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**Big data models of bank risk
contagion**

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Abstract

A very important area of financial risk management is systemic risk modelling, which concerns the estimation of the interrelationships between financial institutions, with the aim of establishing which of them are more central and, therefore, more contagious/subject to contagion.

The aim of this paper is to develop a systemic risk model which, differently from existing ones, employs not only the information contained in financial market prices, but also big data coming from financial tweets.

From a methodological viewpoint, we propose a new framework, based on graphical models, that can estimate systemic risks with models based on two different sources: financial markets and financial tweets, and suggest a way to combine them, using a Bayesian approach.

From an applied viewpoint, we present the first systemic risk model based on big data, and show that such a model can shed further light on the interrelationships between financial institutions.

This can help predicting the level of returns of a bank, conditionally on the others, for example when a shock occurs in another bank, or exogeneously.

Keywords: Financial Risk Management, Graphical models, Systemic risks, Twitter data analysis

1 Introduction

Systemic risk models address the issue of interdependence between financial institutions and, specifically, consider how bank default risks are transmitted among banks.

The study of bank defaults is important for two reasons. First, an understanding of the factors related to bank failure enables regulatory authorities to supervise banks more efficiently. In other words, if supervisors can detect problems early enough, regulatory actions can be taken, to prevent a bank from failing and, therefore, to reduce the costs of its bail-in, faced by shareholders, bondholders and depositors; as well as those of its bail-out, faced by the governments and, ultimately, by the taxpayers. Second, the failure of a bank very likely induces failures of other banks or of parts of the financial system. Understanding the determinants of a single bank failure may thus help to understand the determinants of financial systemic risks, were they due to microeconomic, idiosyncratic factors or to macroeconomic imbalances. When problems are detected, their causes can be removed or isolated, to limit “contagion effects”.

Most research papers on bank failures are based on financial market models, that originate from the seminal paper of Merton (1974), in which the market value of bank assets, typically modelled as a diffusion process, is matched against bank liabilities. Due to its practical limitations, Merton’s model has been evolved into a reduced form (see e.g. Vasicek, 1984), leading to a widespread diffusion of the resulting approach, and the related implementation in regulatory models.

The literature on systemic risk is very recent and follows closely the developments of the financial crisis, started in 2007. A comprehensive review is provided in Benoit et al. (2015). Specific measures of systemic risk have been proposed, in particular, by Adrian and Brunnermeier (2009), Acharya et al. (2010) and Brownless and Engle (2011). All of these approaches are built on financial market data, on the basis of which they lead to the estimation of appropriate quantiles of the estimated loss probability distribution of a financial institution, conditional on a crash event on the financial market. A different approach, explicitly geared towards estimation of the interrelationships among all institutions, is based on network models, and has been proposed in Billio et al. (2012).

Here we shall follow this latter approach, and add a stochastic framework, based on

graphical models. We will thus be able to derive, on the basis of market price data on a number of financial institutions, the network model that best describes their interrelationships and, therefore, explains how systemic risk is transmitted among them. In particular, we will consider categorical network models, which have not yet been employed in the framework of systemic risk.

It is well known that market prices are formed in complex interaction mechanisms that, often, reflect speculative behaviours rather than the fundamentals of the companies to which they refer. Market models and, specifically, financial network models based on market data may, therefore, reflect "spurious" components that could bias systemic risk estimation. This weakness of the market suggests to enrich financial market data with data coming from other, complementary, sources. Indeed, market prices are only one of the evaluations that are carried out on financial institutions: other relevant ones include ratings issued by rating agencies, reports of qualified financial analysts, and opinions of influential media.

Most of the previous sources are private, not available for data analysis. However, summary reports from them are now typically reported, almost in real time, in social networks and, in particular, in financial tweets. Therefore, big data offers the opportunity to extract from them very useful evaluation data that can complement market prices and that can, in addition, "replace" market information when not available (as it occurs for banks that are not listed). To extract from tweets data that can be assimilated to market prices, their text has to be preprocessed using semantic analysis techniques. In our context, if financial tweets on a number of banks are collected daily, it becomes possible to express, using semantic analysis, a daily "sentiment" towards them that expresses, for each day, how each considered bank is, on average, being evaluated by tweeterers.

In this paper we propose how to model semantic based tweet data, so to compare and integrate them with market data, within the framework of graphical network models and, in particular, using Bayesian models. We thus propose a novel usage of twitter data, aimed at assessing systemic risk with a graphical model built on daily variation of bank "sentiment".

The novelty of this paper is twofold. From a methodological viewpoint, we propose a framework, based on graphical models, that can estimate systemic risks with models

based on two different sources: financial markets and financial tweets, and suggest a way to combine them, using a Bayesian approach.

From an applied viewpoint, we present a systemic risk model based on big data, and show that such a model can shed further light on the interrelationships between financial institutions.

The rest of the paper is organised as follows: in Section 2 we introduce our proposal; in Section 3 we apply our proposal to financial and tweet data on the Italian banking market and, finally, in Section 4 we present some concluding remarks.

2 Methodology

We introduce the graphical network models that will be used to estimate relationships between N banks, both with market and tweet data.

Direct relationships between banks can be measured by their partial correlation, that expresses the direct influence of a bank on another. Partial correlations can be estimated assuming that the observations follow a graphical Gaussian model, in which Σ is constrained by the conditional independences described by a graph (see e.g. Lauritzen, 1996).

More formally, let $x = (x_1, \dots, x_N) \in R^N$ be a N -dimensional random vector distributed according to a multivariate normal distribution $\mathcal{N}(\mu, \Sigma)$. Without loss of generality, we will assume that the data are generated by a stationary process, and, therefore, $\mu = 0$. In addition, we will assume throughout that the covariance matrix Σ is not singular.

Let $G = (V, E)$ be an undirected graph, with vertex set $V = \{1, \dots, N\}$, and edge set $E = V \times V$, a binary matrix, with elements e_{ij} , that describe whether pairs of vertices are (symmetrically) linked between each other ($e_{ij} = 1$), or not ($e_{ij} = 0$). If the vertices V of this graph are put in correspondence with the random variables X_1, \dots, X_N , the edge set E induces conditional independence on X via the so-called Markov properties (see e.g. Lauritzen, 1996).

In particular, the pairwise Markov property determined by G states that, for all $1 \leq i < j \leq N$:

$$e_{ij} = 0 \iff X_i \perp X_j | X_{V \setminus \{i,j\}}; \tag{1}$$

that is, the absence of an edge between vertices i and j is equivalent to independence between the random variables X_i and X_j , conditionally on all other variables $x_{V \setminus \{i,j\}}$.

Let the elements of Σ^{-1} , the inverse of the variance-covariance matrix, be indicated as $\{\sigma^{ij}\}$, Whittaker (1990) proved that the following equivalence also holds:

$$X_i \perp X_j | X_{V \setminus \{i,j\}} \iff \rho_{ijV} = 0 \quad (2)$$

where

$$\rho_{ijV} = \frac{-\sigma^{ij}}{\sqrt{\sigma^{ii}\sigma^{jj}}} \quad (3)$$

denotes the ij -th partial correlation, that is, the correlation between X_i and X_j , conditionally on the remaining variables $X_{V \setminus \{i,j\}}$.

Therefore, by means of the pairwise Markov property, and given an undirected graph $G = (V, E)$, a graphical Gaussian model can be defined as the family of all N -variate normal distributions that satisfies the constraints induced by the graph on the partial correlations, as follows:

$$e_{ij} = 0 \iff \rho_{ijV} = 0 \quad (4)$$

for all $1 \leq i < j \leq N$.

Stochastic inference in graphical models may lead to two different types of learning: structural learning, which implies the estimation of the graphical structure G that best describes the data and quantitative learning, that aims at estimating the parameters of a graphical model, for a given graph. In the systemic risk framework, we are mainly interested in structural learning. Structural learning can be achieved by choosing the graphical structure with maximal likelihood, or its penalised versions, such as AIC and BIC. Here we follow the backward selection procedure implemented in the software R and, specifically, in the function `glasso` from package `glasso`.

For the aim of structural learning, we now recall the expression of the likelihood of a graphical model.

For a given graph G , consider a sample X of size n . For a subset of vertices $A \subset N$, let Σ_A denote the variance-covariance matrix of the variables in X_A , and define with S_A the corresponding observed variance-covariance sub-matrix. Similarly, in the discrete case, let

θ_A denote cell probability of the contingency table of the variables in X_A , and define with n_A the corresponding observed table counts.

When the graph G is decomposable (and we will assume so) the likelihood of the data, under the graphical model, nicely decomposes as follows (see e.g. Dawid and Lauritzen, 1993):

$$p(x|\Sigma, G) = \frac{\prod_{C \in \mathcal{C}} p(x_C|\Sigma_C)}{\prod_{S \in \mathcal{S}} p(x_S|\Sigma_S)}, \quad (5)$$

where \mathcal{C} and \mathcal{S} respectively denote the set of cliques and separators of the graph G .

For a continuous graphical model we then have:

$$P(x_C|\Sigma_C) \propto |\Sigma_C|^{-n/2} \exp\left[-\frac{1}{2} \text{tr}(S_C(\Sigma_C)^{-1})\right] \quad (6)$$

and similarly for $P(x_S|\Sigma_S)$.

Operationally, a model selection procedure compares different G structures by calculating the previous likelihood substituting for Σ its maximum likelihood estimator under G . For a complete (fully connected) graphical Gaussian model such an estimator is simply the observed variance-covariance matrix. For a general (decomposable) incomplete graph, an iterative procedure, based on the clique and separators of a graph, must be undertaken (see e.g. Lauritzen, 1996). Here we will follow a Bayesian model selection procedure, based on the Markov Chain Monte Carlo algorithm described in Giudici and Green (1999).

Through model selection, we obtain a graphical model that can be used to describe relationships between banks and, specifically, to understand how risks propagate in a systemic risk perspective. More precisely, in our context, we select one graphical model for each given data source: one from market data and one from tweet data.

Besides comparing the two models, it is quite natural to aim at integrating them into a single model. This task can be achieved within a Bayesian framework, as follows.

We first specify a prior distribution for the parameter Σ and for θ . Dawid and Lauritzen (1993) propose a convenient prior for both of them, respectively the hyper inverse Wishart and hyper Dirichlet distributions.

Hyper inverse Wishart can be obtained from a collection of clique specific marginal inverse Wishart as follows:

$$l(\Sigma) = \frac{\prod_{C \in \mathcal{C}} l(\Sigma_C)}{\prod_{S \in \mathcal{S}} l(\Sigma_S)}, \quad (7)$$

where $l(\Sigma_C)$ is the density of an inverse Wishart distribution, with hyperparameters T_C and α , and similarly for $l(\Sigma_S)$. For the definition of the hyperparameters here we follow Giudici and Green (1999) and let T_C and T_S be the submatrices of a larger "scale" matrix T_0 of dimension $N \times N$, and choose $\alpha > N$.

Dawid and Lauritzen (1996) and Giudici and Green (1999) show that, under the previous assumptions, the posterior distribution of the variance-covariance matrix Σ is a hyper Wishart distribution with $\alpha + n$ degrees of freedom and a scale matrix given by:

$$T_n = T_0 + S_n \tag{8}$$

where S_n is the sample variance-covariance matrix.

The previous result can be used to combine market data with tweet data, assuming that the former represent "data" and the latter "prior information" in a Bayesian prior to posterior analysis.

To achieve this task we recall that, under a complete, fully connected graph, the expected value of the previous inverse Wishart is:

$$E(\Sigma|X) = T_n = (T_0 + S_n)/(\alpha + n) \tag{9}$$

and, therefore, the Bayesian estimator of the unknown variance covariance matrix, the a posteriori mean, is a linear combination between the prior (tweet) mean and the observed (market) mean.

When the graph G is not complete, a similar result holds locally, at the level of each clique and separator.

The previous results suggest to use the above posterior mean as the variance-covariance matrix of a complete graph on which to base (backward) model selection, thereby leading to a new selected graphical model, based on a "mixed" data source, that contains both financial and tweet data, in proportions determined by the quantities α and n .

We now derive our bank risk contagion model, based on the partial correlation coefficients estimated conditionally on the selected graphical models. Recall that the partial correlation coefficient can be expressed as the geometric mean between two regression coefficients, in a multiple regression model. More formally:

$$\rho_{ijV} = \rho_{jiV} = \sqrt{a_{ijV} \cdot a_{jiV}}. \quad (10)$$

where a_{ijV} and a_{jiV} are, respectively, the regression coefficient of the multiple regression of x_i on all other V variables (including x_j) and the regression coefficient of the multiple regression of x_j on all other V variables (including x_i).

Our proposed contagion model is based on what usually assumed in scoring models: the score of a bank, assuming to be Gaussian, can be obtained from its rating class, through the inversion of the cumulative distribution. Formally:

$$Z_i^0 = \phi^{-1}(1 - \pi_i)$$

with $\pi_i =$ default probability.

We extend this assumption by subtracting from the classical score the contagion effect, expressed as a linear combination of the scores of the connected banks, with weights that are the partial correlation coefficients. More formally, we assume that:

$$Z'_i = \phi^{-1}(1 - \pi_0) - \sum_{j \in \text{neigh}(i)} a_{ij|\text{rest}} \phi^{-1}(1 - \pi_i)$$

where $a_{ij|\text{rest}}$ is the partial correlation coefficient between variables X_i and X_j given all the others (rest).

The default probabilities π_i can be calculated from available company ratings, or from scoring models (as in Cerchiello and Giudici, 2015). The partial correlation coefficients, $a_{ij|\text{rest}}$ will instead be obtained from our proposed Bayesian models by means of a Markov Chain Monte Carlo simulation algorithm for graphical Gaussian model selection and averaging.

A final consideration is about the dimension of a bank. The contagion effect of a large bank is indeed higher than that of a smaller one. For this reason we have modified the contagion formula as follows:

$$Z'_i = \phi^{-1}(1 - \pi_0) - \sum_{j \in \text{neigh}(i)} a_{ij|\text{rest}} \phi^{-1}(1 - \pi_i) \frac{A_j}{A_{\text{tot}}}$$

where A_j is the asset value of bank j and A_{tot} is the total asset value of all the considered banks.

3 Application

In this section we consider the application of our proposed methodology. For reasons of information homogeneity we concentrate on a single banking market: the Italian banking system, characterised by a large number of important banks, dominating the economy of the country, in a rapidly changing environment. We focus on large listed banks, for which there exists daily financial market data, that we would like to compare and integrate with tweet data.

The list of banks that we consider, along with their total assets at the end of the last quarter of 2013 (in Euro), a measure of bank size, is contained in Table 1. Banks are described by their stock market code (ticker).

Table 1 about here.

Bank Name	Ticker	Total Assets
UniCredit	UCG	926827
Intesa Sanpaolo	ISP	673472
Banca Monte dei Paschi di Siena	BMPS	218882
Unione di Banche Italiane	UBI	132433
Banco Popolare	BP	131921
Mediobanca	MB	72841
Banca popolare Emilia Romagna	BPE	61637
Banca Popolare di Milano	PMI	52475
Banca Carige	CRG	49325
Banca Popolare di Sondrio	BPSO	32349
Credito Emiliano	CE	30748
Credito Valtellinese	CVAL	29896

Table 1: List of considered listed Italian Banks

For each bank we consider the daily return, obtained from the closing price of financial markets, for a period of 148 consecutive days from July 2013 to February 2014, as follows:

$$R_t = \log(P_t/P_{t-1}),$$

where t is a day, $t - 1$ the day that precedes it and P_t (P_{t-1}) the corresponding closing price of that bank in that day.

For the same period, we have crawled Twitter, using the package `TwitterR`, available open source within the R project environment, and chosen all tweets that contain, besides one of the banks in Table 1, a keyword belonging to a financial taxonomy, that we have built, based on our knowledge of which balance sheet information may affect systemic risk. Each obtained tweet has been transformed into a sentiment class, with categories ranging from 1 to 5. The higher the category, the more positive the sentiment (or value) that the tweet assigns to the bank under analysis.

Table 2 describes our proposed taxonomy, along with the frequency and average sentiment associated to each keyword in our considered database.

Table 2 about here

Item	Frequency*100	Average Sentiment
Commissions	0.03	2.67
Labour costs	1.49	3.21
Deposits	0.08	2.83
Interbank	0.14	2.19
Management	28.58	3.01
Interest margin	4.91	2.79
Subsidiaries	0.99	3.02
Capital	35.67	3.07
Loan losses	0.73	2.90
Loans	10.11	2.93

Table 2: Taxonomy proposed and descriptive sentiment analysis

For each bank we have then calculated a sentiment daily variation, that mimics market

returns, as follows:

$$S_t = \log(T_t/T_{t-1}) \tag{11}$$

where t is a day, $t - 1$ the day that precedes it, and T_t is the corresponding average daily sentiment on that bank for that day.

From a descriptive viewpoint, we expect the market and the tweet "returns" to show some degree of correlation although, given their different informational content, we do not expect such correlation to be very high. Table 3 reports, for each bank, the correlation between financial returns and sentiment returns.

Table 3 about here.

Bank Ticker	Correlation
UCG	0.31
ISP	0.23
BMPS	0.16
UBI	0.33
BP	0.16
MB	0.27
BPE	0.24
PMI	0.26
CRG	0.09
BPSO	0.08
CE	0.14
CVAL	0.22

Table 3: Correlation between financial and sentiment returns

From Table 3 note that correlations are low, especially for smaller banks, that have less tweet information, and this was expected. A detailed inspection of each bank tweet data reveals that banks of similar size show a higher correlation when more information is

disclosed. This explains, for example, the difference in the market-sentiment correlation of UCG and ISP, that between CRG and PMI as well as that between BPSO and CVAL.

We now consider the selected graphical model, obtained by means of (backward) model selection from the mixed data source, obtained by averaging the complete variance-covariance matrices of financial and tweet data, as described in the previous Section.

Figure 1 reports the selected model. For the sake of simplicity, and without loss of generality, we have taken $\alpha = n$ so that the market and the tweet data component have equal weights. In the graph, each bank is indicated with its ticker code.

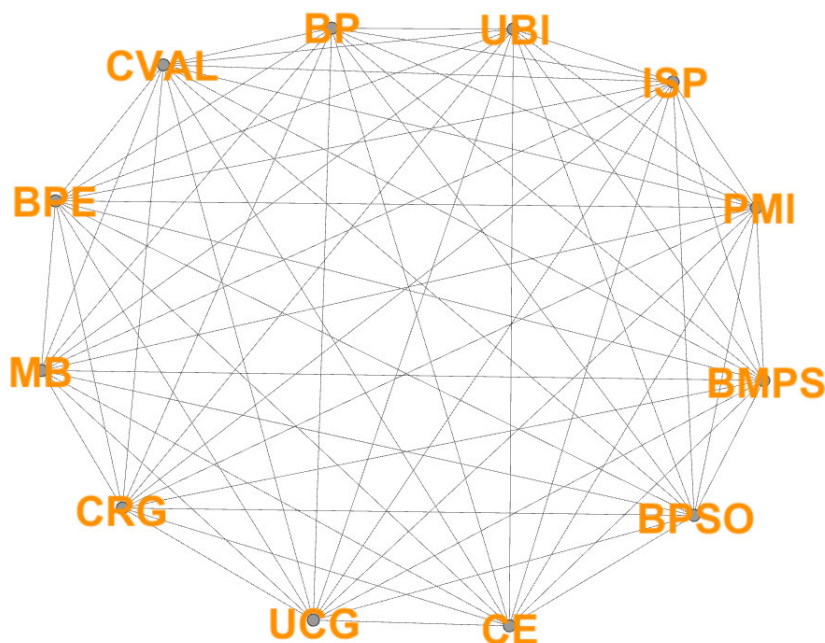


Figure 1: Bayesian gaussian graphical model on mixed data

The graph in Figure 1 shows that banks are all correlated with each other, and this is reasonable, in a period in which country effects prevail in the determination of share returns. The results in Figure 1 can be better interpreted looking at the corresponding estimated partial correlation coefficients, that we report in Table 4.

Table 4 indicates the partial correlation coefficients obtained by model averaging the graphical Gaussian models selected by the MCMC procedure. We also report, as a systemic risk measure for each bank, the weighted degree, obtained by summing all partial

correlations of a bank with the other institutions.

Table 4: Partial correlations and systemic risk measures based on the selected mixed graphical Gaussian model

Bank	UCG	MB	UBI	ISP	CVAL	CE	BP	BPSO	PMI	BPE	BMPS	CRG	Weighted degree
UCG	1.00	0.01	0.16	0.41	-0.11	-0.04	0.19	0.05	0.09	0.11	0.01	-0.01	0.81
MB	0.01	1.00	0.20	0.03	-0.11	-0.04	0.26	-0.07	0.08	0.26	0.08	-0.03	0.67
UBI	0.16	0.20	1.00	0.18	0.11	0.10	-0.05	0.10	-0.06	0.05	0.08	0.03	0.90
ISP	0.41	0.03	0.18	1.00	-0.09	0.02	-0.00	-0.01	0.13	0.01	0.01	0.00	0.70
CVAL	-0.11	-0.11	0.11	-0.09	1.00	-0.01	0.25	0.00	0.07	0.06	0.12	0.06	0.35
CE	-0.04	-0.04	0.10	0.02	-0.01	1.00	-0.02	0.09	0.08	0.14	-0.05	-0.04	0.24
BP	0.19	0.26	-0.05	0.00	0.25	-0.02	1.00	0.03	0.16	0.23	-0.01	-0.01	1.02
BPSO	0.05	-0.07	0.10	-0.01	0.00	0.09	0.03	1.00	0.15	0.04	0.00	0.05	0.43
PMI	0.09	0.08	-0.06	0.13	0.07	0.08	0.16	0.015	1.00	0.10	0.14	0.04	0.97
BPE	0.11	0.26	0.05	0.01	0.006	0.14	0.23	0.04	0.10	1.00	0.02	0.06	1.09
BMPS	0.01	0.08	0.08	0.01	0.12	-0.05	-0.01	0.00	0.14	0.02	1.00	0.11	0.51
CRG	-0.06	-0.03	0.03	0.00	0.06	-0.04	-0.01	0.05	0.04	0.06	0.11	1.00	0.20

Table 4 indicates, in a clear way, which are the most systemic banks: BPE, BP, PMI, UBI, , followed by the large banks UCG and ISP. Table 4 is also very useful to draw "stress test" analysis, such as: of UCG returns drop by 100 basis points, each of the eleven connected banks drop, on average, by 6.75 basis points, with a total impact on the system of 81 basis point. A similar drop in a smaller and relatively isolated bank, such as CE, causes a total impact of only 24 basis points.

Finally, Table 5 shows how contagion modifies the idiosyncratic PD obtained from the credit rating.

Bank	Original PD	Contagious PD	Delta %
UCG	0.0020	0.0064	+220%
UBI	0.0020	0.0059	+195%
MB	0.0020	0.0030	+50%
ISP	0.0020	0.0086	+330%
CVAL	0.0076	0.0055	-28%
CE	0.0076	0.0073	-4%
BP	0.0076	0.1450	+91%
BPSO	0.0020	0.0025	+25%
PMI	0.0076	0.1450	+108%
BPE	0.0076	0.1340	+76%
BMPS	0.0020	0.0024	+20%
CRG	0.0076	0.0070	-8%

Table 5 indicates that the contagion effect is stronger for larger banks, such as ISP and UCG, both highly correlated to each other, and also for UBI, that is correlated with both ISP and UCG.

4 Conclusions

In this paper we have shown how big data and, specifically, tweet data, can be usefully employed in the field of financial systemic risk modelling and, specifically, by means of categorical graphical models.

The paper shows how to combine tweet based systemic risk networks with those obtained from financial market data, using the a posteriori Bayesian mean of the parameter of interest (the variance-covariance matrix, in the continuous case, or the contingency table cell probabilities, in the categorical case).

We believe that our proposal can be very useful to estimate systemic risk and, therefore, to individuate the most contagious/subject to contagion financial institutions. This because it can compare and integrate two different, albeit complementary, sources of information:

market prices and twitter information. In addition, categorical models seem to provide an overall risk measure that is more selective.

Another important value of the model is its capability of including in systemic risk networks institutions that are not publicly listed, using the tweet component alone: a relevant advantage for banking systems a where many banks are not listed.

5 Authors contributions

The article is the result of the close collaboration between the authors; however PC supervised the statistical part; PG the economic part and GN the computational part.

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