High Trend Inflation and Passive Monetary Detours

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# 135 (03-17)

Via San Felice, 5  
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http://epmq.unipv.eu/site/home.html  
March 2017
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Abstract

According to the long-run Taylor principle (Davig and Leeper, 2007), a central bank can deviate to a passive monetary policy and still obtain determinacy if a sufficiently aggressive monetary policy is expected for the future. Does this principle hold true when both monetary and fiscal policies can switch and there is positive trend inflation? We find that passive monetary detours are no longer possible when trend inflation is high, whatever fiscal policy is in place. This has important policy implications in terms of flexibility and monetary-fiscal authorities coordination.

Keywords: trend inflation, monetary-fiscal policy interactions, Markov-switching, determinacy.

JEL classification: E5.

1 Introduction

After years from the trough of the Great Recession, the Great Moderation seems just a distant memory. With the major economies stuck at the zero lower bound, there have been different proposals to leave this impasse. The challenge is to increase inflation expectations in order to reduce real rates. The most common proposals entail an active role for fiscal policy and the suggestion, by some influential economists, to increase the inflation target. Were these proposals put at work, would it always be possible to return to an era such as the Great Moderation? This would be characterised by a

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monetary led policy regime, where the central bank respects the Taylor principle while the government implements the fiscal adjustments necessary to stabilise debt.

To answer this question we extend the work by Ascari et al. (2016), which studies determinacy under monetary-fiscal interactions in a Markov-switching model, to include trend inflation. While that paper modifies Davig and Leeper (2007) placing fiscal policy in the foreground, here we want to check whether the long run Taylor principle holds once trend inflation, typically omitted in these analyses of changing policy regimes, is introduced. Ascari and Ropele (2009) find that an increase in trend inflation makes the determinacy area shrink. Is the magnitude of this reduction still compatible with Davig and Leeper’s results about the long run Taylor principle? In other words, is the enlarged determinacy region found by Davig and Leeper offset by a higher level of trend inflation? We here want to study if and how an increase in trend inflation would affect the long-run Taylor principle and the consequent policy implications.

Our work can be considered as an extension of Coibion and Gorodnichenko (2011) to a monetary and fiscal regime switching setting. How does the presence of trend inflation change the determinacy conditions when even fiscal policy is taken into account in a Markov-switching model? And, is it still true that the reduction in trend inflation during Volcker’s mandate, as Coibion and Gorodnichenko maintain, was a key factor behind the Great Moderation? Would have a strong commitment to achieve low and stable inflation been sufficient to go out from the Great Inflation?

This paper contributes to the recent growing literature on monetary-fiscal policy interactions (see Davig and Leeper, 2006, 2011; Bianchi, 2012; Bianchi and Melosi, 2013; Bianchi and Ilut, 2014) adding determinacy analysis and trend inflation. Foerster (2016), like us, considers inflation target switching in a model with predetermined variables but, assuming full price indexation, shows that it does not affect determinacy. Our model differs from his in that we do not have indexation and, especially, because we consider regime switches of fiscal policy too. Our model is the same as the one by Florio and Gobbi (2015) but in a regime switching context; theirs, on the contrary, is a fixed-coefficient model with learning to study the effects of trend inflation and transparency on expectations anchoring under different monetary-fiscal mixes.

The main finding of the paper is that passive monetary detours are no longer possible when trend inflation is high. And this is true both under a constantly passive fiscal regime or when this fluctuates

1Davig and Leeper (2007) analyse regime changes in monetary policy with an always passive fiscal policy. We here apply the terminology in Leeper (1991). Active monetary (AM) policy arises when the response of the nominal interest rate to inflation is more than one-to-one. Otherwise, we have passive monetary (PM) policy. Analogously, passive fiscal (PF) policy occurs when taxes respond sufficiently to debt to prevent its explosion; otherwise we have active fiscal (AF) policy. In many fixed-coefficient models, a unique bounded equilibrium requires one active and one passive policy.
between active and passive. The impossibility of switching from an active to an accommodating
monetary policy regime has relevant implications for the conduct of monetary policy itself.

The paper proceeds as follows. Section 2 describes the New Keynesian model with trend inflation
and regime switching in both monetary and fiscal policy as well as the methodology employed. Section
3 illustrates the results about determinacy and their policy implications. Section 4 concludes.

2 Model and methodology

The model is the most basic New Keynesian model with fiscal policy. The non-linear model equations
are:

\[ 1 = \beta E_t \left( \frac{Y_t - G}{Y_{t+1} - G} \frac{R_t}{\Pi_{t+1}} \right), \] (1)

\[ \phi_t \left( 1 - \alpha \Pi_t^{\theta-1} \right)^{1-\theta} = \frac{\mu \theta (1-\alpha)^{1-\theta}}{\theta - 1} Y_t + \alpha \beta E_t \left[ \phi_{t+1} \Pi_{t+1}^{\theta-1} \left( 1 - \alpha \Pi_{t+1}^{\theta-1} \right)^{1-\theta} \right], \] (2)

\[ \phi_t = \frac{Y_t}{Y_t - G} + \alpha \beta E_t \left[ \Pi_{t+1}^{\theta-1} \phi_{t+1} \right], \] (3)

\[ \frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + G - \tau_t, \] (4)

\[ \tau_t = \tau \left( \frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{\mu_{\tau,t}}, \] (5)

\[ R_t = R \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\gamma_{\pi,t}} e^{\mu_{\pi,t}}. \] (6)

Equation (1) is a standard Euler equation for consumption, where \( Y_t \) is output, \( R_t \) the nominal
interest rate, \( \Pi_t \) the gross inflation rate and \( G \) government spending, which is assumed to be exogenous
and constant. Equations (2) and (3) describe the evolution of inflation in the non-linear model. \( \phi_t \)
is an auxiliary variable (equal to the present discounted value of expected future marginal revenues)
that allows us to write the model recursively. Equation (4) is the government’s flow budget constraint,
where \( b_t = B_t/P_t \) is real government debt. We follow Leeper (1991) in using lump-sum taxes, i.e., \( \tau \),
which are set according to the fiscal rule (5): taxes react to the deviation of lagged real debt from
its steady-state level \( (b) \) according to the parameter \( \gamma_{\tau,t} \). Equation (6) describes monetary policy. It
is a simple Taylor rule whereby the central bank reacts to the deviations of current inflation from
the target level \( (\bar{\Pi}) \) according to the parameter \( \gamma_{\pi,t} \). A variable without the time index (i.e., \( \tau \), \( b \)
and \( R \)) indicates the value at the steady state. \( \beta \) is the intertemporal discount factor; \( \theta \) is the Dixit-
Stiglitz elasticity of substitution between goods; and \( \alpha \) is the Calvo probability that a firm is unable
to optimise its price.

The key parameters of our analysis are $\gamma_{\pi,t}$ and $\gamma_{\tau,t}$, which describe the time-varying stance of monetary and fiscal policy, respectively. We assume that these parameters follow an underlying two-state Markov process and are equal to $(\gamma_{\pi,i}, \gamma_{\tau,i})$ when the economy is in regime $i$, for $i = 1, 2$. The transition probabilities of going from regime $i$ to regime $j$ are denoted by $p_{ij}$. Thus, $p_{ii}$ is the probability of remaining in regime $i$, and $p_{ij} = 1 - p_{ii}$.

2.1 Solution method and determinacy criterion

As our model includes fiscal policy, we need to account for the dynamics of government debt, which is an endogenous state variable. To do so, we employ the perturbation method developed by Foerster et al. (2015) (henceforth FRWZ) that allows us to solve for the minimal state variable (MSV) solutions of a Markov-switching model in the presence of predetermined variables. In our previous paper (Ascari et al., 2016), we used the same approach to find the rational expectations solutions of a similar model in which zero trend inflation is assumed.\(^2\) The FRWZ method retrieves all the MSV solutions corresponding to a given parametrization. We need therefore to apply a stability criterion in order to understand whether a given solution is stable or not. In the context of Markov switching models, the concept of mean square stability (MSS) - proposed by Costa et al. (2005) and Farmer et al. (2009) - is a straightforward choice as it reduces the stability analysis to checking a simple algebraic condition entailing the autoregressive roots of state variables and the transition probabilities.

Therefore, any given parameter configuration can either lead to: (i) determinacy, when a unique stable solution exists; (ii) indeterminacy, when multiple stable solutions exist; or (iii) explosiveness, when no stable solutions exist. In what follows we seek to explore the parameter space to identify the regions corresponding to these three cases.

3 Results

Our results build on Ascari et al. (2016). In that paper the concepts of globally active (or passive) and globally switching policies were introduced to explain the determinacy properties of the model under Markov-switching. A “globally passive” (or active) fiscal (or monetary) policy admits a deviation into active (or passive) fiscal (or monetary) policy in one of the regimes, but this deviation needs to be modest. If the deviation is instead substantial, the fiscal (or monetary) policy is labelled as “globally

\(^2\)Refer to that paper for more details on our application of the FRWZ method. More on the solution method in the Appendix.
To guarantee the existence of a unique equilibrium, monetary and fiscal policies need to be globally balanced: a globally active monetary policy needs to be coupled with a globally passive fiscal policy and globally switching monetary policies with globally switching fiscal policies.

### 3.1 Determinacy under positive trend inflation

We extend Ascari et al. (2016) to a framework with positive trend inflation. As in that paper, we concentrate (mainly) on the case where one of the two regimes is AM/PF. That is the benchmark mix in the New Keynesian literature and the policy regime that, according to many, prevailed during the post-1984 Great Moderation era in the U.S. Figure 1 reports the monetary frontiers, that is the combinations of monetary policy coefficients \((\gamma_{\pi 1} \text{ and } \gamma_{\pi 2})\) in the two regimes that deliver determinate equilibria, for different levels of trend inflation \((0, 2, 4, 6\%)\) when fiscal policy stays passive in both regimes (with \(\gamma_{\tau} = 0\) and \(p_{11} = p_{22} = 0.95\)). This case, with fiscal policy in the background, is the most common case analysed in the monetary policy literature.

In the first panel, when trend inflation is zero, the well-known Davig and Leeper’s (2007) long run Taylor principle holds: a passive monetary policy, which is indeterminate in a static context, could return determinacy if, in the other regime, monetary policy is sufficiently aggressive. Or, in Ascari et al.’s (2016) words, given an AM/PF regime, determinacy is preserved if monetary policy deviates modestly from an AM behavior becoming to a certain extent passive. With trend inflation at 2% we get the same figure. However, as trend inflation increases going to 4% or higher, we get two important points. The first, not new in the literature, is that the Taylor principle breaks down as trend inflation rises. As you can see from the bottom panels, with trend inflation equal to 4%, in order to have determinacy, it does not suffice to have \(\gamma_{\pi} > 1\), as the Taylor principle prescribes, but the central bank must be much more hawkish to inflation and, the more so, the more trend inflation increases. The second, as far as we know entirely new, is that as trend inflation increases, not only the Taylor principle but even the long-run Taylor principle breaks down. In other words, with high trend inflation (higher than 2%) and an always passive fiscal policy in the two regimes, monetary policy can never be passive rather, to return determinacy, it must be very active. One can not go from a double passive regime to an AM/PF one and still have determinacy. We think this result has important policy implications. We will defer their discussion to the next section.

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3 For a graphical inspection of the modest and substantial variation see Figure 1b in Ascari et al. (2016). We report, for convenience, that figure in the Appendix, together with the determinacy analysis under zero trend inflation.

4 This is the reason why we do not deal with the case of an always active fiscal policy. Results, however, are easily extensible even to that case.

The same comments apply when fiscal policy, rather than being constantly passive, deviates just modestly from being passive in one of the two regimes. Not surprisingly, since fiscal policy is globally passive, this case returns monetary policy frontiers for different levels of trend inflation qualitatively similar to those in Figure 1 (see Figure A2 in the Appendix). However, note that now these results

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6 We concentrate on this case because the parameters estimated for passive and active fiscal regimes, especially for samples including the seventies, show values consistent with Ascari et al. (2016) definition of modest deviation in fiscal policy (see Chung et al., 2007; Davig and Leeper, 2007; Bianchi, 2012; ?; ?; ?). The “globally switching fiscal” case is left to the appendix, where we will go through our results to analyse if and how they change under that specification.

7 This is not surprising since from Ascari et al. (2016) we know that in the presence of modest deviations, the resulting
arise in a - though modestly - switching fiscal policy context. So, with trend inflation at 4% or higher, one can never switch from PM/AF to AM/PF, two determinate regimes under fixed-coefficients, and maintain determinacy.

**Result** With $\alpha = 0.75$, once the inflation target is high (> 2%), if the economy is in a passive monetary regime, there is no chance to reach a determinate equilibrium even if agents expect an active monetary stance under the other regime. The long run Taylor principle never holds.

### 3.2 Policy Implications

**Lack of flexibility** Central bank’s flexibility to accommodate short-run disturbances is a desirable aspect in the conduct of monetary policy. Davig and Leeper (2007, p. 618) note that, although the stabilizing properties of an active monetary policy make it desirable, central banks “also desire the flexibility to respond to developments that may entail a departure from the Taylor principle”. This departure could be either short or long-lasting, depending on the length of the period when active inflation stabilization is de-emphasized to the benefit of other short-run objectives. Woodford (2001, p. 671), for example, stresses how this kind of flexibility could be of use in periods of fiscal dominance: “regimes (...) in which other goals of central bank policy are subordinated to the goal of assisting in the financing of the government budget.” Our model shows that this flexibility is seriously impaired by high trend inflation. A rise in the inflation target would make the central bank less flexible since it could never depart from an active monetary policy. Furthermore, as the inflation target rises, the central bank must be more and more hawkish toward inflation to get determinacy and this is true irrespective of the fiscal regime in place.

**Monetary-fiscal coordination problems** Davig and Leeper (2006) find policy coordination problems to be irrelevant because, despite periods of double active or double passive monetary and fiscal policies that under fixed-coefficients lead, respectively, to explosive and indeterminate solutions, the expectations of stable policy mixes would suffice to get a determinate equilibrium. We find this is not the case with high trend inflation. With a trend inflation higher than 2%, the economy can never visit a double passive regime and get determinacy even if a stable policy mix is expected in the future. The only possibility is going from a double active regime to an AM/PF one. Therefore coordination problems of monetary and fiscal policies become a fact in the presence of high trend inflation.

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8 The following results hold for $\alpha \geq 0.75$.
9 See on this point even Bianchi and Melosi (2012).
Switching from the Great Inflation to the Great Moderation  The literature that blames bad policy for high inflation in the seventies ascribes this to a central bank that did not respect the Taylor principle. Many believe that in the late 1979 there was a regime change with monetary policy switching from passive to active. In the presence of an unchanging (passive) fiscal policy, this implies the shift from a double passive, hence indeterminate in fixed coefficient policy mix, to a determinate AM/PF one, that contributes decisively to the advent of a Great Moderation era. This result, both if derived in a fixed coefficient context (Lubik and Schorfheide, 2004; Clarida et al., 2000) or in a Markov-switching one (Davig and Leeper, 2007; Bianchi, 2012; Baele et al., 2015), is based on the assumptions of always passive fiscal policy and zero trend inflation. Coibion and Gorodnichenko (2011) re-examine these results considering, more realistically, a positive trend inflation in the United States for that period. Building on the finding that the Taylor principle does not guarantee determinacy as trend inflation rises (see Hornstein and Wolman, 2005; Kiley, 2007; Ascari and Ropele, 2009), Coibion and Gorodnichenko claim that the switch to determinacy at the end of the 1970s was due, in large part, to the substantial reduction in the level of trend inflation during the Volcker tenure.10

However, it is becoming more and more common the idea that even fiscal policy could have changed going from the Great Inflation to the Great Moderation. In these last years many examine this possibility employing models with regime switching changes in both monetary and fiscal policies. Markov-switching regressions suggest the shift, for those years, from an active fiscal policy to a passive one. In other words, the pre-Volcker era is found to be consistent with a PM/AF regime (Favero and Monacelli, 2005; Davig and Leeper, 2006, 2011; Bianchi, 2012; Bianchi and Ilut, 2014) (Sims, 2011; Bianchi and Melosi 2013). As a result, the rise of inflation in that period could be ascribed to a lack of fiscal discipline given by a non-Ricardian policy. However, according to Bianchi and Ilut (2014), “If in the 70s agents had been confident about moving to the AM/PF regime, the Great Inflation would not have occurred”.11

When agents expect an AM/PF regime in the future and trend inflation is low, our model finds determinacy under both the policy mixes that, according to the literature, could have prevailed during the seventies. In particular, looking at the first two panels of Figure 1, one can realise that, provided

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10Trend inflation, then, could make indeterminate even equilibria where policymakers satisfy the Taylor principle. According to Orphanides (2002), this was precisely the case before the advent of Volcker in the United States.

11See on this point even Bianchi (2012, 2013) and Bianchi et al. (2013) who claim that if agents are aware of the possibility of a return to the AM/AF mix, a fiscal imbalance would not be inflationary. However, according to Ascari et al. (2016), under zero trend inflation, the expectation of an absorbing AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects and inflation. (It is not necessary because with a modest deviation from an AM/PF regime there are no wealth effects (even when there are not absorbing regimes). It is not sufficient because with a substantial deviation there are wealth effects, hence a spike in inflation in the PM/AF regime, even when the AM/PF mix is an absorbing one.]
monetary policy is not too passive, one can go from a double passive mix to an AM/PF and still have determinacy. The same is true when the regime shifts from PM/AF to AM/PF (Figure A1).

However, as the previous Section shows, the presence of high trend inflation changes dramatically determinacy areas. With high trend inflation, as in the 70s, if agents had been confident about the advent in the early 80s of the AM/PF regime, both if they were under a PM/PF regime or under a PM/AF one, equilibrium determinacy could not have been reached. The possibility to avoid indeterminacy, once in a passive regime, expecting an active monetary policy in the future, depends on trend inflation at the time of the conjecture (very high during the great inflation, indeed). With high (greater than 2%) trend inflation this is not feasible. The only way to have determinacy would have been reducing trend inflation. As CG (2011) maintain, our model confirms a lower level of trend inflation to be a key factor behind the Great Moderation.

**Escaping the Great Recession** The recent financial crisis has spurred some popular economists (Rogoff, 2008; Blanchard et al., 2010; Ball, 2013) to suggest an increase in the inflation target to a value in the 4-6% range. The main motivation behind this proposal is to help the economies, stuck at their zero lower bound, to decrease real rates in order to go out from the recession.

According to Davig and Leeper (2011), the regime that probably best describes policy behavior during the early years of the Great Recession is the PM/AF one. This is the same policy mix invoked by Bianchi and Melosi (2015) as a way to escape the Great Recession by inflating debt away.\textsuperscript{12} Were this the regime in place, our model suggests that increasing the inflation target, as suggested, from 2% to 4%, could not be a good idea. Once in a PM/AF regime, with trend inflation equal to 4%, it would not be possible to go back to an AM/PF regime and, at the same time, reach a determinate equilibrium. With high trend inflation, flexibility would be lost.

4 Conclusions

Once monetary-fiscal interactions are taken into account, the role of a low trend inflation as a key factor to switch towards a Great Moderation era is confirmed. We find that an increase in trend inflation voids the long run Taylor principle: even if agents expect an active monetary stance for the future, there is no chance to reach a determinate equilibrium if the economy is in a passive monetary regime. Furthermore, as the inflation target rises, the central bank must be more and more hawkish toward inflation to get determinacy and this is true irrespective of the fiscal regime in place. This has

\textsuperscript{12}More on this in the Appendix.
important policy implications in terms of flexibility and monetary-fiscal authorities coordination. A rise in the inflation target would make the central bank less flexible since it could never depart from an active monetary policy. Furthermore, with high trend inflation, the expectations of stable policy mix in the future would not suffice to get a determinate equilibrium.

As a consequence, we argue that a strong commitment to react heavily to inflation for the future would not have been sufficient, in any case, to go out from the Great Inflation. The possibility to avoid indeterminacy, once in a passive regime, expecting an active monetary policy in the future, depends on trend inflation at the time of the conjecture and with an inflation as high as in the seventies this is not a way to go. This is true whatever fiscal policy is in place. The only way to have determinacy would have been reducing trend inflation.

As for the proposal to increase the inflation target as a way to overcome the zero lower bound during the Great Recession, we find that it could seriously impair the return to an expected AM/PF regime, once and if the passive monetary regime would be abandoned. This, again, is true whatever fiscal policy is in place.


5 Appendix

5.1 Solution method

We employ the perturbation method developed by FRWZ. Using their notation, our model can be written as:

\[ \mathbb{E}_t f(y_{t+1}, y_t, x_{t-1}, \varepsilon_{t+1}, \varepsilon_t, \theta(s_{t+1}), \theta(s_t)) = 0 \]

where \( x_t = b_t \) and \( y'_t = [Y_t, \Pi_t, \phi_t, R_t]' \) are the predetermined and non-predetermined variables, respectively, and \( \theta'(s_t) = [\gamma_\Pi(s_t), \gamma_\tau(s_t)]' \) is the vector of parameters that switch according to the Markov-switching process \( s_t \). We look for recursive solutions in the form

\[ x_t = h_{s_t}(x_{t-1}, \varepsilon_t, \chi) \quad (7) \]
\[ y_t = g_{s_t}(x_{t-1}, \varepsilon_t, \chi) \quad (8) \]

perturbed around the non-stochastic steady state \([x, y']\). Note that in our model the solutions are regime-dependent while the steady state is not. The perturbation method in FRWZ allows to check the existence and uniqueness of the solution to the first order approximation of the dynamic system.

Under regime \( i \), the first order Taylor expansions of the solutions are

\[ b_t \approx b + h_{i,b}(b_{t-1} - b) + h_{i,\varepsilon}\varepsilon_t + h_{i,\chi}\chi, \quad y_t \approx y + g_{i,b}(b_{t-1} - b) + g_{i,\varepsilon}\varepsilon_t + g_{i,\chi}\chi, \]

with the partial derivatives evaluated at the steady state. The derivatives of \( \mathbb{E}_t f \) are equal to zero and depend on the unknown coefficients \( h_{i,b}, h_{i,\varepsilon}, h_{i,\chi}, g_{i,b}, g_{i,\varepsilon}, g_{i,\chi} \). \( h_{i,b} \) and \( g_{i,b} \) are necessary to perform the determinacy analysis. FRWZ show that the \( h_{i,b} \) and \( g_{i,b} \) are the roots of a separated system.


of quadratic equations, that they propose to solve by using Groebner basis to find all the possible solutions. Note that $h_{i,b}$ and $g_{i,b}$ characterise the response of the endogenous variables, predetermined and non-predetermined variables respectively, to the state variable in the policy functions. Once all the admissible solutions are find, a stability criterion needs to be imposed to select the stable ones. The criterion is the concept of mean square stability (MSS) proposed by Costa et al. (2005) and Farmer et al. (2009). MSS requires the existence of:

$$\lim_{t \to \infty} E_0 \left( \begin{bmatrix} x_t \\ y_t \end{bmatrix} \right), \quad \text{and} \quad \lim_{t \to \infty} E_0 \left( \begin{bmatrix} x_t \\ y_t \\ x_t' \end{bmatrix} \right)$$ (9)

In our context with 2 regimes and 1 state variable, the condition for MSS constrains the values of the autoregressive roots in the state variable policy function in the two regimes. In particular, the solution $(h_{1,b}, h_{2,b})$ is MSS if the following matrix has all its eigenvalues inside the unit circle:

$$\begin{bmatrix} p_{11}h_{1,b}^2 & (1 - p_{22})h_{2,b}^2 \\ (1 - p_{11})h_{1,b}^2 & p_{22}h_{2,b}^2 \end{bmatrix}$$ (10)

To have mean square stability (MSS), thus, $h_{1,b}, h_{2,b}$ should satisfy these conditions:

$$\left| h_{1,b}^2 h_{2,b}^2 (p_{11} + p_{22} - 1) \right| < 1$$ (11)

$$p_{11}h_{1,b}^2 (1 - h_{2,b}^2) + p_{22}h_{2,b}^2 (1 - h_{1,b}^2) < 1 - h_{1,b}^2 h_{2,b}^2$$ (12)

Hence, any given parameter configuration could lead to: (i) determinacy, that admits a unique stable solution; (ii) indeterminacy, that admits multiple stable solutions; (iii) explosiveness, that admits no stable solutions. In what follows we want to identify the determinacy region in the parameter space that is, all those parametrisation for which a unique MSS solution exists.

\textsuperscript{13}Davig and Leeper (2007) employs a more restrictive concept of stability: bounded stability, which requires bounded paths and thus rules out temporarily explosive paths in one of the two regimes. See Farmer et al. (2009) for a discussion in the context of MS-DSGEs.
5.2 Determinacy under zero trend inflation

5.2.1 Fixed-coefficient case

The log-linearized model is a trivariate dynamic system in the two jump variables \(\hat{y}_t\) and \(\hat{\pi}_t\) and the predetermined variable \(\hat{b}_t\):

\[
\frac{1}{c} \hat{Y}_t = \frac{1}{c} E_t \hat{Y}_{t+1} - \left( \hat{R}_t - E_t \hat{\Pi}_{t+1} \right)
\]

(13)

\[
\hat{\Pi}_t = \frac{\lambda}{c} \hat{Y}_t + \beta E_t \hat{\Pi}_{t+1}
\]

(14)

\[
\hat{b}_t = \frac{1}{\beta} \left( 1 - \frac{\tau}{b} \gamma_r \right) \hat{b}_{t-1} - \frac{1}{\beta} \hat{\Pi}_t + \hat{b} \hat{R}_t + \frac{1}{\beta} \sigma_r u_{r,t}
\]

(15)

where \(\hat{R}_t\) is given by the monetary policy rule: \(\hat{R}_t = \gamma \pi \hat{\Pi}_t + \sigma_m u_{m,t}\), and \(\hat{b}\) and \(\hat{c}\) are the steady state debt-to-GDP and consumption-to-GDP ratios, respectively. It is useful here to recall the necessary and sufficient conditions for determinacy of the REE in a fixed coefficient model. Using the renowned Leeper (1991) taxonomy, fiscal policy is said to be passive if the fiscal rule guarantees debt stabilisation in (15), that is if:

\[
\left| \frac{1}{\beta} \left( 1 - \frac{\tau}{b} \gamma_r \right) \right| < 1
\]

(16)

In case of passive fiscal policy, it is easy to show that the following conditions have to hold to yield determinacy:

\[
\gamma_{\pi} > 1
\]

(17)

and

\[
\gamma_{\pi} > \frac{\beta - 1}{\lambda}.
\]

(18)

The first condition is the Taylor principle and it implies the second, which then becomes redundant. Still following Leeper, monetary policy is labelled active if it satisfies the Taylor principle, otherwise is labelled as passive. Hence, the famous result by Leeper (1991) follows: in the presence of passive fiscal policy, monetary policy needs to be active, i.e., \(\gamma_{\pi} > 1\), to yield determinacy.

Conversely, in case of active fiscal policy, i.e., (16) does not hold, then monetary policy should be passive to guarantee determinacy: \(\gamma_{\pi} < 1\). In this case, the REE is non-Ricardian, such that a change in lump-sum taxation has real effects, and the so-called “fiscal theory of the price level” holds.

Summing up, in a fixed coefficient model as in Leeper (1991), the determinacy region is defined by the following conditions: Active Monetary (AM) / Passive Fiscal (PF): \(\gamma_{\pi} > 1\) and \((1-\beta)\frac{b}{\gamma_r} < \gamma_r < (1+\beta)\frac{b}{\gamma_r}\) or Active Fiscal (AF) / Passive Monetary (PM): \(\gamma_{\pi} < 1\) and \((1-\beta)\frac{b}{\gamma_r} < \gamma_r < (1+\beta)\frac{b}{\gamma_r}\). The REE
equilibrium is indeterminate under PM/PF and explosive under AM/AF.

5.2.2 Regime switching case

Applying the FRWZ method, we derive the following system for the general case with \( p_{11}, p_{22} < 1 \):

\[
g_{1,\pi,b} \left\{ 1 + \lambda \gamma_{1,\pi} - h_{1,b} p_{11} \left( \beta + 1 + \lambda \right) + \beta h_{1,b}^2 \right\} + (1 - p_{11}) \beta h_{1,b} h_{2,b} (1 - p_{22}) g_{1,\pi,b} \]

\[
+ g_{2,\pi,b} (1 - p_{11}) h_{1,b} \left[ \beta h_{1,b} p_{11} - (\beta + 1 - \beta h_{2,b} + \lambda) \right] = 0
\]

\[
g_{2,\pi,b} \left\{ 1 + \lambda \gamma_{2,\pi} - h_{2,b} p_{22} \left( \beta + 1 + \lambda \right) + \beta h_{2,b}^2 \right\} + (1 - p_{22}) \beta h_{1,b} h_{2,b} (1 - p_{11}) g_{2,\pi,b} \]

\[
+ g_{1,\pi,b} (1 - p_{22}) h_{2,b} \left[ \beta h_{2,b} p_{22} - (\beta + 1 - \beta h_{1,b} + \lambda) \right] = 0
\]

with

\[
g_{1,\pi,b} = \frac{1}{\beta} \left( \frac{1}{\beta} \gamma_{1,\pi} - h_{1,b} \right) \]

\[
g_{2,\pi,b} = \frac{1}{\beta} \left( \frac{1}{\beta} \gamma_{2,\pi} - h_{2,b} \right)
\]

and where the 4 unknowns are the coefficients \( h_{1,b}, h_{2,b}, g_{1,\pi,b} \) and \( g_{2,\pi,b} \). Recall that debt \( b_t \) is the state variable of the system, \( h_{i,b} \) is the response of debt to its lag in regime \( i \), and \( g_{i,\pi,b} \) is the response of inflation to the lagged debt in regime \( i \). Determinacy obtains when a unique pair \( (h_{1,b}, h_{2,b}) \) satisfies the MSS conditions 11 and 12.

Figure A1 shows how the number of stable solutions varies depending on the combinations of the monetary \( (\gamma_{\pi,2}) \) and the fiscal \( (\gamma_{\tau,2}) \) coefficients for the second regime, given an AM/PF regime 1 \( (\gamma_{\pi,1} = 1.5; \gamma_{\tau,1} = 0.2) \). Notably, there are two regions in the \( (\gamma_{\pi,2}, \gamma_{\tau,2}) \) space that return determinacy: an upper-right zone and a lower-left one. In the upper-right zone, determinacy emerges when the second regime is AM/PF too. However, there is determinacy even if the second regime deviates from the AM/PF mix. In other words, in order to have determinacy, fiscal and monetary policy in the second regime are not constrained to be, respectively, passive and active. Rather, they can now vary “modestly” and be (to a certain extent), respectively, active and passive. It can be shown that this effect is more pronounced the lower is \( p_{22} \). The modest changes for fiscal and for monetary policy are illustrated, respectively, by the dashed and solid arrowed lines in the figure.

Consider now what happens in the lower-left zone. In this case, in order to have determinacy fiscal
Figure A1: Determinacy areas given an AM/PF regime 1.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness. The solid lines with arrows indicate a modest deviation from active monetary policy; the dashed lines with arrows indicate a modest deviation from passive fiscal policy.

and monetary policy in the second regime are constrained to be, respectively, “more than just” active and “more than just” passive with respect to Leeper’s (1991) conditions. Hence, both monetary policy and fiscal policy must deviate “substantially” from the previous AM/PF regime.

5.3 The monetary frontiers when fiscal policy switches

5.3.1 A modest fiscal policy deviation

Figure A2 depicts the monetary frontiers for different levels of trend inflation when fiscal policy shifts from active in regime 1 ($\gamma_{\tau,1} = 0$) to passive in regime 2 ($\gamma_{\tau,2} = 0.2$), with $p_{11} = p_{22} = 0.95$. This is the case of a modest deviation from the passive fiscal regime that entails a globally passive fiscal policy.\(^{14}\) As Ascari et al. (2016) make clear, under this specification, since fiscal policy is globally passive, there are no expectational wealth effects in the AM/PF regime.

5.3.2 A substantial fiscal policy deviation

Figure A3 shows the monetary frontier for the globally switching fiscal case, that is when there is a substantial switch in the stance of fiscal policy across the two regimes. Again, we consider the outcome for different levels of trend inflation. We report the case of a switch from (very) active fiscal policy in

\(^{14}\)With Ascari et al. (2016)’s calibration you have a modest fiscal deviation from a PF regime when $-0.02 < \gamma_{\tau} < 0.02$.  

Figure A2: The monetary policy frontier when fiscal policy switches modestly.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

regime one \((\gamma_{\tau,1} = -0.05)\) to the usual fiscal passive regime two \((\gamma_{\tau,2} = 0.2)\). Under this specification, since fiscal policy is switching and passes through an AF regime, there are wealth effects even in the AM/PF regime.

If we concentrate on the switch to an AM/PF regime 2, we realise that when trend inflation is zero determinacy is attained when regime 1 is PM/AF. As trend inflation becomes higher determinacy gradually vanishes. Under this globally switching fiscal case, for a trend inflation higher than 4%,
Figure A3: The monetary policy frontier when fiscal policy switches substantially.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

central bank’s flexibility is impaired (it would not be possible, once in a passive monetary regime, to avoid indeterminacy expecting an AM/PF regime in the future\textsuperscript{15}) and coordination problems are confirmed. Hence, if the inflation target is raised from 2% to 4%, as suggested during the Great Recession to overcome the zero lower bound problem, once in a PM/AF regime, it would not be possible to return to an AM/PF.

However, with this policy setting, the proposal by Bianchi and Melosi (2014) to escape the Great Inflation.

\textsuperscript{15}This, once again, confirms the central role played by trend inflation’s reduction when exiting the Great Inflation.
Recession acquires relevance. Once at the zero lower bound, they claim, the Great recession could be escaped by generating an increase in inflation expectations via a PM/AF mix that inflates debt away. Suppose that suggestion was accepted and agents were convinced of a future switch to an AM/PF regime. Fiscal imbalances would be inflationary if the necessary fiscal adjustments were not made. From Ascari et al. (2016), we know that a modest deviation from the AM/PF regime would entail these fiscal adjustments and, given the absence of wealth effects, would avoid the spike in inflation. On the contrary, in the global switching case, if the switch from the AM/PF mix is substantial (or modest but long-lasting), the economy could escape the Great recession engineering, through the presence of wealth effects, higher inflation that, in turn, drops real rates helping to stimulate the economy.