ISSN: 2281-1346

Department of Economics and Management

DEM Working Paper Series

Bail in or Bail out? The Atlante example from a systemic risk perspective

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 # 124 (06-16)

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http://epmq.unipv.eu/site/home.html

June 2016

Bail in or Bail out? The Atlante example from a systemic risk perspective

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June 15, 2016

Abstract

Giudici and Parisi (2016) have proposed a novel econometric approach that measures systemic risk as a probabilistic "add-on" to the idiosyncratic probability of default of an economic sector (sovereign, corporate or bank). In this contribution we extend their approach to financial insitutions and, doing so, we investigate the relative advantage, in terms of systemic risk, of a bail-in versus a bail-out scenario. We apply our methods to the Italian bail-out private intervention scheme named Atlante. The results show that the bail-out of a troubled bank and, specifically of the Banca Popolare di Vicenza, is more convenient for the smaller, safer and highly correlated banks.

1 Systemic risk

The study of systemic risk is particularly problematic, because of the high number of dimensions that can be included: accordingly, different perspectives have been adopted and, therefore, different econometric measurement models have been used and applied to a variety of data, in different geographical regions and periods. For simplicity, we have chosen two main discriminant factors, thus dividing systemic risk models into three main categories: bivariate models, causal models and network models. While the first two explicitly deal with the time-dimension, in an endogenous or in an exogenous way, the latter focuses on the cross-sectional dimension.

Bivariate Models. From a chronological viewpoint, the first systemic risk measures have been proposed for the financial sector, in particular by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015). On the basis of market share prices, these models consider systemic risk as endogenously determined and calculate, as in the classical market risk approach, appropriate percentiles of the estimated loss probability distribution of a bank, conditional on an extreme event in the financial market.

The above described methodology is useful to identify the most *systemically important* institutions, since its bivariate nature allows the derivation of conditional default probabilities or losses during shock events in the reference market, possibly caused by other institutions. However, it does not address the issue of how risks are transmitted between different institutions, in a multivariate framework.

Causal Models. A different stream of research considers systemic risks as exogenous factors and has been proposed, among others, by Chong et al. (2006), Longstaff (2010) and Shleifer and Vishny (2010), who examined the impact of monetary policies on default probabilities for the banking sector, with a particular focus on crisis periods. More general causal models, proposed by Duffie and Lando (2001), Lando and Nielsen (2010), Koopman et al. (2012), Betz et al. (2014) and Duprey et al. (2015), explain whether the default probability of a bank, a country, or a company depends on a set of exogenous risk sources, thus combining idiosyncratic with systematic factors.

While powerful from an early warning perspective, causal models, similarly to bivariate ones, concentrate on single institutions rather than on the economic system as a whole and, therefore, underestimate systemic sources of risk arising from contagion effects within the system.

Network Models. In order to address the multivariate nature of systemic risk, researchers have recently proposed financial network models, able to combine the rich structure of network models (see, e.g., Lorenz et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who derive connectedness measures based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013), Ahelegbey et al. (2015) and Giudici and Spelta (2016) extend such methodology introducing correlation network models, while Das (2015) derives a systemic risk decomposition into individual and network contributions.

Financial network models are very useful for identifying the most important contagion channels in a cross-sectional perspective, thus identifying the most *vulnerable* institutions. However, since they are built on cross-sectional data, they can not be used as predictive models in a time-varying context. Moreover, the importance of each institution only depends on its position in the graph, and not on its specific risk.

Our proposal. Bivariate and causal models explain whether the risk of a bank, a company, or of a country, is affected by a market crisis event or by a set of exogenous risk factors; financial network models explain whether the same risk depends on contagion effects. Giudici and Parisi (2016) improve all these three classes of models, in the context of country risk, introducing multivariate stochastic processes and combining them with correlation network models, thus "correcting" individual default probabilities into a total default probability that takes contagion into account. Doing so, they merge the advantages of bivariate models (endogeneity and non-linearity), causal models (predictive capability) and correlation networks (contagion channels).

In this work we propose to extend the previous approach at the micro level: in particular, we combine default probabilities of financial institutions with correlation networks, thus deriving time-dependent measures able to explain to what extent the default probability of each financial institution is affected by a contagion effect that comes from the variation in the default probabilities of the other institutions.

The proposed methodology will be employed to analyse the main differences between bail-in and bail-out scenarios, that may occur in case one financial institution is close to its default point. In particular, we will simulate two alternatives: a) the "troubled" institution defaults, thus affecting the other banks in the system through

contagion propagation; b) the "troubled" institution is helped by the other banks in the system through a capital-lending operation. In the first situation, that we will call the bail-in scenario, the troubled bank's default affects its neighbours through a shock in their default probabilities derived from contagion effects: however, after a while the bank system will reach a new equilibrium, without the defaulted bank and, thus, affected by less contagion risk. In the second situation, that we will call the bail-out scenario, the troubled bank does not default and, thus, does not affect the others through a shock in their default probabilities; however, it continues being part of the banking system, so that all the other banks in the network will still be affected by the high contagion risk derived from its presence.

The design of these two scenarios allows to establish which banks in the system would benefit from a bail-out, rather than from a bail-in scenario. Precisely, each bank can choose the scenario that leads to the lowest total probability of default.

The above described research design will be firstly applied to the stylised case of three banks, of which two are safe and one troubled, and will then be extended to the Italian banking system. This is a particularly interesting case study, since in early 2016 Italian banks have organised themselves by supporting an equity fund, called Atlante, which has among its main aims the recapitalisation of "troubled" financial institutions. Each bank has decided, on voluntary basis, whether to allocate capital in the Atlante fund: as a result, a medium size lender, Banca Popolare di Vicenza, that had been found strongly under capitalised by the European Central Bank regulatory supervisor, has been recapitalised with the help of most of the banks in the system, thus avoiding bail-in.

Through simulation exercises, we will examine wether the choice of each bank (to take part in a bail-out or not) can be considered as the best one from a systemic risk perspective; in particular, we will examine whether and by how much the advantage of choosing a bail-out rather than a bail-in scenario depends on the default probability and on the size of the safe banks, on the default probability of the troubled bank, on the correlations with it and, last, on the correlations between the safe banks.

Our results can be summarised as follows. First, in the stylised setting of three banks, the simulation results reveal that the smaller or the safer a bank is, the larger the advantage of choosing a bail-out scenario. The advantage increases with the correlation with the troubled bank; it decreases with the correlation between the safe banks and it decreases with the default probability of the troubled bank.

Second, the application to the real Italian case reveals that some banks (BPM, CVAL, UBI) will benefit more from a bail-out scenario, some others (CRG, MPS, BAPO, BPER, CREDEM, POPSO) will benefit more from a bail-out and the remaining ones (UCG, ISP, MDL, MB) are practically "neutral".

2 Methodology

Consider a set V of N banks, with elements $m \in V = \{1, ..., N\}$. For each bank m we introduce a measure for its expected losses, derived as the product between its capital C^m and its default probability PD^m, in the worst case situation of a null recovery rate:

$$EL^m = C^m \cdot PD^m. \tag{1}$$

In the following, we show that such expected losses can be used to build correlation networks that transmit contagion between different banks. The final result will be a total default probability, TPD^m, able to incorporate bank-specific PDs and further contagion components.

2.1 Contagion effects

Let A be the correlation matrix between the expected losses of the N banks in the system, based on the following structure:

$$\mathrm{Corr}[EL^m, EL^n] = \rho_{mn}. \tag{2}$$

The correlation matrix A can be employed to derive correlation networks between banks (following Billio, 2012; Ahelegbey et al., 2015; Giudici and Spelta, 2016). However, such correlations can be misleading because they take into account bivariate (marginal) relationships, which may be spurios. For this reason we propose to employ conditional (partial) correlations, different from bivariate ones as they are adjusted by the presence of all the other institutions in the system. Let A^{-1} be the inverse of the correlation matrix, with elements a^{mn}. The partial correlation coefficient $\rho_{mn|S}$ between variables EL^m and EL^n, conditional on the remaining variables in V, can be obtained as:

$$\rho_{mn|S} = \frac{-a^{mn}}{\sqrt{a^{mm}a^{nn}}}. \tag{3}$$

In order to better explain partial correlations and their differences with respect to marginal ones, we now report a useful and interesting property. For any two elements $\{m, n\} \in V$, set $S = V \setminus \{m, n\}$ and suppose, similarly as in Giudici and Parisi (2016), to express the dependence between expected losses through multiple linear models in the following way:

$$\begin{cases} EL^m = a^m + \sum_{n \neq m} a_{mn|S} EL^n; \\ EL^n = a^n + \sum_{m \neq n} a_{nm|S} EL^m. \end{cases} \tag{4}$$

It can be shown that the partial correlation coefficient between EL^m and EL^n, given all the other $N-2$ measures, can be interpreted as the geometric average between the multiple linear coefficients in (4):

$$|\rho_{mn|S}| = |\rho_{nm|S}| = \sqrt{a_{mn|S} \cdot a_{nm|S}}. \tag{5}$$

Note that in case of only two components ($S = \varnothing$), equation (4) becomes:

$$\begin{cases} EL^m = a_m + a_{mn} EL^n \\ EL^n = a_n + a_{nm} EL^m, \end{cases} \tag{6}$$

from which the marginal correlation coefficient ρ_{mn} can be derived as the geometric average between the coefficients in (6):

$$|\rho_{mn}| = |\rho_{nm}| = \sqrt{a_{mn} \cdot a_{nm}}.$$

We propose to build a correlation network based on partial correlations rather than on marginal correlations. To achieve this aim we introduce an undirected graph

$G = (P, E)$, with a vertex set $P = V = \{1, ..., N\}$ and an edge set $E = P \times P$. Such edge set is defined by binary elements e_{mn} that describe whether pairs of vertices are (symmetrically) linked to each other ($e_{mn} = 1$) or not ($e_{mn} = 0$), depending on whether the partial correlation coefficient between the corresponding pair of variables is equal to zero or not.

2.2 Total default probability

The probability of default derived in (1) is bank-specific, as it is assumed independent from the default probability of other institutions: in our view this is an unrealistic assumption, since different banks are interrelated and depend on each other, as can be easily found looking at comovements between market returns. We thus propose to evolve the PD into a total default probability, TPD, able to incorporate both sector-specific and contagion components. For each bank m define a "total" expected loss TEL^m, expressed as a linear function of a "baseline" loss EL^m, which depends exclusively on the bank m, and of a further component, which depends on the loss measures EL^n of the other banks $n \neq m$:

$$TEL^m = EL^m + \sum_{n \neq m} a_{mn|S} EL^n. \tag{7}$$

By substituting the coefficients $a_{mn|S}$ with their geometric averages $\rho_{mn|S}$ (obtained from the inverse of the correlation matrix A^{-1}, according to (5)), we obtain that:

$$TEL^m = EL^m + \sum_{n \neq m} \rho_{mn|S} EL^n, \tag{8}$$

on which we place the following economics constraints:

$$\begin{cases} TEL^m = \min(C^m, TEL^m) & \text{if } TEL^m > 0, \\ TEL^m = \max(0, TEL^m) & \text{if } TEL^m < 0. \end{cases} \tag{9}$$

Note that, in analogy with the baseline expected loss, the total expected losses can be expressed as the product between the capital C^m and a default probability, that we name TPD^m (total default probability). Dividing the expression in (8) by C^m, the latter becomes:

$$TPD^m = PD^m + \sum_{n \neq m} \rho_{mn|S} \cdot PD^n \cdot \frac{C^n}{C^m}, \tag{10}$$

which shows that TPD^m "adds" to the standard PD a component that depends on contagion effects of the PD of other banks, "mediated" by the correlations and the relative capitalisation sizes.

We can now introduce the time dimension. Each bank, in fact, should be able to evaluate systemic risks in a long-term perspective. We can think of a discrete timeline, made up by a number M of key events: $T = \{t_1, ..., t_M\}$. Each bank can

evaluate which is its default probability, after the occurrence of those events, by aggregating its TPD over time as follows:

$$TPD^m_{t_j} = 1 - \prod_{\substack{t_i \in T \\ i \leq j}} (1 - TPD^m_{t_i}). \tag{11}$$

3 Application: a stylised banking system

In this section we compare, by means of a simulation study, the total default probabilities of each bank under two hypothesis: (a) one bank in the system defaults (bail-in scenario); (b) one bank in the system is in a troubled situation because of its high bank-specific PD, but at some point it is saved by the other banks through a solidarity process of capital loan (bail-out scenario). For each bank, the best scenario will be the one with the lowest TPD.

As seen in the previous Section, each total probability of default depends on a set of variables: its default probability, the default probabilities of the other banks, the correlation structure between the banks, and the relative capital sizes. To better understand the dependence of TPD on all these variables, we first propose a stylised simulation exercise. Let us consider a system composed by three banks B^1, B^2 and B^3, with the last one being in a troubled situation, as shown in Figure 1.

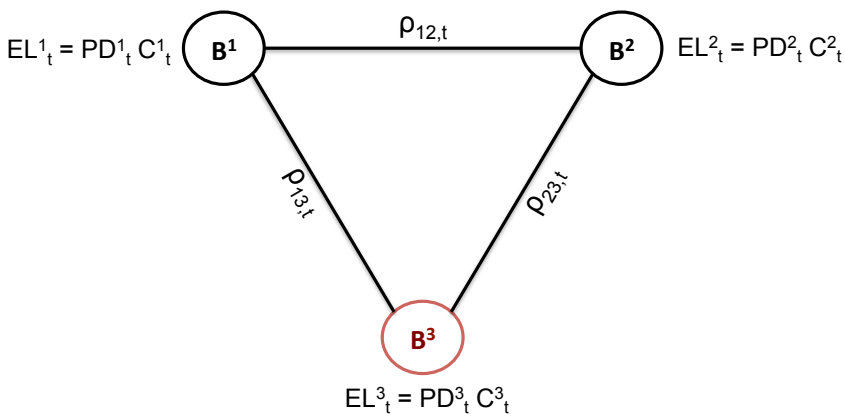

Figure 1: Simulated correlation structure between two "safe" banks, B^1 and B^2, and a "troubled" bank B^3, at a certain time t. All banks are associated to their expected losses, and links between each other are based on the partial correlation coefficients ρ_{mn}.

At any time t, all the three banks have an expected loss (EL^m), calculated as the product between their capital (C^m) and their default probability (PD^m): in addition, they are all (directly) correlated to each other through the partial correlation coefficients ρ_{mn}.

In order to understand whether Bank 1 (B^1) and Bank 2 (B^2) will benefit more from saving bank 3 (B^3) rather than from letting it default, we need to add the time component, so to derive the time evolution of the total default probabilities under the two hypothesis. We consider for simplicity, to have three times, t_0, t_1 and t_2, as shown in Figure 2.

According to Figure 2, we suppose to firstly observe the bank system at t_0: at this point, one bank (B^3) reveals to be risky, because of a high default probability. At the following time t_1, two events can occur: a) the bank B^3 defaults, in a Bail-in

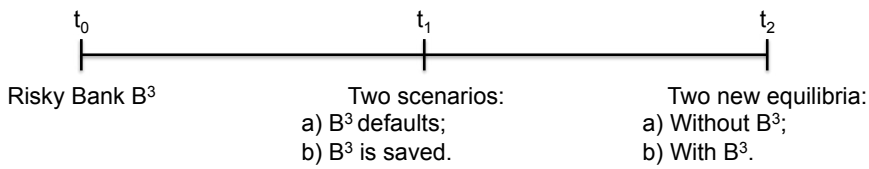

Figure 2: Simulated time evolution of three banks, under two scenarios. At time t_1 two events can occur: a) B^3 defaults, or b) B^3 is saved by B^1 and B^2. At time t_2 the bank system will reach a new equilibrium, without or with B^3, respectively if event a) or b) has verified.

scenario; b) the bank B^3 is "saved" by the other two banks in the system, B^1 and B^2, through a capital-lending operation, in a Bail-out scenario. Finally, at time t_2, the bank system will reach a new equilibrium: a) without B^3 in case it has defaulted; b) with B^3 in case it has been saved by the other banks. In the following Sections we will analyse the two scenarios and compare them in terms of TPD, for the two "safe" banks in the system.

3.1 Bail-in scenario

Let us assume, without loss of generality, that each bank keeps the same amount of capital over time: in other words, $C^m_{t_0} = C^m_{t_1} = C^m_{t_2}$. Moreover, assume that the two safe banks, B^1 and B^2, maintain the same default probability through time: $PD^{1,2}_{t_0} = PD^{1,2}_{t_1} = PD^{1,2}_{t_2}$. Finally, the risky bank B^3 is characterised by its default probability at time t_0, $PD^3_{t_0}$, while in the following time $PD^3_{t_1} = 1$, as it defaults and, then, disappears.

Marginal and, consequently, partial correlation coefficients can be derived from the correlation matrix between the expected losses: in particular, we suppose that the shock B^1 and B^2 receive at time t_1, due to the default of B^3, depends on the correlations between the two safe banks and the risky bank observed in the time just before t_1. For this reason, we will use the same correlation coefficients calculated at t_0 for propagating the default shock at t_1. After B^3 has defaulted, the bank system will be composed of only two banks, B^1 and B^2, which will reach a new equilibrium: the new correlation matrix will thus be a 2×2 rather than a 3×3 matrix, and from its inverse the partial correlation coefficients can be derived. A summary of the involved variables can be observed in Table 1.

		t_0	t_1	t_2
Capital	B^1	C^1	C^1	C^1
	B^2	C^2	C^2	C^2
	B^3	C^3	C^3	-
PD	B^1	PD^1	PD^1	PD^1
	B^2	PD^2	PD^2	PD^2
	B^3	$PD^3_{t_0}$	$PD^3_{t_1} = 1$	-
Marg. Corr.	B^1			
	B^2	A_{t_0} (3×3)	A_{t_0} (3×3)	A_{t_2} (2×2)
	B^3			
Part. Corr.	B^1			
	B^2	$[(A_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$	$[(A_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$	$[(A_{t_2})^{-1}]^{mn} = \rho^a_{mn\|S,t_2}$
	B^3			

Table 1: Time evolution of the variables that determine the total default probabilities of the three banks in the system, under the hypothesis that bank 3 defaults at time t_1 (scenario a, bail-in).

Consistently with equation (10), at each time t_j we can calculate the total default

probability of each bank. By substituting the variables summarised in Table 1 in equation (10), the following results can be obtained:

$$t_0: \quad \begin{cases} TPD^{1,a}_{t_0} = PD^1 + \rho_{12|S} \cdot PD^2 C^2/C^1 + \rho_{13|S} \cdot PD^3_{t_0} C^3/C^1 \\ TPD^{2,a}_{t_0} = PD^2 + \rho_{12|S} \cdot PD^1 C^1/C^2 + \rho_{23|S} \cdot PD^3_{t_0} C^3/C^2 \\ TPD^{3,a}_{t_0} = PD^3_{t_0} + \rho_{13|S} \cdot PD^1 C^1/C^3 + \rho_{23|S} \cdot PD^2 C^2/C^3 \end{cases} \tag{12}$$

$$t_1: \quad \begin{cases} TPD^{1,a}_{t_1} = PD^1 + \rho_{12|S} \cdot PD^2 C^2/C^1 + \rho_{13|S} \cdot C^3/C^1 \\ TPD^{2,a}_{t_1} = PD^2 + \rho_{12|S} \cdot PD^1 C^1/C^2 + \rho_{23|S} \cdot C^3/C^2 \\ TPD^{3,a}_{t_1} = 1 \end{cases} \tag{13}$$

$$t_2: \quad \begin{cases} TPD^{1,a}_{t_2} = PD^1 + \rho^a_{12|S,t_2} \cdot PD^2 C^2/C^1 \\ TPD^{2,a}_{t_2} = PD^2 + \rho^a_{12|S,t_2} \cdot PD^1 C^1/C^2 \end{cases} \tag{14}$$

Such total default probabilities can be aggregated over time, according to equation (11), in order to obtain one "overall" default probability, $TPD^{m,a}_T$ for each bank. These results will then be compared with the equivalent results obtained from the bail-out scenario ($TPD^{m,b}_T$).

3.2 Bail-out scenario

In case B^1 and B^2 decide to "save" B^3 through a capital-lending operation, we assume, without loss of generality, a proportional capital allocation. More precisely, suppose B^3 needs an amount of capital X in order to be saved: the other two banks in the system will lend, respectively, a fraction X^1 and X^2 of their capital, proportionally to their capital dimensions, as follows:

$$\begin{cases} X^1 = X \frac{C^1}{C^1+C^2}, \\ X^2 = X \frac{C^2}{C^1+C^2}. \end{cases} \tag{15}$$

Consistently with 15, at time t_1 and t_2 the total amount of capital of bank 1 and of bank 2 is reduced by the amounts X^1 and X^2, while the capital of bank 3 is increased by an amount X. For what concerns default probabilities, we suppose that, as in the bail-in scenario, B^1 and B^2 maintain their PD over time: the difference lies in PD^3, since B^3 now does not default, and is thus characterised by a default probability $PD^3_{t_2} \neq 1$. As bank 3 has been helped with a recapitalisation, we can reasonably suppose that its default probability at time t_2 will be different than before. In particular, in the worst scenario, B^3 will have the same PD as before $(PD^3_{t_2} = PD^3_{t_1}) = PD^3_{t_0})$ but, in general, we can impose the constraint $PD^3_{t_2} \leq PD^3_{t_0}$.

Marginal and partial correlations can be derived as in the bail-in scenario, with the only difference being that now, at time t_2, the correlation matrix is a 3×3 matrix since B^3 is still part of the banking system. In this last case, we can safely assume that the correlation matrix remains the same as in t_0. The involved variable are summarised in Table 2.

Consistently with equation (10), at each time t_j we can calculate the total default probability of each bank. By substituting the variables summarised in Table 1 in

		t_0	t_1	t_2
Capital	B^1	C^1	$C^1(1-\frac{X}{C^1+C^2})$	$C^1(1-\frac{X}{C^1+C^2})$
	B^2	C^2	$C^2(1-\frac{X}{C^1+C^2})$	$C^2(1-\frac{X}{C^1+C^2})$
	B^3	C^3	C^3+X	C^3+X
PD	B^1	PD^1	PD^1	PD^1
	B^2	PD^2	PD^2	PD^2
	B^3	$PD^3_{t_0}$	$PD^3_{t_0}$	$PD^3_{t_2} \leq PD^3_{t_0}$
Marg. Corr.	B^1			
	B^2	A_{t_0} (3×3)	A_{t_0} (3×3)	A_{t_0} (3×3)
	B^3			
Part. Corr.	B^1			
	B^2	$[(A_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$	$[(A_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$	$[(A_{t_0})^{-1}]^{mn} = \rho_{mn\|S}$
	B^3			

Table 2: Time evolution of the variables used for estimating the total default probabilities of the three banks in the system, under the assumption that bank 3 is saved by the other two through a capital-lending operation at time t_1 (scenario b, bail-out).

equation (10), the following results can be obtained:

$$t_0\colon \quad \begin{cases} TPD^{1,b}_{t_0} = PD^1 + \rho_{12|S}\cdot PD^2C^2/C^1 + \rho_{13|S}\cdot PD^3_{t_0}C^3/C^1 \\ TPD^{2,b}_{t_0} = PD^2 + \rho_{12|S}\cdot PD^1C^1/C^2 + \rho_{23|S}\cdot PD^3_{t_0}C^3/C^2 \\ TPD^{3,b}_{t_0} = PD^3_{t_0} + \rho_{13|S}\cdot PD^1C^1/C^3 + \rho_{23|S}\cdot PD^2C^2/C^3 \end{cases} \tag{16}$$

$$t_1\colon \quad \begin{cases} TPD^{1,b}_{t_1} = PD^1 + \rho_{12|S,t_0}\cdot PD^2C^2/C^1 + \rho_{13|S}\cdot PD^3_{t_0}\frac{C^3+X}{C^1+C^2-X}\frac{C^1+C^2}{C^1} \\ TPD^{2,b}_{t_1} = PD^2 + \rho_{12|S}\cdot PD^1C^1/C^2 + \rho_{23|S}\cdot PD^3_{t_0}\frac{C^3+X}{C^1+C^2-X}\frac{C^1+C^2}{C^2} \\ TPD^{3,b}_{t_1} = PD^3_{t_0} + \frac{C^1+C^2-X}{(C^3+X)(C^1+C^2)\left[\rho_{13|S}\cdot PD^1C^1+\rho_{23|S}\cdot PD^2C^2\right]} \end{cases} \tag{17}$$

$$t_1\colon \quad \begin{cases} TPD^{1,b}_{t_2} = PD^1 + \rho_{12|S,t_0}\cdot PD^2C^2/C^1 + \rho_{13|S}\cdot PD^3_{t_0}\frac{C^3+X}{C^1+C^2-X}\frac{C^1+C^2}{C^1} \\ TPD^{2,b}_{t_2} = PD^2 + \rho_{12|S}\cdot PD^1C^1/C^2 + \rho_{23|S}\cdot PD^3_{t_0}\frac{C^3+X}{C^1+C^2-X}\frac{C^1+C^2}{C^2} \\ TPD^{3,b}_{t_2} = PD^3_{t_2} + \frac{C^1+C^2-X}{(C^3+X)(C^1+C^2)\left[\rho_{13|S}\cdot PD^1C^1+\rho_{23|S}\cdot PD^2C^2\right]} \\ PD^3_{t_2} \leq PD^3_{t_0} \end{cases} \tag{18}$$

As in the bail-in scenario, such total default probabilities can be aggregated over time, so $TPD^{m,b}_T$ can be obtained and compared to $TPD^{m,a}_T$.

3.3 Bail-in vs bail-out scenario

According to the previous equations, the final default probabilities for each bank m, conditional on their previous survival, can be summarised as follows:

$$\begin{cases} TPD^{m\neq 3,a}_T = 1-(1-TPD^{m,a}_{t_0})\cdot(1-TPD^{m,a}_{t_1})\cdot(1-TPD^{m,a}_{t_2}), \\ TPD^{3,a}_T = 1, \\ TPD^{m,b}_T = 1-(1-TPD^{m,b}_{t_0})\cdot(1-TPD^{m,b}_{t_1})\cdot(1-TPD^{m,b}_{t_2}). \end{cases} \tag{19}$$

Since banks 1 and 2 have to decide wether to help bank 3 or not, from a systemic risk perspective they are interested in analysing the difference between TPD^a_T and TPD^b_T. In particular, if $TPD^{m,a}_T - TPD^{m,b}_T > 0$, then saving bank 3 (bail-out

scenario) decreases the total probability of default of bank m with respect to the bail-in scenario; on the contrary, if $TPD_T^{m,a} - TPD_T^{m,b} < 0$, than letting bank 3 default (bail-in scenario) decreases the total probability of default of bank m, with respect to the bail-out scenario.

In this section we compute such differences simulating alternative scenarios. We consider two large banks and a smaller one, with $C^1 = 40$, $C^2 = 20$ and $C^3 = 4$ (billion euro of capitalisation), with $X = C^3$ in case of bail-out scenario. Correlation coefficients are sampled from Gaussian distributions: $\rho_{mn} \sim \mathcal{N}(\mu_{\rho_{mn}}, \sigma^2_{\rho_{mn}})$, with $\mu_{\rho_{12}} = \mu_{\rho_{13}} = \mu_{\rho_{23}}$, and unit variances. Baseline default probabilities are also sampled from Gaussian distributions: $PD^m \sim N(\mu_{PD^m}, \sigma^2_{PD^m})$, with $\mu_{PD^1}, \mu_{PD^2} = 0.01, 0.03, 0.05, 0.07$ and unit variances.

Last, the default probability of bank 3 at t_2 (in the bail-out scenario), is simulated according to different values of $PD^3_{t_2}$:

$$\begin{cases} PD^3_{t_j} \sim N(\mu_{PD^3_{t_j}}, \sigma^2_{PD^3}), \\ \mu_{PD^3_{t_0,t_1}} = 0.10, \\ \mu_{PD^3_{t_2}} \sim \mathcal{U}([0, 0.10]), \end{cases} \tag{20}$$

with a unit variance. The resulting differences in TPD, as a function of the sampled PDs, are shown in Figure 3. As seen before, the higher the difference, the more convenient the bail-out is, with respect to the bail-in.

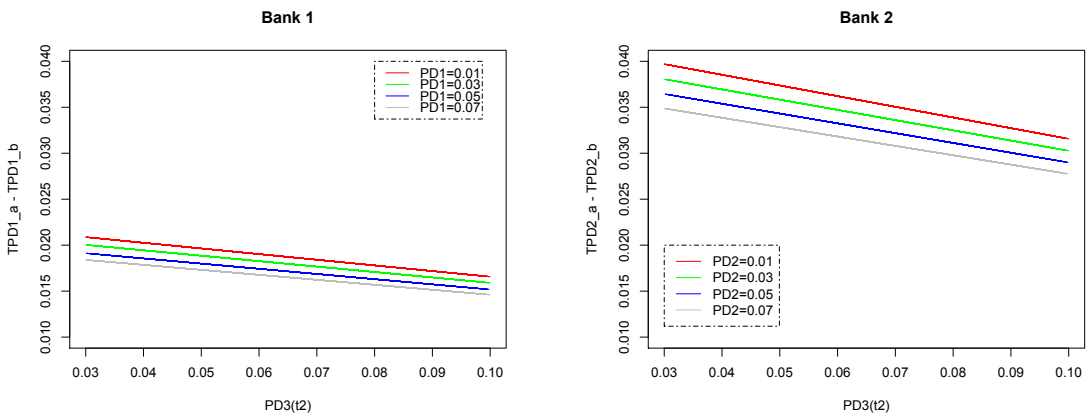

Figure 3: Monte Carlo simulated differences between the total default probabilities in case of bail-in and bail-out for bank 1 (left) and bank 2(right), plotted as functions of $\mu_{PD^3_{t_2}}$.

The results plotted in Figure 3 can be summarised and interpreted according to both the different dimensions and the default probabilities of the two banks. First, in this special case of all positive correlations, it is always convenient for both banks to help bank 3 without letting it default. Secondly, by comparing the two graphs, it is clear that the smaller bank B^2 has a larger advantage of helping B^3 and that such advantage is positively dependent on the decreasing dimension of the "safe" banks. Thirdly, both graphs represent four lines according to four different values for $\mu_{PD^{1,2}}$: by comparing such four lines, the result is that the safer a bank is, the larger the advantage of helping the troubled bank B^3.

The previous simulation is based on the hypothesis of fixed and positive $\mu_{\rho_{mn}}$: however, this is an extremely simplified assumption, especially under the bail-in

scenario. It is common knowledge, in fact, that when a bank is in default, or when the banking system faces a crisis period, correlations between them vary. In order to take this more realistic scenario into account, we now sample $\mu_{\rho_{mn}}$ as well, uniformly over the possible range:

$$\begin{cases} \rho_{mn} \sim \mathcal{N}(\mu_{\rho_{mn}}, \sigma^2_{\rho_{mn}}), \\ \mu_{\rho_{mn}} \sim \mathcal{U}([-1, 1]), \end{cases} \tag{21}$$

The resulting differences in TPD, as a function of the sampled correlations, and of the PD of the safe banks, keeping $\mu_{PD^3_{t_2}} = 0.10$, are shown in Figure 4.

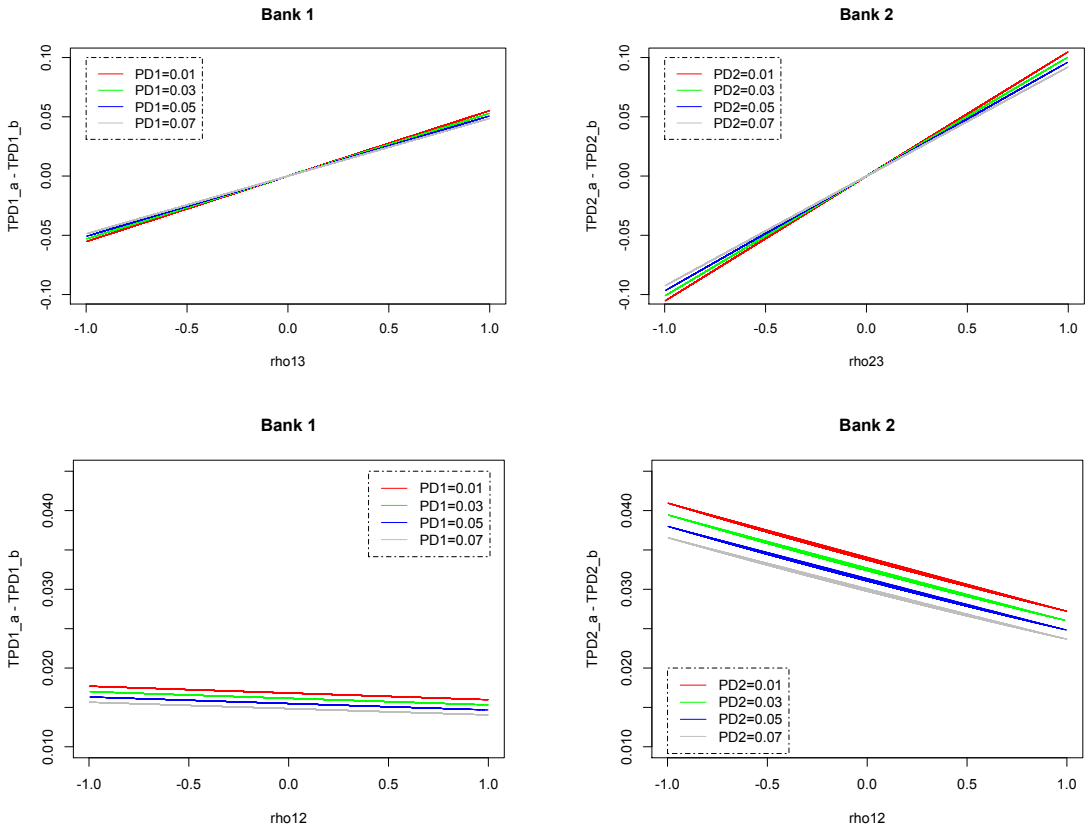

Figure 4: Monte Carlo simulated differences between the total default probabilities in case of bail-in and bail-out for bank 1 (left) and bank 2(right), plotted as functions of $\mu_{\rho_{13}}$ (top-left), $\mu_{\rho_{23}}$ (top-right) and $\mu_{\rho_{12}}$ (bottom).

Figure 4 represents the advantage/disadvantage of helping bank 3 as a function of the correlations between bank 1 and bank 3 (top-left), between bank 2 and bank 3 (top-right) and between the two safe banks 1 and 2 (bottom, left referred to bank 1, right referred to bank 2). By looking at the top two graphs it is first clear that the smaller the bank is, the stronger the dependence on correlations. Second, in case of positive correlations with B^3, the bail-out scenario is better, and the smaller or the safer a bank is, the larger the advantage. On the contrary, in case of negative correlations the bail-in scenario is preferred, and the advantage increases with the dimension of a bank.

The two bottom graphs show the relative convenience of the two scenarios in terms of the impact of the correlation between the two safe banks B^1 and B^2. It

reveals that the impact is not particularly significant for large banks (such as bank 1), while it slightly changes the results referred to small banks: the weaker the correlation between bank 1 and bank 2 is, the bigger the advantage of helping bank 3.

By jointly reading Figures 3 and 4, the results show that, overall, B^1 and B^2 should prefer the bail-in rather than the bail-out scenario only in case of negative partial correlations. In addition, the convenience for the bail-out situation is a decreasing function of the default probabilities of the safe banks, a decreasing function of the dimension of the safe banks in system, and an increasing funtion of the correlation of safe banks with the troubled one.

4 Application: the Italian banking system

We now apply the proposed methodology to the Italian banking system. We consider the largest Italian banks, in terms of capitalisation, and consistently with recent events we focused our attention to Banca Popolare di Vicenza as the "troubled" bank in the system. The banks considered in the sample, together with their default probabilities, are listed in Table 3.

Ticker	$\mu_{PD}(\%)$	C (Mln)
MPS	7.23	1905.90
BPER	2.22	2433.00
BPM	3.45	2859.10
POPSO	2.68	1504.30
BAPO	3.89	2249.10
CRG	4.23	613.60
CREDEM	2.23	2084.10
CVAL	3.82	759.60
ISP	1.87	41200.30
MB	2.42	5818.40
UBI	3.06	3343.70
UCG	2.41	19838.90
MDL	1.70	5246.40
POPVIC	7.23	145.16

Table 3: Largest Italian banks in terms of their capitalisation, associated to their default probabilities (expressed in percentage points) and amounts of capital (expressed in terms of their stock market capitalisation, millions of euro).

The bank-specific PDs have been obtained in different ways. In particular, for Monte dei Paschi di Siena (MPS), Banco Popolare di Milano (BPM), Banco Popolare (BAPO), Istituto San Paolo (ISP), Mediobanca (MB), Unione Banche Italiane (UBI) and Unicredit (UCG) we employed the PDs obtained from the Credit Default Swaps, extracted from Markit, considering the time period January-April 2016 (since the introduction of the European Banking Union Bail-in rule to the announcement of the "Atlante" bail-out fund). More precisely, consistently with Parisi and Giudici (2016), we calculated $PD_t^m = 1 - \exp^{-(y_t^m - S_t)}$, where $y_t^m - S_t$ represents CDS spreads at time t, with respect to a risk free rate. In the table, for each bank we report the average default probabilities over the whole considered period.

For the banks Banca Popolare dell'Emilia Romagna (BPER), Banca Popolare di Sondrio (POPSO), Banca Carige (CRG), Credito Emiliano (CREDEM), Credito Valtellinese (CREVAL) and Banca Mediolanum (MDL), which do not have CDS data in Markit, we used the default probabilities calculated by the Risk Management Institute of the National University of Singapore, calibrated with those implied by the Fitch ratings. Note that the troubled Banca Popolare di Vicenza does not have either CDS data or a PD from the Singapore Institute; however, its Fitch rating is equal to that of MPS,and, therefore, its default probabilities have been set equal to PD^{MPS}.

Concerning bank capitalisations, they have been extracted from the Borsa Italiana database. The values in the Table refer to the closing value at 15th of April 2016, when the bail-out of Banca Popolare di Vicenza by the Atlante fund was announced.

Multiplying daily capitalisation values with daily PDs we have calculated the expected losses of the banks, as in (1). We have then derived the partial correlations and, thus, the partial correlation network. The results are depicted in Figure 5.

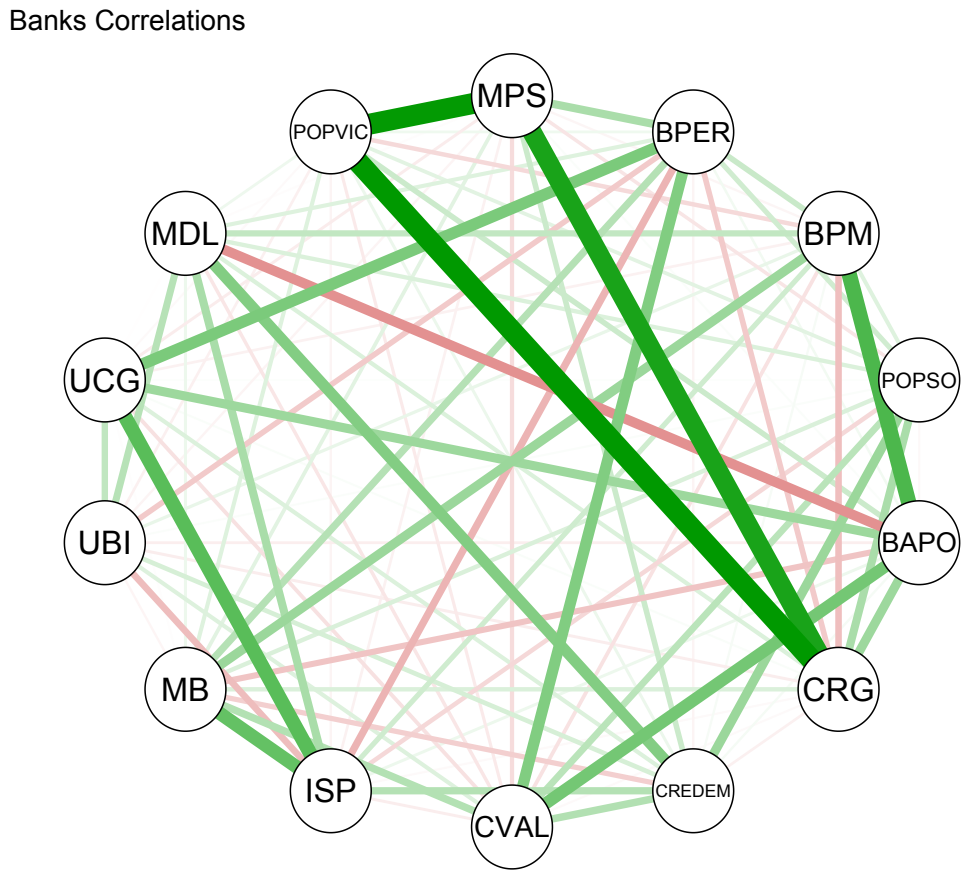

Figure 5: Correlation network between Italian banks, based on the partial correlations between their expected losses.

The network proposed in Figure 5 shows both positive (green lines) and negative (red lines) partial correlations: the ticker the line, the stronger the connection. It is interesting to observe that positive correlations prevail: in particular, they are strong between "troubled" banks such as MPS and CRG. Note that, by construction, the

correlation between POPVIC and CRG is equal to that between MPS and CRG, whereas that between MPS and POPVIC (which should be 1 by contruction) has been set equal to that between MPS and CRG.

We now consider the calculation of the difference between the TPD, respectively, in the bail-out and bail-in scenarios, as in the previous Section. As a capital value $C^m_{t_0}$, we consider the capitalisation values reported in Table 3. In case of bail-out (scenario b) at time t_1, such amounts are decreased by a fraction X^m, as follows:

$$\begin{cases} C^{m\neq POPVIC}_{t_1} = C^m \left(1 - \frac{X}{\sum\limits_{m\neq POPVIC} C^m}\right), \\ C^{POPVIC}_{t_1} = C^{POPVIC}_{t_0} + X, \end{cases} \tag{22}$$

with $X = 1500$ Mlns euros.

For robustness purposes, correlations are now extracted from a normal distribution, $\rho_{mn|S} \sim \mathcal{N}(\mu_{\rho_{mn|S}}, \sigma^2_{\rho_{mn|S}})$, with $\mu_{\rho_{mn|S}}$ and $\sigma^2_{\rho_{mn|S}}$ calculated using the available expected losses time-series. Similarly, default probabilities are $PD^m \sim N(\mu_{PD^m}, \sigma^2_{PD^m})$, with μ_{PD^m} as in Table 3.

For the default probability of POPVIC at time t_2 we assume an ample range of variation, as follows:

$$\begin{cases} PD^{POPVIC}_{t_j} \sim N(\mu_{PD^{POPVIC}_{t_j}}, \sigma^2_{PD^{POPVIC}}), \\ \mu_{PD^{POPVIC}_{t_2}} \sim \mathcal{U}([0.03, 0.3]). \end{cases} \tag{23}$$

Consistently with the previous Section, we now calculate the differences between the aggregated default probabilities at the end of the time period (t_2). The results are shown in Figure 6.

Figure 6 clearly shows that some banks would benefit from helping and saving POPVIC through the Atlante equity fund, while some others would not. In particular, Banca Carige (CRG) and Monte dei Paschi di Siena (MPS) show extremely negative values of $TPD^a_T - TPD^b_T$: this result is due to their high idiosyncratic default probabilities, as well as to their strong positive connections with each other. More precisely, we can interpret this by stating that, in case POPVIC defaults, they would not be damaged too much by contagion effects, since their PD is already high; on the contrary, in the following time-period t_2 they would benefit from POPVIC not being in the banking system any more. In other words, the benefit coming from not being connected to a "troubled" bank any more at time t_2 is much greater than the damage deriving from a shock caused by POPVIC's default at time t_1.

Other banks that would benefit from letting POPVIC default are: Banca Popolare di Sondrio (POPSO), Banco Popolare (BAPO), Credito Emiliano (CREDEM) and Banca Popolare dell'Emilia Romagna (BPER). All these banks present a low correlation with POPVIC and a high correlation with the other banks in the system: the combination of these two factors prevails over the effect derivimg from their relatively low PD and size.

A different situation regards Credito Valtellinese (CVAL), Banca Popolare di Milano (BPM) and UBI Banca, which would benefit from helping POPVIC through the Atlante fund. This results can be explained by remembering that these banks are either small, have a low PD, or are highly correlated with POPVI, and would thus be damaged by its default.

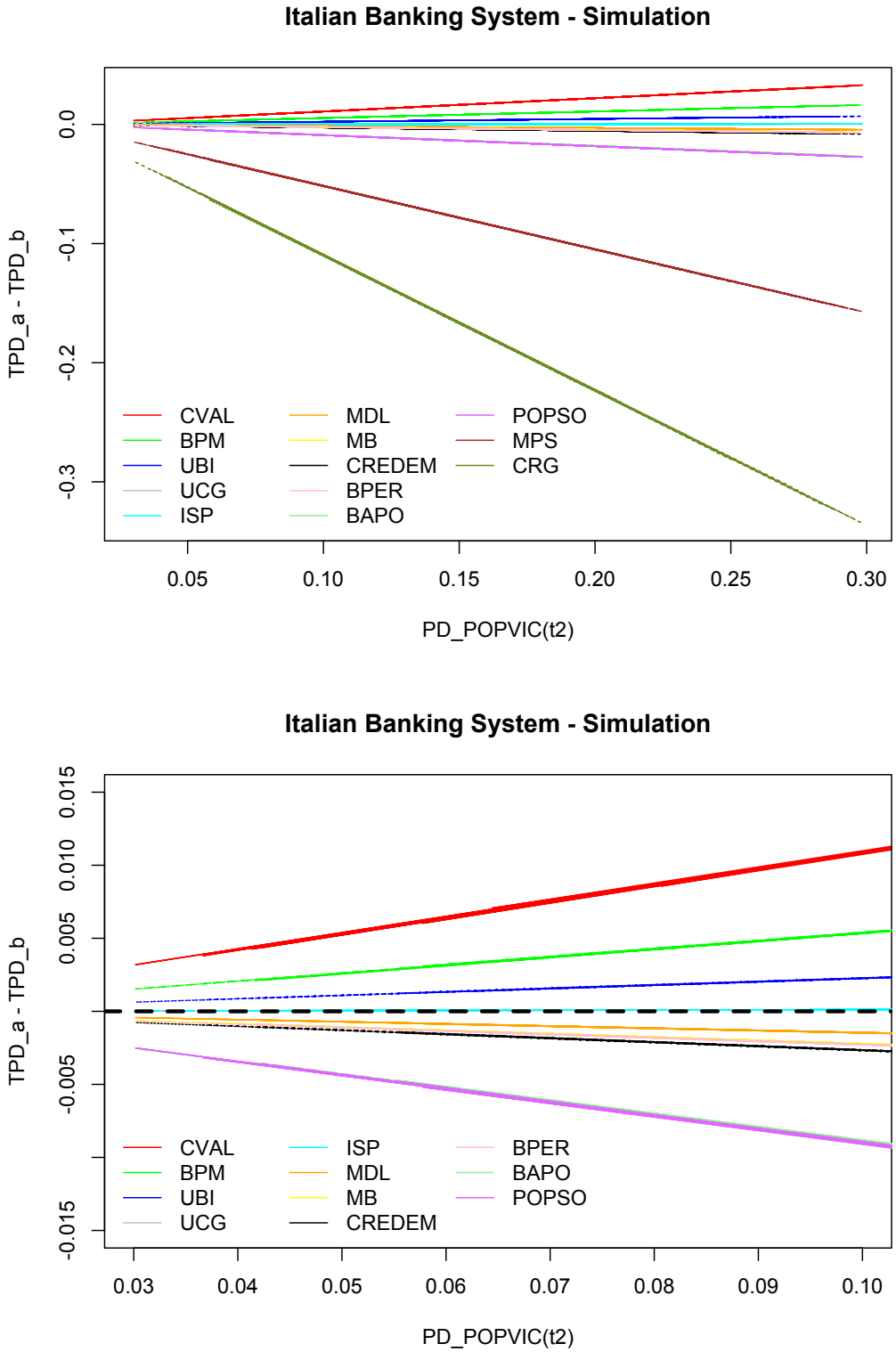

Figure 6: Differences between the total default probabilities in case of bail-in and bail-out for Italian banks, plotted as functions of $\mu_{PD_{t_2}^{POPVIC}}$. Left: $\mu_{PD_{t_2}^{POPVIC}} \sim \mathcal{U}([0.03, 0.3])$. Right: $\mu_{PD_{t_2}^{POPVIC}} \sim \mathcal{U}([0.03, 0.10])$

The remaining banks, Intesa San Paolo (ISP), Unicredit (UCG), Mediobanca (MB) and Mediolanum (MDL) are almost neutral, because of their large size and/or

their low correlation with POPVIC.

Figure 6 also reveals an interesting result: none of the lines crosses the *x-axis*, meaning that the default probability of POPVIC at time t_2 does not affect the other banks' choice of taking or not taking part of the Atlante fund. The increase in $PD_{t_2}^{POPVIC}$ instead only affects their degree of advantage or disadvantage. For those who benefit from joining Atlante, such a benefit increases as $PD_{t_2}^{POPVIC}$ increases. On the other side, for those banks which would benefit from letting POPVIC default, such a benefit increases as $PD_{t_2}^{POPVIC}$ increases.

In order to build a more realistic scenario, we now propose to stress partial correlations in case of bail-in. We consider only positive increases in correlation levels, as follows:

$$\begin{cases} \mu^a_{rho_{mn|S,t_1}} = \mu_{rho_{mn|S,t_0}} + f \cdot |\mu_{rho_{mn|S,t_0}}|, \\ f = 10\%, 20\%, 30\%, 50\%. \end{cases} \tag{24}$$

The results are shown in Figure 7.

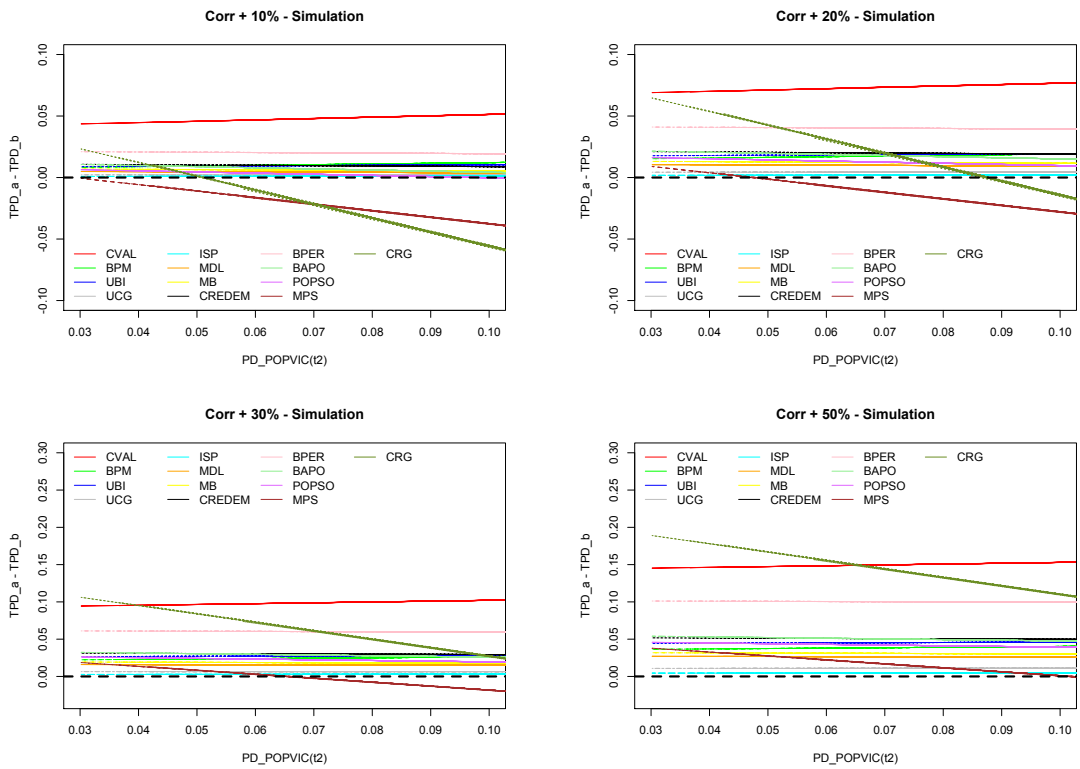

Figure 7: Differences between the total default probabilities in case of bail-in and bail-out for Italian banks, plotted as functions of $\mu_{PD_{t_2}^{POPVIC}}$ and according to stressed partial correlations. Top-left: $f = 10\%$. Top-right: $f = 20\%$. Bottom-left: $f = 30\%$. Bottom-right: $f = 50\%$.

Figure 7 overall reveals that an increase in partial correlations increases the advantage of choosing the bail-out rather than the bail-in scenario, quite homogeneously across banks. Furthermore, when partial correlations become higher and higher, the survival probability of POPVIC becomes less important in the choice of the bail-in or the bail-out scenario: when correlations are high and even if the survival probability of Banca Popolare di Vicenza is small, all the banks in the system would benefit more from the bail-out rather than from the bail-in scenario.

Ticker	**Cap (Mln)**	**Atlante (Mln)**	$\Delta Cap(\%)$	$C^b_{t_1}$ **(Mln)**
MPS	1906	-50	-2.62%	1855.9
BPER	2433	-100	-4.11%	2333
BPM	2859	-100	-3.50%	2759.1
POPSO	1504	-50	-3.32%	1454.3
BAPO	2249	-50	-2.22%	2199.1
CRG	614	-20	-3.26%	593.6
CREDEM	2084	0	0.00%	2084.1
CVAL	760	-60	-7.90%	699.6
ISP	41200	-1000	-2.43%	40200.3
MB	5818	0	0.00	5818.4
UBI	3344	-200	-5.98%	3143.7
UCG	19839	-1000	-5.04%	18838.9
MDL	5246	-50	-0.95%	5196.4
POPVIC	145	+1500		1510.11

Table 4: Biggest Italian banks in terms of their capitalisation, associated to the capital they transferred to the Atlante fund (expressed in Mln euros), their capital change (expressed in percentage points) and their final capital amount (expressed in Mln euros).

An interesting remark should be added for interpreting Monte dei Paschi and Banca Carige: as partial correlations increase, they may prefer choosing the bail-out scenario, but such a choice depends on $PD^{POPVIC}_{t_2}$. In other words, we can identify specific values of $PD^{POPVIC}_{t_2}$, that we will call $PD^{cut-off,m}_{t_2}$ such that if $PD^{POPVIC}_{t_2} < PD^{cut-off,m}_{t_2}$, than Bank m should choose the bail-out scenario; conversely, if $PD^{POPVIC}_{t_2} > PD^{cut-off,m}_{t_2}$, than Bank m should choose the bail-in scenario.

In the previous Section we have used equation (22) in order to homogeneously and proportionally distribute the capital amount X POPVIC needs in order to not default. Indeed, we know the real amounts X^m of capital that each bank has transferred to the Atlante fund, by the 29th of April, 2016. They are shown in Table 4.

Table 4 reveals that only two banks have not taken part of the Atlante fund: Credito Emiliano and Mediobanca. This choice is consistent with our results (Figure 6), but we underline that if partial correlations increase even by only 10%, than even these two banks would benefit from joining Atlante. Furthermore, Credito Valtellinese and UBI Banca are the banks that transferred the biggest fractions of their capital amount to Atlante, and this choice is again consistent with our simulated results (Figures 6 and 7). Carige and Monte dei Paschi have decided to invest in the Atlante fund even if our model suggest they would not benefit: their choices can thus be explained not in terms of systemic risk, but by other strategic factors (such as the possible acquisition of their non performing loans).

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