central banks are those larger in size, consistently with the economic literature (as shown, in particular, in the recent paper by [24]).

Future applied research may include the extension of the model to different types of hierarchies of financial institutions (e.g. listed or unlisted; large or
450 small).

In addition, it would be interesting to insert in our proposed models further sources data: for example, analysts opinions, ratings and other sources of soft information.

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Table 1: The considered banks

Bank Name	Code	Country	T.Assets
HSBC Holdings	HSBA	UK	2040674.62
Deutsche Bank	DBK	Germany	2012329.00
BNP Paribas	BNP	France	1907290.00
Credit Agricole	ACA	France	1842361.00
Barclays	BARC	UK	1782411.78
Royal Bank of Scotland	RBS	UK	1569494.14
Banco Santander	SAN	Spain	1269628.00
Societe Generale	GLE	France	1250696.00
ING Groep NV	INGA	Netherlands	1168632.00
Lloyds Banking Group	LLOY	UK	1105756.67
UBS	UBSN	Switzerland	1041208.74
UniCredit	UCG	Italy	926827.50
Credit Suisse Group	CSGN	Switzerland	764250.28
Nordea Bank AB	NDA	Sweden	677309.00
Intesa Sanpaolo	ISP	Italy	673472.00
Banco Bilbao Vizcaya Argentaria	BBVA	Spain	637785.00
Commerzbank	CBK	Germany	635878.00
Natixis-BPCE Group	KN	France	528370.00
Standard Chartered	STAN	UK	482417.01
Danske Bank	DANSKE	Denmark	466755.96
Dexia	DEXB	Belgium	357210.00
DnB ASA	DNB	Norway	308173.44
Skandinaviska Enskilda Banken	SEB.A	Sweden	285875.07
Svenska Handelsbanken	SHB.A	Sweden	277776.39
KBC Groep NV	KBC	Belgium	256886.00
CIC Credit Mutuel Group	CC	France	235732.00
Banca Monte dei Paschi di Siena	BMPS	Italy	218882.20
Swedbank	SWED.A	Sweden	215194.90
Erste Group Bank	EBS	Austria	213824.00
Banco de Sabadell	SAB	Spain	161547.10
Banco Popular Espanol	POP	Spain	157618.10
Raiffeisen Bank International	RBI	Austria	136116.00
Unione di Banche Italiane	UBI	Italy	132433.70
Banco Popolare	BP	Italy	131921.40
Allied Irish Banks	AIB	Ireland	122516.00
National Bank of Greece	ETE	Greek	104798.80
Banco Comercial Portuguese	BCP	Portugal	89744.00
Banco Espirito Santo	BES	Portugal	83690.80
Wustenrot and Wurttembergische	WUW	Germany	77192.90
Mediobanca	MB	Italy	72841.30
Piraeus Bank	TPEIR	Greek	70406.20
Eurobank Ergasias	EUROB	Greek	67653.00
Banca popolare Emilia Romagna	BPE	Italy	61637.80
Alpha Bank	ALPHA	Greek	58357.40
Bankinter	BKT	Spain	58165.90
Banca Popolare di Milano	PMI	Italy	52475.00
Banca Carige	CRG	Italy	49325.80
PKO Bank Polski	PKO	Poland	47308.75
Aareal Bank	ARL	Germany	45734.00
Pohjola Bank Oyj	POH1S	Finland	44623.00
Banco BPI	BPI	Portugal	44564.60
Bank Pekao	PEO	Poland	36909.55
OTP Bank	OTP	Hungary	34694.23
Jyske Bank	JYSK	Denmark	34585.96
Banca Popolare di Sondrio	BPSO	Italy	32349.10
IKB Deutsche Industriebank	IKB	Germany	31593.70
Komerční Banka	BAAKOMB	Czech Republic	31295.88
Credito Emiliano	CE	Italy	30748.70
Credito Valtellinese	CVAL	Italy	29896.10

Table 2: Linear regression of country means on macroeconomic variables

<i>Dependent variable:</i>	
Country mean Return	
GDP	0.470*** (0.066)
Un	-0.056*** (0.012)
Inf.	-0.872*** (0.305)
Constant	0.701*** (0.148)
Observations	1,220
R ²	0.060
Adjusted R ²	0.058
Residual Std. Error	2.286 (df = 1216)
F Statistic	25.957*** (df = 3; 1216)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3: Linear Regressions of bank returns on balance sheet ratios

	<i>Dependent variable:</i>			
	share			
	IT	DE	FR	ES
Leverage				0.340 (0.223)
Tier1	0.218** (0.094)			
Coverage	-0.170* (0.099)		0.833 (0.511)	0.547** (0.236)
Impairments		-0.384*** (0.116)	-0.497* (0.293)	-0.394*** (0.148)
Assets _{std}		1.480** (0.671)	0.574* (0.328)	
NIM				-0.914** (0.455)
ROAA				
Liquid			0.040*** (0.015)	0.059* (0.035)
Loans		0.121** (0.047)		
Constant	-1.249 (0.823)	-3.733* (1.887)	-0.648 (0.600)	-0.692 (1.329)
Observations	240	100	100	120
R ²	0.028	0.116	0.079	0.106
Adjusted R ²	0.019	0.088	0.040	0.066
Residual Std. Error	2.500 (df = 237)	2.357 (df = 96)	1.699 (df = 95)	1.966 (df = 114)
F Statistic	3.376** (df = 2; 237)	4.199*** (df = 3; 96)	2.042* (df = 4; 95)	2.694** (df = 5; 114)

Note:

*p<0.1; **p<0.05; ***p<0.01

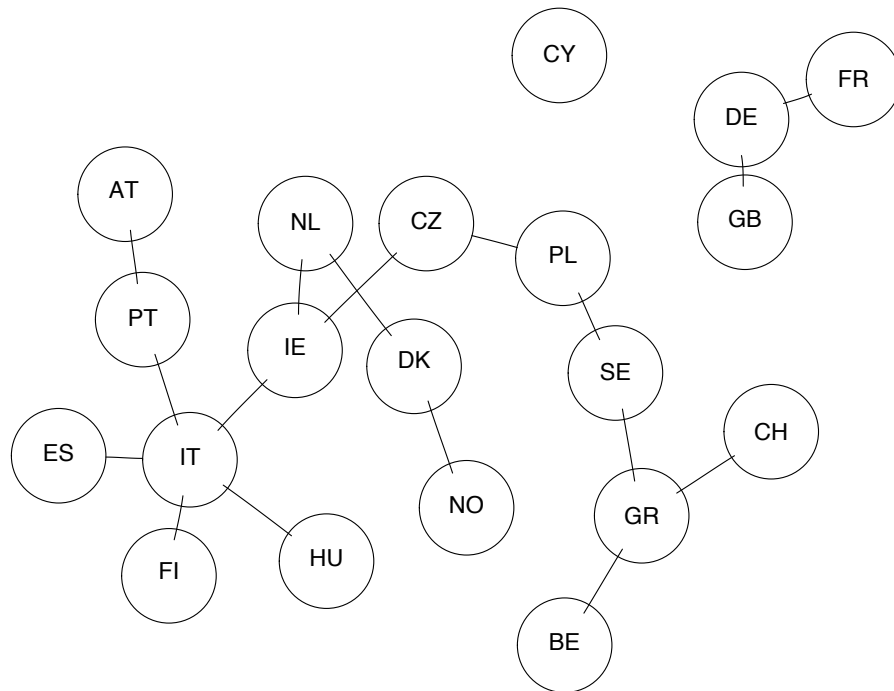


Figure 1: Selected graphical model between country mean returns

- | | | | | |
|--------------|---------------|------------------|------------------|--------------|
| (AT) Austria | (BE) Belgium | (CH) Switzerland | (DE) Germany | (DK) Denmark |
| (ES) Spain | (FI) Finland | (FR) France | (GB) UK | (GR) Greece |
| (HU) Hungary | (IE) Ireland | (IT) Italy | (NL) Netherlands | (NO) Norway |
| (PO) Poland | (PT) Portugal | (SE) Sweden. | | |

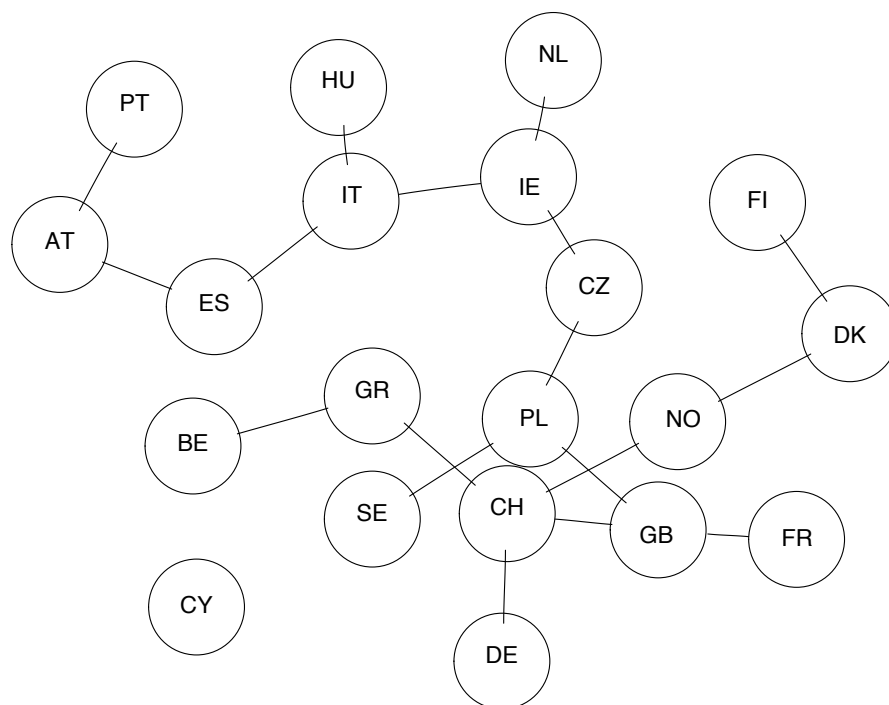


Figure 2: Selected graphical model between country mean returns, conditional on macroeconomic variables

(AT) Austria	(BE) Belgium	(CH) Switzerland	(DE) Germany	(DK) Denmark
(ES) Spain	(FI) Finland	(FR) France	(GB) UK	(GR) Greece
(HU) Hungary	(IE) Ireland	(IT) Italy	(NL) Netherlands	(NO) Norway
(PT) Portugal	(SE) Sweden.			

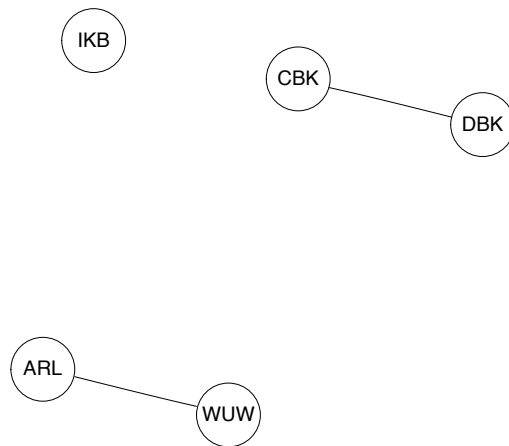


Figure 3: Selected graph of Germany bank returns
(ARL=Aareal Bank) (CBK=Commerzbank) (DBK=Deutsche Bank)
(IKB=IKB Deutsche Industriebank) (WUW=Wustenrot and Wurttembergische)

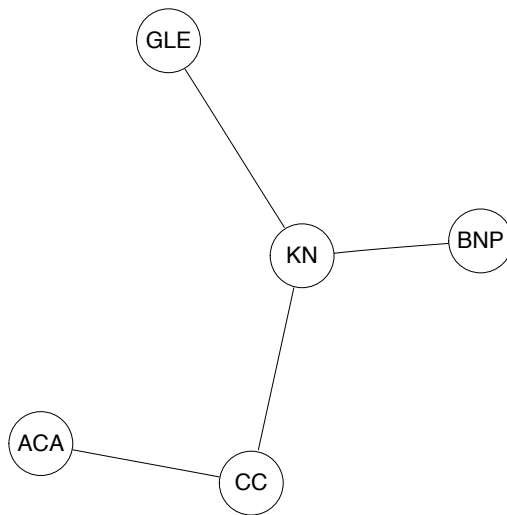


Figure 4: Selected graph of France bank returns
(ACA=Credit Agricole) (BNP=BNP Paribas) (CC=Credit Industriel et Commercial)
(GLE=Societe' Generale) (KN=Natixis)

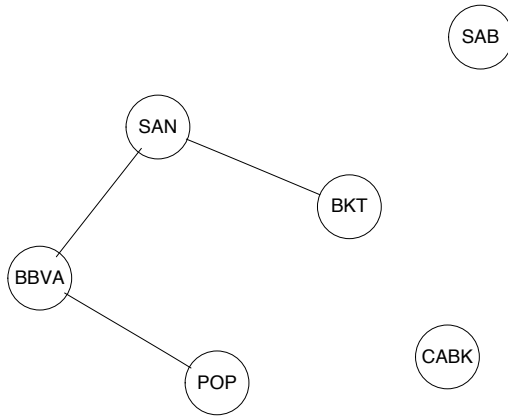


Figure 5: Selected graph of Spain bank returns

(BKT=Bankinter) (BBVA=Banco Bilbao Vizcaya Argentaria) (CABK=Caixabank)
 (POP=Banco Popular Espanol) (SAB=Banco de Sabadell) (SAN=Banco Santander)

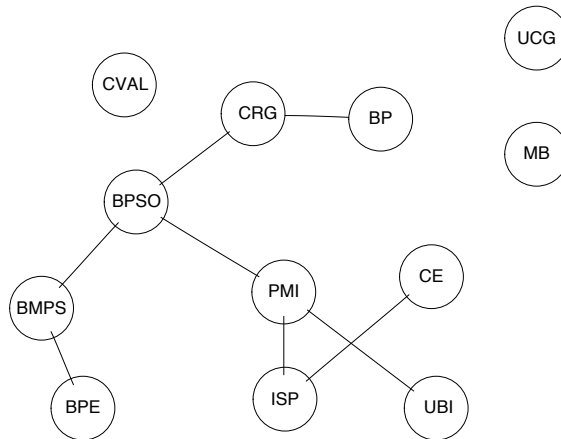


Figure 6: Selected graph of Italy bank returns

(BMPS=Banca Monte dei Paschi di Siena) (BP=Banco Popolare) (BPE=Banca popolare dell'Emilia Romagna)
 (BPSO=Banca Popolare di Sondrio) (CE=Credito Emiliano) (CRG=Banca Carige)
 (CVAL=Credito Valtellinese) (ISP=Intesa Sanpaolo) (MB=Mediobanca)
 (PMI=Banca Popolare di Milano) (UBI=Unione di Banche Italiane) (UCG=UniCredit)

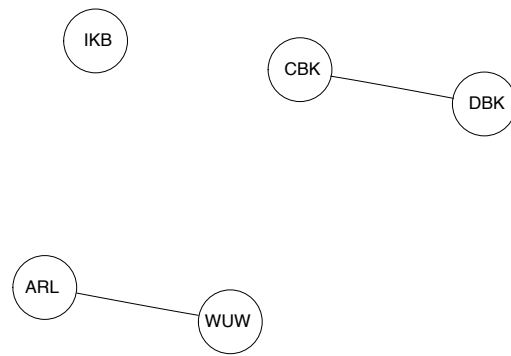


Figure 7: Selected graph of Germany bank residuals
(ARL=Aareal Bank) (CBK=Commerzbank) (DBK=Deutsche Bank)
(IKB=IKB Deutsche Industriebank) (WUW=Wstenrot and Wrtembergische)

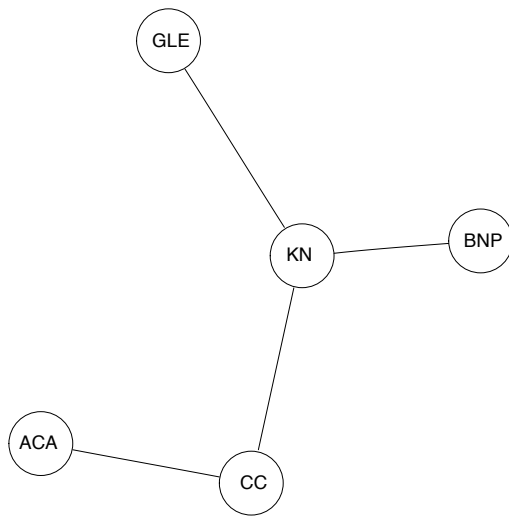


Figure 8: Selected graph of France bank residuals

(ACA=Crdit Agricole) (BNP=BNP Paribas) (CC=Credit Industriel et Commercial)

(GLE=Societe Generale) (KN=Natixis)

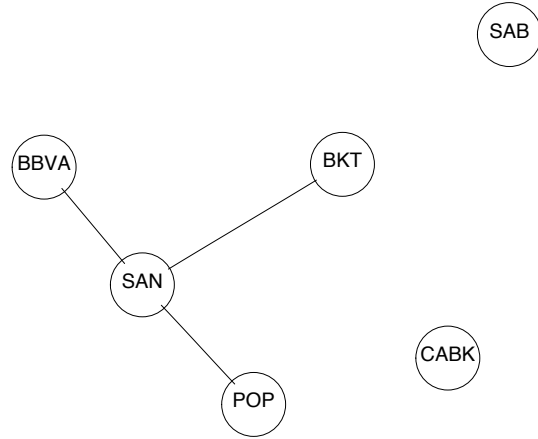


Figure 9: Selected graph of Spain bank residuals
 (BKT=Bankinter) (BBVA=Banco Bilbao Vizcaya Argentaria) (CABK=Caixabank)
 (POP=Banco Popular Espanol) (SAB=Banco de Sabadell)
 (SAN=Banco Santander)

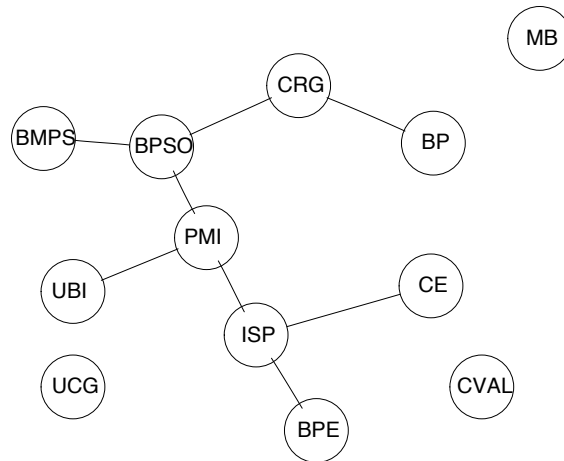


Figure 10: Selected graph of Italy bank residuals
 (BMPS=Banca Monte dei Paschi di Siena) (BP=Banco Popolare) (BPE=Banca popolare dell'Emilia Romagna)
 (BPSO=Banca Popolare di Sondrio) (CE=Credito Emiliano) (CRG=Banca Carige)
 (CVAL=Credito Valtellinese) (ISP=Intesa Sanpaolo) (MB=Mediobanca) (PMI=Banca Popolare di Milano)
 (UBI=Unione di Banche Italiane) (UCG=UniCredit)