ISSN: 2281-1346

Department of Economics and Management

DEM Working Paper Series

Vertical Differentiation With Optimistic Misperceptions And Information Disparities

Alberto Cavaliere
(Università di Pavia and IEFE - Università Bocconi)

Giovanni Crea
(Università di Pavia)

 # 137 (05-17)

Via San Felice, 5
I-27100 Pavia
http://epmq.unipv.eu/site/home.html

May 2017

Vertical Differentiation With Optimistic Misperceptions And Information Disparities

A. Cavaliere*   G. Crea†

Abstract

We consider vertical differentiation with quality uncertainty and information disparities, in a duopoly where products have credence attributes and a minimum quality standard exists. Optimistic misperceptions further relax price competition but uninformed consumers may be cheated in equilibrium due to minimum product differentiation when informed consumers buy low quality goods. Optimistic misperceptions turn out to be an incentive for product differentiation when informed consumers buy high quality goods, even if the real quality differentiual is always lower than expected by uninformed consumers.Increasing the share of informed consumers may counterbalance the effect of optimism on equilibrium prices but in the meantime reduce the incentives for product differentiation.

Key words: Asymmetric information, Brand premium, Quality uncertainty

JEL Codes: L15, L13, D82

*Corresponding author. Dipartimento di Scienze Economiche e Aziendali, Università di Pavia and IEFE-Università Bocconi, alberto.cavaliere@unipv.it, Tel +39 0382986477, via S. Felice, 5, 27100 Pavia, ITALY

†Dipartimento di Scienze Economiche e Aziendali, Università di Pavia, giovanni.crea@unipv.it

1 Introduction

In markets where products are vertically differentiated (Gabsewicz and Thisse, 1979; Shaked and Sutton, 1982), consumers may be uncertain about the quality differential provided by high quality firms and then consider if this differential is worth the price premium they should pay for it. If products are experience goods, ex-post consumption may provide more precise information to consumers. Firms can establish a reputation for high quality, as shown by Shapiro (1982,1983). If products are credence goods, as in case of drugs, chemicals or products sold as green goods, many consumers may lack the expertise to ascertain the quality differential, even after purchase[1]. In that case reputation may not be effective as a mechanism to convey information about product quality, as shown by experimental evidence (Dulleck et al. 2011).

Actually for products classified as credence goods, consumers may even not know what is the minimum and the maximum quality that a firm can potentially provide. Accordingly it may be difficult for consumers even to assign a probability distribution to the quality choice.Therefore consumers may carry out purchase decisions according to misperceptions about product quality. Consumers may overestimate the quality differential provided by the seller. For example brand loyalty may imply that consumers overestimate the quality provided by one brand with respect to similar products.In this last case firms may profit from misperceptions and misinformation by charging excessive prices to consumers: actually high perceived quality may coexist with minimal product differentiation. Competition between generics and branded pharmaceuticals is a typical example; optimistic consumers may continue to buy branded drugs, though equivalent to generics. According to a recent empirical analysis (Bronnenberg et al. 2015), brand premia due to quality misperceptions imply an additional cost of $44 billion per year for US consumers

In the meantime some consumers may be well informed about real quality differentials and willing to pay a price-premium for products that deserve it. Better information may derive either

[1]Credence goods were firstly introduced by Darby and Karni (1973) considering repair services or medical treatments where consumers, do not even know what they need, if not asisted by the diagnosis of an expert. However the literature has extended this definition to consider goods vertically differentiated by process attributes In that cases "consumers know what they need , but observe neither what they get nor the utility derived from what they get" (Dullek et al. 2011, p. 527).

by consumer expertise, costly information gathering activities or better education. For example informed consumers may be able to distinguish a real environmental commitment from strategic greenwashing, disposing of expertise, education or precise consumer reports provided by associations like Greenpeace.The different shopping behavior of informed and uninformed consumers has been recently considered by Bronneneberg et al. (2015) concerning health products sold in the US. By considering the choice of informed experts, like pharmacists, physicians and better educated consumers, they find informed consumers are more likely to buy store brands than national brands, showing that price-premia payed to national brands depend on optimistic misperceptions.

Information disparities between consumers in a model of vertical product differentiation were firstly introduced by Cavaliere (2005), just considering the price competition stage and then neglecting both the quality choice and the cost of quality provision.In this paper we extend the analysis to include the quality choice by firms, when providing higher quality requires a costlier effort. We then analyse the case of a duopoly with vertically differentiated products, uncertainty about quality differentials, optmistic misperceptions and information disparities.

Consumers are split between uninformed and informed purchasers. Uninformed consumers over-estimate the quality differential provided by the high quality firm. As a minimum quality standard (MQS) is imposed by the Government, even uninformed consumers expect that any product sold in the market at least complies with the standard.(we show that such an expectation is fulfilled in equilibrium). The firm providing higher quality goods claims over-compliance with respect to the MQS. But over-compliance implies the supply of a credence attribute, so that consumers choice is exposed to optimistic misperceptions. Informed consumers are on the contrary aware of the real quality differential.

To the extent that both higher education and the willingness to pay for information gathering activities are correlated with income, one particular feature of our model is that uninformed and informed consumers are not randomly distributed in the population of consumers. Information disparities are correlated with the distribution of the willingness to pay for quality[2]. Therefore, by assumption, the higher the willingness to pay for quality the higher the likelihood that a consumer

[2]As in turn the willingness to pay for quality can be typically correlated with income in vertical differentiation model (cfr. for example Tirole 1989)

is informed (such an assumption implies that if a consumer i, with a willingness to pay for quality θ_i is informed any consumer j with a willingness to pay $\theta_j > \theta_i$ will be informed as well). Information in some cases may then lead consumers with higher income to buy low quality goods to avoid ripoffs, a result that would not occur in vertical differentiation models with complete information.

We do not analyze information decisions by consumers. These decisions are exogenous to the model. However we can analyze quality and price competition between firms for the full range of information disparities, i.e. for any split between informed and uninformed consumers that can affect demand functions. Competition between firms is represented by a two stage game, in the first stage the two firms compete in qualities, given the market split between informed and uninformed consumers In the second stage price competition takes place.

It is worthwhile to notice that the interest od our analysis lies in the coexistence of uninforemd and informed consumers. If we just considered uninformed consumers then, given that high quality cannot be observed by any consumer and considering that high quality goods are costlier to produce with respect to low quality goods, it would be optimal for both firms to provide the MQS. Therefore in equilibrium we would observe minimum product differentiation though firms would be able to charge higher equilibrium prices consistent with optimistic misperceptions. We would then observe moral hazard by the high quality firm. On the contrary our model can account for different types of equilibria with different pricing and product differentiation strategies, according to the behavior of uninformed and informed consumers.

We can account for different types of market demands when most consumers are uninformed and when most consumers are informed. The first case may be closer to market reality. The second one may be worthwhile to be considered as a benchmark for policy reasons, as far as the effect of information provision policies can be evaluated.

In case most consumers are uninformed and buy both low quality and high quality goods, we consider two main equilibria, depending on the behavior of informed consumers.

If informed consumers buy high quality goods brand loyalty must be supported by some real product differentiation. In equilibrium optimistic misperceptions contribute to raise both equilibrium prices. As optimistic misperceptions distort price upwards, price competition is further relaxed with respect to the full information case. Optimistic misperceptions then represent an incentive for product differentiation, even though the real level of quality provided by the high quality firm is always lower with respect to the expected quality by uninformed consumers. In the meantime real quality increases both with expected quality and with the share of uninformed consumers. Therefore uninformed consumers are cheated in equilibrium but their misperceptions represent an incentive for product differentiation, as more and more informed consumers buy high quality goods and the more optimistic are uninformed consumers.

If informed consumers buy low quality goods,still optimistic misperceptions contribute to relax price competition, as equilibrium prices depend on expected quality by uninformed consumers. However we obtain a result of minimum differentiation, given that high quality goods are just bought by uninformed consumers, that are cheated by the high quality firm which is providing a quality level close to the MQS This type of equilibrium can well represent the outcome of competition between branded drugs and generics, with the latter being bought both by the richest population (due to their information) and by the poorest consumers (due to the lower price). Actually product equivalence could not support any significant differentiation in qualities, but mispercetions about the quality differential still lead to brands sold at higher prices than generics. Therefore uninformed consumers that buy high quality goods are cheated in equilibrium. However as equilibrium prices also depend asymmetrically on the share of informed consumers the latter contribute to counterbalance the effect of optimistic misperception on prices, with a reduction in the price of high quality goods and a comparable increase in the prices of low quality goods.

Equilibria with a majority of informed consumers may be interesting to analyze in order to consider the effect of information policies on product differentiation affected by optimistic misperceptions. When informed consumers are the majority, and buy both low quality goods and high quality goods, equilibrium prices are not affected by optimisitic misperceptions and just depend on the real quality differential. However we can still consider two types of equilibria, according to the behavior of the minority of uninformed consumers If misperceptions are such that uninformed consumers buy high quality goods, the majority of informed consumers contributes to reduce equilibrium prices that depend on the real quality differential. But again equilibrium prices also depend asymmetrically on the share of informed consumers with a balancing effect. An increase of informed consumers contributes to reduce the price of high quality goods (and to raise the price of low quality goods). Product differentiation decreases with more informed consumers, consistently with a decrease of equilibrium prices with respect to the full information equilibrium. Therefore information contributes to eliminate consumer cheating but incentives for product differentiation are negatively affected.

On the contrary with less optimistic misperceptions the second type of equilibrium holds, where uninformed consumers are not mislead and buy low quality goods, as well as some informed consumers with an higher willingness to pay. Then high quality goods are just bought by the richest consumers as in a standard model of vertical product differentiation. Actually in this case our model collapses to vertical differentiation with complete information, provided the share of informed consumers is high enough.

To the best of our knowledge our model is the first one to analyze the case of pure vertical differentiation with consumers' misperceptions, information disparities, and endogenous quality. Previous contributions include Bester (1998) considering a model of horizontal and vertical differentiation where quality is both endogenous and uncertain for consumers and prices can be a quality signal, but information disparities are not analyzed. Garella and Petrakis (2007) consider both information disparities, consumers' misperceptions and endogenous quality but in an oligopolistic setting with imperfect substitutes, according to the Dixit-Spence-Bowley approach. With respect to us they can consider randomly distributed misperceptions but not in a framework of pure vertical differentiation. According to a strand of literature informed consumers can exert a positive externality on uninformed ones and affect the incentive of firms to provide higher quality products: Chan and Leland (1982), Cooper and Ross (1984) and Wolinsky (1983), in the framework of perfect competition and monopolistic competition, also show that higher prices can signal higher quality. In the framework of vertical differentiation, the signaling function of prices (and advertising) when quality is uncertain, has been considered by Fluet and Garella (2002), Hertzendorf and Overgaard (2001) and Daughety and Reinganum (2008). However in these models quality is exogenously given and there are no information disparities. Gabszewicz and Resende (2012) consider price competition in the case of credence goods - as we do - but without considering the quality choice. Moreover they introduce asymmetric information about quality by assuming that consumers do not know which firms sells which quality, building on the previous analysis of Gabszewicz and Grilo (1992).

Bonroy and Constantatos (2008) follow this same approach to address the issue of voluntary versus mandatory labels in credence good markets. Information provision policies are also considered by Brouhle and Khanna (2007) in a duopoly with vertical differentiation and imperfect information about quality. Quality is endogenous in their model, but consumers' heterogeneity depends on their beliefs about the accuracy of information provision, which directly affects consumers utility.

The paper is structured as follows. In section 2 we present the basic model In section 3 we consider demand functions In section 4 we introduce equilibrium analysis. In section 5 we carry out equilibrium analysis for the most interesting cases Section 8 concludes.

2 The Basic Model

We consider a market with N consumers. Each consumer buys one unit of the product (we shall assume that the market is completely covered). Consumer preferences can be represented by the following quasi-linear utility function (Mussa and Rosen, 1978):

$$U = \theta q - P$$

The willingness to pay for quality is represented by θ,which is uniformly distributed between $\underline{\theta}$ and $\bar{\theta}$ with $\bar{\theta} = \underline{\theta} + 1$ and density $f(\theta) = 1$. P is the market price and q represents product quality, which can be low (q_L) or high (q_L)[3]. There is a minimum quality standard q_0, enforceable by the government; thus $q_L \geq q_0$ and q_0 is common knowledge. Consumers have rational expectations about the low quality product, as they expect that $q_L = q_0$ (such an expectations is fulfilled in equilibrium). High quality is perfectly known to the producers but is unknown to the consumer, unless it is informed. Uninformed consumers are uncertain about the quality differential. Due to the existence of a minimum quality standard they can exclude that $q_H < q_0$ but hold consumers' misperceptions about the quality differential which is provided by the firm claiming to sell high quality products. However we assume that each uninformed consumer has the same expectation q_E concerning high quality. As we do not put further restrictions on q_H and q_E, we consider the case where $q_E > q_H$, i.e. uninformed consumers are characterized by optimistic misperceptions

[3]The vertical differentiation model with complete information we make reference to is presented by Tirole(1989).

As to the distinction between informed and uninformed consumers we split the market in two parts, following the distribution of θ. Consumers with a willingness to pay for quality $\theta \geq \theta^*$ are informed and then observe q_H. Consumers characterized by a willingness to pay $\theta < \theta^*$remain uninformed; and make purchase decisions on the basis of an expectation q_E. Therefore, the greater is θ^* and the lower is the share of informed consumers. In what follows we shall not put any restriction on the value of θ^* except that $\underline{\theta} \leq \theta^* \leq \bar{\theta}$. Therefore demand functions will be shaped accordingly. The timing structure of the model can be described in the following way:

1.In the first stage the market is split between uninformed and informed consumers, according to consumers heterogeneity about θ, which is exogenously given

2.In the second stage firms, taking consumers information and expectations about the quality differential as given, choose the quality level

3.In the third stage firms, given their decisions concerning quality, compete in prices.

In the market there are two firms that can produce either a good of quality q_L or a good of quality q_H. Firms are perfectly informed about both product qualities. Let firm one specialize in the production of the good of quality q_L and firm two specialize in the production of quality q_H, so that we can label firm one as L and firm two as H. We do not consider fixed production cost as we neglect the entry stage and we normalize to zero the variable cost of production. But we suppose that providing higher qualities implies higher efforts. Therefore we consider the cost of quality as αq^2, with $\alpha q_L^2 < \alpha q_L^2$. By considering the cost of quality as the cost of the greater effort of providing high quality goods we can well consider cases where firms should respect a minimum quality standard but can put greater efforts in quality control or any other activity which improves product quality. Low quality goods are sold at price P_L and high quality goods are sold at price P_H. As we assume that the market is covered we suppose that in equilibrium $P_L^* \leq q_L\underline{\theta}$.

In order to define market demand for q_L and q_H we start from the definition of the marginal consumer, who is indifferent between buying from firm L or from firm H. .However in this model informed consumers observe the true quality q_H while uninformed consumers just have an expectation about quality: q_E. Both consumers expect that $q_L = q_0$.Thus we are led to define two types of marginal consumer. The first one is the uninformed marginal consumer θ', who is defined by the following equality: $\theta q_0 - P_L = \theta q_E - P_H$ giving

$$\theta' = \frac{P_H - P_L}{q_E - q_0}$$

Let us call $\Delta_E = q_E - q_0$ the expected quality difference perceived by uninformed consumers. Then uninformed consumers, with a willingness to pay $\theta \geq \theta'$ (and $\theta \leq \theta^*$) choose the high quality product while uninformed consumers with a willingness to pay $\theta \leq \theta'$ (and $\theta \leq \theta^*$) choose the low quality product

The second marginal consumer is the informed one θ'':

$$\theta'' = \frac{P_H - P_L}{q_H - q_0}$$

and let us call $\Delta = q_H - q_0$ the true quality differential, only known to informed consumers. Then informed consumers with a willingness to pay $\theta \geq \theta''$ (and $\theta \geq \theta^*$) choose the high quality product while informed consumers with a willingness to pay $\theta \leq \theta''$(and $\theta \leq \theta^*$) choose the low quality product.

However the definition of demand functions for the low quality and high quality products requires further assumptions on the parameters of the model. For each market splitting between informed and uninformed consumer, i.e. for each location of θ^* with respect to θ' and θ'', market demands can change accordingly. Furthermore, when considering the respective locations of the marginal consumers θ' and θ'' across the market, we are necessary led to distinguish two main cases. Either $\theta' < \theta''$ or $\theta' > \theta''$. Given P_H, P_L and q_0, the sign of the previous inequality only depends on the relationship between q_E and q_H. Actually if $q_H < q_E$, uninformed consumers are **optimistic**. In the optimistic case $\theta' < \theta''$[4]

3 Market Demands

Equilibrium analysis needs a definition of demand functions, in our model they are given by

$$D_i(P_H, P_H, \Delta_E, \Delta, \theta^*) \; i = L, H$$

[4]Actually in thi same framework also the case of pessimistic misperceptions could be analyzed as in Cavaliere Crea (2016).

(from now on D_L and D_H). We can define demand functions through the following steps. We start by considering alternative locations for θ^* in the space $\left[\underline{\theta}, \bar{\theta}\right]$, with respect to the location of θ' and θ'',(remembering that $\theta' \leq \theta''$ always holds due to optimistic misperceptions) At this step we assume that prices and quality differentials Δ_E and Δ are given. For each case we can find restrictions on price domains and expressions for the segments of market demands corresponding to these restrictions (cases A.1-A.10, in Appendix II).

However further assumptions about Δ_E and Δ need to be introduced to consider the full range of price domains consistent with market segments previously defined.(as in cases A1-A10). Actually the share of informed-uninformed consumers can vary together with Δ_E and Δ.In the second step we shall then consider the variations in Δ_E and Δ, by looking at the ratio $\frac{\Delta_E}{\Delta}$ telling us how much optimistic uninformed consumers can be. One should also consider that the expected quality differential Δ_E cannot be unbounded. As also the willingness to pay for quality cannot be greater then $\bar{\theta}$,restrictions on $\frac{\Delta_E}{\Delta}$ depending on on $\underline{\theta}, \bar{\theta}$,and θ^* appear to be sensible in the framework of this model. As a result we shall be able to restrict the definitions of market demands to four alternative cases. Restrictions will concern the ratio $\frac{\Delta_E}{\Delta}$ and be consistent with variations in $\underline{\theta}, \bar{\theta}$ and θ^*.In the last step, for each of these four cases we consider the sequence of price domains consistent with the market segments previously defined and finally obtain demand functions in each of the four cases.

By considering alternative orderings of the price domains previously found to define demand segments, we can obtain the following restrictions on $\frac{\Delta_E}{\Delta}$, that define four alternative couples of demand functions:

$$\begin{aligned} A.a)1 &\leq \frac{\Delta_E}{\Delta} \leq Min\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\}; A.b)\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}}; \\ A.c)\frac{\theta^*}{\underline{\theta}} &\leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}; A.d)Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}} \end{aligned}$$

Considering that $\frac{\Delta_E}{\Delta} \geq 1$, in case A.a the previous restrictions allow for Δ_E strictly close to Δ, (i.e $\frac{\Delta_E}{\Delta} \sim 1$). This implies consumers that are only slightly optimistic and by chance expect a quality differential close to the real one. In case A.d we can observe the highest ratio, with "over-optimistic" consumers (i.e $\frac{\Delta_E}{\Delta} \sim \frac{\bar{\theta}}{\underline{\theta}}$). In between these two extremes, we find intermediate cases A.b and A.c In cases A. b and A.c the restrictions are such that we can respectively state that most consumers are uninformed, (as $\theta^* \geq \sqrt{\underline{\theta}\bar{\theta}}$)[5] or most consumers are informed (as $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$) [6]. We concentrate our attention on these cases, as optimistic misperceptions are significant in both cases, while consumers' information makes the difference.

3.1 Demand Functions in Case (A.b): Most Consumers are Uninformed

In order to define the price domains of the demand function we consider the following price ordering for $P_L : P_H - \underline{\theta}\Delta_E \geq P_H - \theta^*\Delta \geq P_H - \bar{\theta}\Delta \geq P_H - \theta^*\Delta_E$, to obtain D_L, and the following price ordering for $P_H : P_L + \theta^*\Delta_E \geq P_L + \Delta\bar{\theta} \geq P_L + \theta^*\Delta \geq P_L + \underline{\theta}\Delta_E$ to obtain D_H. One can check that the previous price orderings can be reduced to: $\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}}$ The restriction ($\frac{\bar{\theta}}{\theta^*} \leq \frac{\theta^*}{\underline{\theta}}$) implies that $\theta^* \geq \sqrt{\underline{\theta}\bar{\theta}}$, i.e the share of informed consumers is smaller with respect to the share of uninformed ones. Given the previous restriction, one can then look for the demand segments corresponding to each price domain, by checking cases A1-A10 listed in Appendix I. We start by defining $D_L(P_L, P_H)$

$$D_L(P_L, P_H) = \begin{cases} \theta' - \underline{\theta} & if \quad P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta' - \underline{\theta} + \theta'' - \theta^* & if \quad P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta \\ \theta' - \underline{\theta} + \bar{\theta} - \theta^* & if \quad P_H - \theta^*\Delta_E \leq P_L \leq P_H - \bar{\theta}\Delta \\ \bar{\theta} - \underline{\theta} & if \quad 0 \leq P_L \leq P_H - \theta^*\Delta_E \end{cases}$$

One can check that the price-domain of the first segment of D_L is consistent with case (A.3). The second segment is consistent with case (A.1) and the third segment with case A.5. With the highest prices L is bought just by uninformed consumer with a low θ and D_L is affected by consumers misperceptions. When P_L decreases we reach the second price domain where also informed consumers (with an higher θ) buy L. Actually the reduction in P_L moves θ'' towards $\bar{\theta}$ until there is a switch fro $\theta'' \leq \theta^*$ to $\theta'' \geq \theta^*$ (from A.3 to A.1) implying that a share of informed consumers switch to L. Their demand is given by $\left(\theta'' - \theta^*\right)$, and depends on the location of θ^*. As

[5]This inequality implies that θ^*must be larger than the geometric mean between the minimum willingness to pay $\underline{\theta}$ and the maximum willingness to pay $\bar{\theta}$. Cfr. Appendix I for the details.

[6]In case A.a sup $\left\{1 \leq \frac{\Delta_E}{\Delta} \leq Min\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\}\right\} = Min\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\}$ can be shown to be consistent both with most consumers being uninformed (as $\theta^* \geq \sqrt{\underline{\theta}(\underline{\theta}+1)}$) and most consumers being (as $\theta^* \leq \sqrt{\underline{\theta}(\underline{\theta}+1)}$) The same conclusion holds for case A.d, where inf $\left\{Max\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}\right\} = Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\}$

$\theta^{''}$reaches $\overline{\theta}$ - due to the continuous decrease of P_L- the third segment is reached, where all informed consumers with an higher θ buy L. In the third segment the decrease of P_L gradually reduces also the share of uninformed consumer with an intermediate θ that sticks to H. As P_L further decreases $\theta^{'}$ moves towards θ^{*}, until all uninformed consumers buy L Then $D_L\left(P_L,P_H\right)=1$.

The demand for the high quality product $D_H\left(P_L,P_H\right)$ then follows (and one can check that it is complementary to $D_L\left(P_L,P_H\right)$):

$$D_H\left(P_L,P_H\right)=\begin{cases}\theta^{*}-\theta' & if \quad P_L+\overline{\theta}\Delta=P_H\leq P_L+\theta^{*}\Delta_E\\ \theta^{*}-\theta'+\overline{\theta}-\theta^{''} & if \quad P_L+\theta^{*}\Delta\leq P_H\leq P_L+\overline{\theta}\Delta\\ \overline{\theta}-\theta' & if \quad P_L+\underline{\theta}\Delta_E\leq P_H\leq P_L+\theta^{*}\Delta\\ \bar{\theta}-\underline{\theta} & if \quad 0\leq P_H\leq P_L+\underline{\theta}\Delta_E\end{cases}$$

Then the first segment is consistent with case A.5. The second segment is consistent with case A.1. The third segment is consistent with case A.3. With the highest prices for H only uninformed consumers which overestimate the quality differential are willing to buy H, as the real quality differential Δ is not large enough to lead informed consumers to buy H. When P_H decreases and the second price domain is reached then also the demand coming from informed consumers with the highest θ adds to the demand by uninformed consumers. Within the second segment, the reduction of P_H implies that $\theta^{''}$moves towards θ^{*}.When $\theta^{''}=\theta^{*}$ the third segment is reached and further reductions of P_H imply a switch from $\theta^{''}\geq\theta^{*}$to $\theta^{''}<\theta^{*}$, which is consistent with case A.3. so that D_H increases further and just $\theta^{'}$can affect it (actually $\theta^{''}<\theta^{*}$ implies that $\theta^{''}$can no more affect market demands). As the reduction of P_H also moves $\theta^{'}$ towards $\underline{\theta}$, when P_H is low enough it occurs that $\theta^{'}=\underline{\theta}$. In this last case all consumers buy H and $D_H\left(P_L,P_H\right)=1$.

Demand functions are then represented in fig 2 and 3 showing their kinked shape which is typical of vertical differentiation models.

3.2 Demand Functions in Case (A.c): Most Consumers Are Informed

In this sub case we assume the following price ordering for P_L in order to define the price domain of $D_L\left(P_L,P_H\right):P_H-\theta^{*}\Delta\geq P_H-\underline{\theta}\Delta_E\geq P_H-\theta^{*}\Delta_E\geq P_H-\bar{\theta}\Delta$ and the following price ordering for P_H in order to define $D_H\left(P_L,P_H\right):P_L+\Delta\bar{\theta}\geq P_L+\theta^{*}\Delta_E\geq P_L+\underline{\theta}\Delta_E\geq P_L+\theta^{*}\Delta$. One can check that the previous inequalities reduce to the following: $\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}$. The restriction $(\frac{\theta^*}{\underline{\theta}} \leq \frac{\bar{\theta}}{\theta^*})$ implies that $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$, i.e the share of informed consumers is larger than the share of uninformed ones. Given the previous restrictions, one can then define demand segments for each price domain, by checking cases A.1-A.10, as defined above

$$D_L\left(P_L, P_H\right) = \begin{cases} \theta^{''} - \theta^* & if \quad P_H - \underline{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta'' - \theta^* + \theta' - \underline{\theta} & if \quad P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta'' - \underline{\theta} & if \quad P_H - \bar{\theta}\Delta \leq P_L = P_H - \theta^*\Delta_E \\ \overline{\theta} - \underline{\theta} & if \quad 0 \leq P_L \leq P_H - \bar{\theta}\Delta \end{cases}$$

The first price domain is consistent with case A.4 The second one is consistent with case A.1 and the third one with A.2 .We can notice that in this sub-case, demand functions, are affected mainly by $\theta^{''}$, i.e by Δ, as most consumers are informed (the only exception being the second segment). With an high P_L, D_L comes from informed consumers with an intermediate θ. Moreover D_L increases with the share of informed consumers. Information leads to choose L even with an high P_L,as Δ is not worth selecting H. With a decrease of P_L, also a share of uninformed consumers with a lower θ: $(\theta' - \underline{\theta})$ buy L . As P_L further decreases θ' moves towards θ^* until $\theta' = \theta^*$ and the third price domain is reached. Then θ'can no more affect D_L.Therefore demand will be given by the whole share of uninformed consumers $(\theta^* - \underline{\theta})$ plus the share of informed consumers finding it convenient to buy L: $(\theta'' - \theta^*)$ to get $D_L = (\theta'' - \underline{\theta})$. For an even lower P_L, θ'' moves towards $\underline{\theta}$ until $\theta'' = \underline{\theta}$ and then $D_L\left(P_L, P_H\right) = 1$.

$D_H\left(P_L, P_H\right)$ then follows and one can easily check that it is complementary to $D_L\left(P_L, P_H\right)$:

$$D_H\left(P_L, P_H\right) = \begin{cases} \overline{\theta} - \theta'' & if \quad P_L + \theta^*\Delta_E \leq P_H \leq P_L + \underline{\theta}\Delta \\ \theta^* - \theta' + \overline{\theta} - \theta'' & if \quad P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta_E \\ \overline{\theta} - \theta'' + \theta^* - \underline{\theta} & if \quad P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \\ \bar{\theta} - \underline{\theta} & if \quad 0 \leq P_H \leq P_L + \theta^*\Delta \end{cases}$$

One can check that price domains of $D_H\left(P_L, P_H\right)$ are respectively consistent with case A.2, A.1 and A.4. All segments of the demand function (but the second one) are affected by the marginal informed consumers θ'' and by θ^*. As θ' disappears from most segments (but the second one), consumers' misperceptions are not affecting market demands. With high prices, H is just purchased by informed consumers with the highest willingness to pay. As P_H decreases then the demand from uninformed consumers with an intermediate θ will add to get $D_H = \left(\theta^* - \theta' + \overline{\theta} - \theta''\right)$. Within the second price domain the decrease of P_H will move θ' towards $\underline{\theta}$, and the third price domain is reached when $\theta' = \underline{\theta}$. This implies that all uninformed consumers (included the "poorest"ones) will demand H. Actually in the third segment $D_H = 1 + \theta^* - \theta''$ implying that D_H increases with a decrease of the share of informed consumers. When P_H further decreases within the third segment, then θ'' moves toward θ^*, until $\theta'' = \theta^*$ and $D_H\left(P_L, P_H\right) = 1$.

Demand functions are then represented in fig 1.6, 1.7

3.3 Demand Functions in Case (A.d) and (A.a)

Even if we do not analyze case A.d, (see Appendix III) we would like to point out that considering the concerned parameter restrictions: $Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$ we can show that this case is consistent either with most consumers being informed or most consumers being uninformed[7]. Actually when checking for the price domains of the demand functions (Appendix II) one can show they are consistent with cases A.4, A.1 and A.5 respectively. However what distinguishes sub-case A.d is the fact that the ratio $\frac{\Delta_E}{\Delta}$ is very high, i.e. consumers are "over-optimistic". Then Δ is likely to be significantly lower than Δ_E. and when P_L decreases across price domains, the increase of θ'' is likely to be such that θ'' moves towards and reaches $\bar{\theta}$ before $D_H\left(P_L, P_H\right) = 1$, implying of course that it is not necessary to reduce P_L too much to persuade informed consumers with the highest θ to purchase L,given the low quality differentialΔ. On the contrary only a decrease of P_H would lead these consumers to switch to H. Therefore when P_H is too high, then H end up being bought just by uninformed consumers with an intermediate θ[8].

Concerning case A.a (see Appendix III) we can point out that also this case, given the parameter restrictions, is consistent either with either most consumers being informed or most consumer being uninformed. Furthermore as $\frac{\Delta_E}{\Delta} \sim 1$,the quality differential is less important then in previous cases

[7](as $Max\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\}$ includes both cases where $\theta^* \leq \sqrt{\bar{\theta}\underline{\theta}}$ and cases where $\theta^* \geq \sqrt{\bar{\theta}\underline{\theta}}$)

[8]Likewise, as Δ_E is likely to be very high, when P_H decreases across price domains then the decrease can be such that θ' moves towards and reaches $\underline{\theta}$ before $D_H\left(P_L, P_H\right) = 1$, implying that it is not necessary to reduce P_H to much to persuade uninformed consumers with the lowest willingness to pay to buy high quality goods.

in shaping the demand functions, as by chance uninformed consumers expect a quality differential close to ith the real one. Therefore this case is less interesting from our point of view.

4 Equilibrium Analysis

In this section we analyse price and quality competition between the two firms, solving the two stage game by backward induction. In the last stage, firms decide on prices, given qualities chosen in the previous stage and information disparities arising by exogneous consumers' decisions. Each firm chooses a strategy that is the best reply to the other seller's strategy. Thus let $\Pi_i(P_i, P_j) = P_i D_i(P_i, P_j)\ i, j = L, H$ denote the profit function of firm i., remembering that we have assumed that firm one sells the low quality product and firm two sells the high quality product

Definition: A price (Nash) equilibrium is a pair (P_L^*, P_H^*) such that no firm has an incentive to change its price unilaterally:

$$\Pi_i(P_i^*, P_j^*) \geq \Pi_i(P_i, P_j^*)\ i, j = L, H$$

In the following sub-sections we shall look for a candidate Nash equilibrium in prices both in the optimistic and the pessimistic case. Being demands piecewise linear, for each configuration of the demand function we can find the candidate Nash equilibrium prices, considering each price domain for each demand function. For each sub-case we can moreover obtain the restrictions on the number of informed consumers that result from checking that the candidate equilibrium prices actually belong to the price domains in question In order to show that the price pairs are indeed a Nash equilibrium we have to check that the last Definition is satisfied[9]. This will be equivalent to checking that the candidate equilibrium prices assure optimisation of the profit functions not only in the price domains considered one at a time, but also in the entire price range characterising each configuration of the demand functions[10].

Given equilibrium prices, we then consider the quality choice in the previous stage, to analyze the degree of product differentiation in equilibrium.Optimal qualities will arise not only by considering the maximization of the profit function evaluated at equilibrium prices found in the last stage but

[9] A complete analysis of equilibria from this point of view can be found in Crea (2015)

[10] For a similar analytical methodology, see Garella and Martinez-Giralt(1989)

also considering the restrictions given by the price domains characterizing each couple of candidate equilibrium prices.

In order to carry out equilibrium analysis with information disparities, a useful benchmark is represented by the case where all consumers are uninformed This case simply follows from the standard vertical differentiation model when considering the uninformed marginal consumer $\theta' = \frac{P_H - P_L}{q_E - q_0}$to define demand functions: $D_L(P_L, P_H, \Delta_E) = (\theta' - \underline{\theta})$ and $D_H(P_L, P_H, \Delta_E) = \left(\bar{\theta} - \theta'\right)$ and obtain profit functions:

$$\Pi_L(P_L, P_H) = P_L\left(\theta' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H(P_L, P_H) = P_H\left(\bar{\theta} - \theta'\right) - \alpha q_H^2$$

Then equilibrium prices depend on the expected quality differential Δ_E:

$$P_L^* = \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

Given consumers misperceptions about the quality differential Δ_E,and considering that q_H is costlier to produce with respect to q_L,but q_H cannot be observed by any consumers, it is optimal both for firm L and firm H to provide the MQS. Therefore in equilibrium we observe minimum product differentiation though firm can charge equilibrium prices consistent with the misperceived quality differential Δ_E.Price competition is then relaxed by misperceived quality differentiation, and we observe moral hazard by firm H.

5 Equilibrium Analysis in case A.d and A.c

We consider equilibrium analysis in case A.d and A.c (see Appendix IV for cases A.a and A.d)

5.1 Case A.b: Most Consumers Are Uninformed

In order to find the candidate equilibrium prices we consider the complementary demand segments of $D_L(P_L, P_H)$ and $D_H(P_L, P_H)$ one at a time:

5.1.1 A.b.1

Given the respective price domain: $P_H - \theta^*\Delta \leq P_L^* \leq P_H - \underline{\theta}\Delta_E$ for D_L and $P_L + \underline{\theta}\Delta_E \leq P_H^* \leq P_L + \theta^*\Delta D_H$ for D_H, demand segments lead to the following profit functions:

$$\Pi_L(P_L, P_H) = P_L\left(\theta' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H(P_L, P_H) = P_H\left(\bar{\theta} - \theta'\right) - \alpha q_H^2 \tag{1}$$

By profit maximization we can then obtain the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3} \tag{2}$$

and equilibrium profits:$\Pi_L^* = \frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} - \alpha q_H^2$

By checking if the candidate equilibrium prices are actually included in the price domains given above, we get a further restriction on θ^*: $\theta^* \geq \frac{\Delta_E\left(2\underline{\theta}+1\right)}{3\Delta}$

By considering that across case A.b most consumers are uninformed (as $\theta^* \geq \sqrt{\underline{\theta}\bar{\theta}}$) in case A.b.1 θ^* should be even greater if $\frac{\Delta_E}{\Delta} \gtrapprox \frac{3}{2}$ (or lower if $\frac{\Delta_E}{\Delta} \lessapprox \frac{3}{2}$). Still most consumers remain uninformed. Given the previous solution for the last stage of the game, we can consider the quality selection stage, where the degree of product differentiation is found by maximizing equilibrium profits with respect to qualities. Considering profit maximization by firm L with respect to q_L we get: $\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta}-\bar{\theta}^2-4\underline{\theta}^2}{9} - 2\alpha q_L \leq 0$

Therefore firm L finds it it optimal to minimize q_L,but due to the existence of a MQS this implies that $q_L^* = q_0$. Concerning firm H,as its profits depend both on the actual level of quality provided (through costs) and on expected quality (through demand) we consider firstly the impact of the quality increase on costs by maximization of the profit function to get: $\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_H$

As revenues depend on expected quality, which is not controlled by the firm, we can get a restriction on q_H by checking that the equilibrium prices P_L^* and P_H^* actually belongs to the respective price intervals. We then get the following restriction on q_H:

$$q_H \geq q_0 + \frac{\Delta_E\left(2\underline{\theta} + 1\right)}{3\theta^*} \tag{3}$$

By jointly considering the previous two results we can obtain a corner solution: $q_H^* = q_0 + \frac{\Delta_E\left(2\underline{\theta}+1\right)}{3\theta^*}$. According to this solution we can state that $q_H^* > q_0$ (there is some product differentiation as $\Delta > 0$) and moreover that $(q_H^* - q_0) < \Delta_E$. As P_H^* depends on Δ_E, but Δ is lower, we can state that consumers of H pay an excessive price premium with respect to the quality differential actually provided. Moreover as also P_L^* depends on Δ_E we can observe that with optimistic misperception price competition is further relaxed with respect to the full information case, Given that all informed consumers buy H ($\theta^* > \theta'$ and $\theta^* = \theta''$) the fact that these consumers are characterized by the highest willingness to pay contributes to explain their choice to stick to high quality even if the price charged is excessive with respect to the quality differential actually provided. Furthermore by considering the expression of q_H^* one can easily check that Δ increases both with Δ_E and with an increase in the share of informed consumers.The product differentiation effort is increasing with expected quality as the more optimistic are uninformed consumers the higher will be equilibrium prices and therefore an increasing level of quality needs to be provided to informed consumers to make product H worthwhile being selected by them. If the share of informed consumers increases then consumers with a lower and lower willingness to pay may consider to buy high quality goods instead of low quality goods. Therefore in order to capture these consumers, an increasing level of quality should be provided by the high quality firm, in order to avoid "deception" that may lead some informed consumers to switch to low quality goods. Therefore in this equilibrium even if a minority of "rich" consumers is informed, as all of them buy H, a positive quality differential is provided by firm H, though $\Delta < \Delta_E$.

Proposition 1 *When most consumers are uninformed and buy both q_L and q_H,while all informed consumers buy q_H, equilibrium prices are distorted upwards and price competition is further relaxed by optimistic misperceptions. The quality differential provided by firm H is lower than expected by uninformed consumers but increases both with the level of expected quality and the share of informed consumers.*

5.1.2 A.b.2

Considering the respective price domain for P_L^*:$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \theta^*\Delta$ and for $P_H^* : P_L + \theta^*\Delta \leq P_H^* \leq P_L + \bar{\theta}\Delta$ and the related demand segments, we get the following profit functions:

$$\Pi_L (P_L, P_H) = P_L \left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H (P_L, P_H) = P_H \left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

Profit maximization leads to the following equilibrium prices:

$$P_L^* = \frac{\Delta_E \Delta \left(1 - \left(\underline{\theta} + \theta^*\right)\right)}{3\left(\Delta_E + \Delta\right)} \qquad P_H^* = \frac{\Delta_E \Delta \left(\underline{\theta} + \theta^* + 2\right)}{3\left(\Delta_E + \Delta\right)}$$

In this case both L and H are bought by uninformed and informed consumers. Then equilibrium prices are affected by all parameters of the model. What is interesting to notice is that an increase in the share of informed consumers (lower θ^*) implies an increase of P_L^* and a reduction of P_H^*. While a decrease of this share (higher θ^*) has the opposite effect. Therefore consumer information affects price competition, with opposite effect on firms. An increasing share of informed consumers implies a reduction of D_H and an increase of D_L with asymmetric effects on equilibrium prices and profits In this case we cannot reach a clear cut conclusion concerning the quality choice at equilibrium, which is discussed in Appendix VI.

5.1.3 A.b.3

Considering the respective price domains for $P_L^* : P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \bar{\theta}\Delta$ and $P_H^* : P_L + \bar{\theta}\Delta \leq P_H^* \leq P_L + \theta^* \Delta_E$ and the corresponding demand segments we get the following profit functions

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\left(\bar{\theta} - \underline{\theta}\right) - \theta^* + \theta'\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\left(\theta^* - \theta'\right) - \alpha q_H^2\right.$$

leading to the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta_E\left(2 - \theta^*\right)}{3} \qquad P_H^* = \frac{\Delta_E\left(1 + \theta *\right)}{3}$$

and equilibrium profits:$\Pi_L^* = \frac{\Delta_E(2-\theta^*)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E(1+\theta^*)^2}{9} - \alpha q_H^2$ Checking if equilibrium prices belongs to the price domain characterizing A.b.3, we get a further restriction on θ^*: $\theta^* \geq \frac{1}{2} + \frac{3\Delta\bar{\theta}}{2\Delta_E}$.

Considering the quality selection stage we get:$\frac{\partial \Pi_L}{\partial q_{\mathrm{L}}} = -\frac{{\theta^*}^2 - 4\theta^* + 4}{9} - 2\alpha q_L \leq 0$ and $\frac{\partial^2 \Pi_L}{\partial q_L^2} = -2\alpha q_L$.

Still leading to $q_{\mathrm{L}}^* = q_0$. Concerning the high quality firm, by considering profit maximization with respect to quality we get: $\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_H$; $\frac{\partial^2 \Pi_H}{\partial q_H^2} = -2\alpha$.

The previous f.o.c. and s.o.c. account for the negative effect of cost on the level of q_H. And by considering the restrictions on equilibrium prices arising from the price domains we obtain

$$q_H \leq q_0 + \frac{\Delta_E\left(2\theta^* - 1\right)}{3\bar{\theta}}$$

Therefore by jointly considering both the f.o.c. and the previous restriction we find that the firm will find it optimal to provide the lowest possible quality: $q_H^* \to q_0$. As in this sub-case (A.b.3) $\theta'' = \bar{\theta}$, and still considering that most consumers are uninformed, then the small share of informed consumers with an high θ finds it optimal to purchase L, being aware of minimum product differentiation. Uninformed consumers with the lowest θ also buy L but just because they cannot afford product H. As only uninformed consumers with an intermediate θ buy H,these consumers are cheated in equilibrium: they pay an higher price to get the same quality q_0.demanded by informed consumers. Therefore there is no real product differentiation in equilibrium, as $q_H^* \to q_0$ However virtual product differentiation implied by optimistic misperceptions still help firms to relax price competition as both P_L^*,and P_H^* are distorted upwards depending on Δ_E. Furthermore as equilibrium prices also depend on θ^* one can check that an increase of informed consumers still leads (as in case A.b.2) to an increase of P_L^*.and a decrease of P_H^* Consumers information counterbalances optimistic misperceptions and then reduces price distortions for consumers of H, but not for consumers of L, given that product H is purchased by informed consumers with an higher θ.By checking also equilibrium profits one can see that more information implies further gains for firm L and further losses for firm H.

Proposition 2 *When most consumers are uninformed and buy both q_L and q_H while all informed consumers buy q_L, equilibrium prices are distorted upwards by optimistic misperceptions but there is minimum product differentiation. Price competition is asymmetrically affected by consumers information, as P_L^* increases with the share of informed consumers and P_H^* decreases with it. Therefore informed consumers exert a positive externality on uninformed consumers that buy H at lower prices.*

5.2 Case A.c, Most Consumers Are Informed

5.2.1 A.c.1

Considering the respective price domains for $P_L^* : P_H - \underline{\theta}\Delta_E \leq P_L^* \leq P_H - \theta^*\Delta$ and $P_H^* : P_L + \theta^*\Delta \leq P_H^* \leq P_L + \underline{\theta}\Delta_E$ and the demand segments defined by the previous price domains we can get the following profit functions:

$$\Pi_L(P_L,P_H)=P_L\left(\theta''-\theta^*\right)-\alpha q_L^2 \qquad \Pi_H(P_L,P_H)=P_H\left((\bar{\theta}-\underline{\theta})-\theta''+\theta^*\right)-\alpha q_H^2$$

leading to the following equilibrium prices:

$$P_L^*=\frac{\Delta\left(1(\bar{\theta}-\underline{\theta})-\theta^*\right)}{3} \qquad P_H^*=\frac{\Delta\left(2(\bar{\theta}-\underline{\theta})+\theta*\right)}{3}$$

and equilibrium profits:$\Pi_L^*=\frac{\Delta(1-\theta^*)^2}{9}-\alpha q_L^2 \qquad \Pi_H^*=\frac{\Delta(2+\theta^*)^2}{9}-\alpha q_H^2$

By considering that equilibrium prices should belong to the above price domains we get a further restriction on θ^*:$\theta^*\leq min\left\{1\,,\,\frac{3\underline{\theta}\Delta_E}{2\Delta}-\frac{\bar{\theta}-\underline{\theta}}{2}\right\}$

Considering then the quality selection stage, by profit maximization in qualities we get:$\frac{\partial\Pi_L}{\partial q_L}=-\frac{{\theta^*}^2-2\theta^*+1}{9}-2\alpha q_L\leq 0$; $\frac{\partial^2\Pi_L}{\partial q_0^2}=-2\alpha$ and $\frac{\partial\Pi_H}{\partial q_{\mathrm{H}}}=\frac{{\theta^*}^2+4\theta^*+4}{9}-2\alpha q_H$

trough the f.o.c. we still get that $q_L=q^\circ$ and the optimal quality level for firm H :

$$q_H^*=\frac{{\theta^*}^2+4\theta^*+4}{18\alpha}$$

Considering the restrictions given by the price domain we can show that the following condition about q_E holds in equilibrium:

$$q_E\geq q_0+\frac{\Delta\left(2\theta^*+1\right)}{3\underline{\theta}}$$

This lower bound, on expected quality, implies that the minority of uninformed consumers with a lower willingness to pay - whose demand segment is given by $(\theta^*-\underline{\theta})$ - is willing to purchase H provided they are sufficiently optimistic. High quality goods are also bought by informed consumers with the greatest θ: $\left(\bar{\theta}-\theta''\right)$. Accordingly L is bought by informed consumers, with an intermediateθ:.$(\theta''-\theta^*)$. As one can check, both demands depend on Δ as well as equilibrium prices Furthermore the higher the share of informed consumers (the lower is θ^*) the higher is P_L^* (as well as Π_L^*) and the lower is P_H^*.(as well as Π_H^*). Therefore consumers information affects price competition in two ways, by reducing equilibrium prices symmetrically when most consumers are informed (as P_L^* and P_H^* depend on $\Delta\leq\Delta_E$) and asymmetrically considering that (as we observed in cases A.b.2 and A.b.3) an increasing share of informed consumers increases P_L^* and decreases P_H^*

With respect to product differentiation we can state that quality competition is softened by the increase in the share of informed consumers (consistently with more price competition) while more vertical differentiation arises if the share of informed consumers shrinks Actually one can easily check that q_H^* is an increasing function of θ^*.

Proposition 3 *When most consumers are informed and buy both q_L and q_H while all uninformed consumers buy q_H, equilibrium prices decrease but price competition is asymmetrically affected by consumers information: P_L^*increases with the share of informed consumers and P_H^* decreases with it. The quality differential provided by firm H is negatively affected by the share of informed consumers. Then informed consumers exert a positive externality on uninformed consumers paying lower prices for q_H and a negative externality concerning the level of q_H^*.chosen by firm H.*

5.2.2 A.c.2

In this case both L and H are bought by uninformed and informed consumers and candidate equilibrium prices are identical to those found in case A.b.2 above (for a complete analysis see appendix V).

5.2.3 A.c.3

Considering the respective price domain for P_L^* : $P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \theta^*\Delta_E$ and for P_H^* : $P_L + \theta^*\Delta_E \leq P_H^* \leq P_L + \bar{\theta}\Delta$ and the related demand segments, we get the following profit functions:

$$\Pi_L (P_L, P_H) = P_L \left(\theta'' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H (P_L, P_H) = P_H \left(\bar{\theta} - \theta''\right) - \alpha q_H^2$$

Leading to the following candidate equilibrium prices:

$$P_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

and equilibrium profits:$\Pi_L^* = \frac{\Delta\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} - \alpha q_H^2$

By considering the price domains we get the following restrictions on θ^*: $\theta^* \leq \frac{\Delta\left(2\underline{\theta}+1\right)}{3\Delta_E}$

Considering then quality competition, by maximization of the respective profit functions we get: $\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L \leq 0$ such that $q_L = q^{\circ}$ and $\frac{\partial \Pi_H}{\partial q_H} = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{9} - 2\alpha q_H = 0$

to get the following interior solution for q_H :

$$q_H^* = \frac{\left(\bar{\theta} - 2\underline{\theta}\right)^2}{18\alpha}$$

Considering price domains we also get the following restriction concerning q_H^*

$$q_H^* \geq q_0 + \frac{3\Delta_E \theta^*}{1 + 2\underline{\theta}}$$

In this sub-case it is worthwhile to consider the previous restrictions on $\theta^* \leq \frac{\Delta\left(2\underline{\theta}+1\right)}{3\Delta_E}$.As across case A.c most consumers are informed due to $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$, the previous restrictions allow for an even larger number of consumers being informed as $\theta^* \leq \frac{\Delta\left(2\underline{\theta}+1\right)}{3\Delta_E} \leq \sqrt{\underline{\theta}\bar{\theta}}$. Furthermore, being $\theta^* < \theta' < \theta''$,: $D_L = (\theta'' - \theta^*) + (\theta^* - \underline{\theta}) = (\theta'' - \underline{\theta})$ and then $D_H = \left(\bar{\theta} - \theta''\right)$, one can see that in this sub-case equilibrium prices and qualities are alike to those we can obtain in a standard model of vertical differentiation with perfect information, exception made for the existence of a MQS.

Proposition 4 *When most consumers are informed and buy both q_L and q_H while all uninformed consumer buy q_L equilibrium prices and qualities collapse to the full information case, provided the share of informed consumers is high enough.*

6 Conclusions

Quality uncertainty has been widely explored in the economic literature. However the interaction between quality uncertainty and vertical product differentiation has received less attention. The case where information disparities overlap with quality uncertainty and moreover quality is endogenous has never been explored in pure vertical differentiation models. Our analysis considers the case when the choice between low quality and high quality goods is affected by consumers misperceptions about the quality differential provided by an high quality firm. Markets for credence goods may be particularly suitable as an example of consumers choice driven by misperceptions. In that case it is reasonable to assume that the Government may set a MQS and that competition may lead some firms to claim the provision of quality levels exceeding the MQS. However market failures due to asymmetric information about quality may persist, considering that reputation cannot work as a substitute for quality information.

By departing from the case where all consumers are uncertain about the quality differential (all consumers are uninformed) we can show how increasing the share of informed consumers can alleviate the moral hazard problem with optimistic misperceptions Furthermore we can consider to what extent informed consumers can exert externalities on uninformed ones, both when the share of informed consumers is low and when this share is high. We can find different types of equilibria where price competition and product differentiation are affected both by consumers misperceptions and consumer information.

We can discriminate between equilibria where the brand premium is justified by ”some” product differentiation or just due to consumers misperceptions. In this last case we find minimum product differentiation and buyers of products perceived as high quality goods are cheated in equilibrium. Equilibria where prices are distorted upwards with respect to the full information case, far from being the result of signalling strategies, are just due to optimistic misperceptions. Lower prices may be consistent with real product differentiation when the share of informed consumers is high and our model collapses to the full infromation case. Interestingly with an increase of expected quality we could observe greater incentive for product differentiation, even if the quality differential is lower than expected by uninformed consumers. More consumer information may lead to less product differentiation, especially considering that the price of high quality goods is driven down by an increase in the share of informed consumers. Low quality firms may then profit more from consumer information, considering that that a larger share of informed consumers drives up the price of low quality goods and drives down the price of high quality goods, with respect to the full information case.

In our analysis consumers information and consumers beliefs are exognously given. However we think that it could be possible to introduce also a further stage of the game with firm entry and to consider sunk costs as R&D, which may affect the real quality differential, or advertising, that could affect consumers beliefs and the expected quality differential. These sunk costs could then become endogenous to the model. In our framework expenditure in persuasive advertising may then be provided a foundation through the analysis of optmisitic misperceptions. Furthermore if persuasive advertising can modify consumers beliefs, an even richer model could be considered where beliefs become endogenous. Finally the case of optimistic consumers can be well adapted to deal with competition in the drug market,as the results of equilibrium analysis show. With slight modifications we can account also for price regulation and information provision to consider competition between generics and branded drugs affected by public policies, still considering our framework as a basis for strategic interaction with quality uncertainty and information disparities.

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Appendix I

Restriction on the share of informed consumer. When we have $\frac{\bar{\theta}}{\theta^*} \leq \frac{\theta^*}{\underline{\theta}}$, we obtain $\theta^* \geq \sqrt{\bar{\theta}\underline{\theta}}$, this inequality implies that θ^* must be larger than the geometric mean of the minimum willingness to pay $\underline{\theta}$ and the maximum willingness to pay $\bar{\theta}$, and considering $\bar{\theta} = \underline{\theta} + 1$ we have the result $\theta^* \geq \sqrt{\underline{\theta}\left(\underline{\theta}+1\right)}$. Using a Laurent expansion for $\theta = \infty$

$\underline{\theta} + \frac{1}{2} - \frac{1}{8\underline{\theta}} + \frac{1}{16\underline{\theta}^2} - \frac{5}{128\underline{\theta}^3} + \mathcal{O}$

So the number of informer consumers is approaching 1/2 of the market size when $\underline{\theta}$ increase.

In the opposite case $\frac{\bar{\theta}}{\theta^*} \geq \frac{\theta^*}{\underline{\theta}}$, we obtain: $\theta^* \leq \sqrt{\underline{\theta}\bar{\theta}}$ so θ^* must be lower than the geometric mean of the minimum willingness to pay $\underline{\theta}$ and the maximum willingness to pay $\bar{\theta}$

Appendix II

A.1) $\underline{\theta} \leq \theta' \leq \theta^* \leq \theta'' \leq \bar{\theta}$. (Cf. Figure 1). Both D_L and D_H are given by the sum of the demand by uninformed and informed consumers: $D_L = \theta' - \underline{\theta} + \theta'' - \theta^*$; $D_H = \theta^* - \theta' + \bar{\theta} - \theta''$. One can then notice that not only uninformed consumer with a lower willingness to pay buy L, but also informed consumers with an higher willingness to pay select L, once they are informed about the quality differential. On the contrary there are consumers - with a comparatively lower willingness to pay - that buy H just because they hold optimistic misperceptions. Considering the previous inequalities we can obtain the following restrictions concerning market prices, which will be useful in defining the price domains of demand functions. Concerning D_L and D_H we get

$$P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta_E \quad \textit{or conversely} \quad P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta_E \tag{4}$$

$$P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta \quad \textit{or conversely} \quad P_L + \theta^*\Delta \leq P_H \leq P_L + \Delta\bar{\theta} \tag{5}$$

A.2) $\underline{\theta} \leq \theta^* \leq \theta' \leq \theta'' \leq \bar{\theta}$. D_L is the sum of the demand by uninformed consumers, $(\theta^* - \underline{\theta})$ and informed consumers, $(\theta'' - \theta^*)$: then $D_L = \theta'' - \underline{\theta}$. D_H depends only on informed consumers: $D_H = \bar{\theta} - \theta''$. In this case even consumers with a lower willingness to pay are informed about the quality differential and buy L. Consumers misperceptions are not affecting neither D_L nor D_H (such a result is due to the location of θ^*, given that $\theta^* \leq \theta'$) .We can obtain the following restrictions about price domains:

$$P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E \, or \, conversely \, P_L + \theta^*\Delta_E \leq P_H \leq P_L + \Delta\bar{\theta} \quad (6)$$

A.3) $\underline{\theta} \leq \theta' \leq \theta'' \leq \theta^* \leq \bar{\theta}$. D_L comes only from uninformed consumers: $D_L(\theta' - \underline{\theta})$. D_H depends both from uninformed consumers, $(\theta^* - \theta')$,and informed consumers: $(\bar{\theta} - \theta^*)$: $D_H = \bar{\theta} - \theta'$. Consumers' misperceptions are then affecting both demands, while consumers' information has no effect (this is due to the location of θ^*, given that $\theta'' \leq \theta^*$). The restrictions on price domains arising from case A.3 are the following:

$$P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \, or \, conversely \, P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \quad (7)$$

A.4) $\theta' \leq \underline{\theta} \leq \theta^* \leq \theta'' \leq \bar{\theta}$. D_L comes only from informed consumers: $D_L = (\theta'' - \theta^*)$. D_H depends both from all uninformed consumers, $(\theta^* - \underline{\theta})$,and informed consumers: $(\bar{\theta} - \theta'')$: $D_H = \theta^* - \underline{\theta} + \bar{\theta} - \theta''$. Consumers' misperceptions are then affecting both demands. The restrictions on price domains are the following: From $\theta^* \leq \theta'' \leq \bar{\theta}$ we get:

$$P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta \, or \, conversely \, P_L + \theta^*\Delta \leq P_H \leq P_L + \bar{\theta}\Delta \quad (8)$$

and from $\theta' \leq \underline{\theta}$ we get:

$$P_H \leq P_L + \underline{\theta}\Delta_E \; or \; P_H - \underline{\theta}\Delta_E \leq P_L \quad (9)$$

A.5) $\underline{\theta} \leq \theta' \leq \theta^* \leq \bar{\theta} \leq \theta''$. D_L comes from uninformed consumers: $(\theta' - \underline{\theta})$ and informed consumers $(\bar{\theta} - \theta^*)$: $D_L = \theta' - \underline{\theta} + \bar{\theta} - \theta^*$.$D_H$ depends only on uninformed consumers,$D_H = \theta^* - \theta'$. Consumers' misperceptions are then affecting both demands. The restrictions on price domains follow, from $\underline{\theta} \leq \theta' \leq \theta^*$ we get:

$$P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta_E \, or \, conversely \, P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta_E \quad (10)$$

and from $\bar{\theta} \leq \theta''$ we get:

$$P_L + \bar{\theta}\Delta \leq P_H \; or \; P_L \leq P_H + \bar{\theta}\Delta \quad (11)$$

A.6) $\theta' \leq \theta'' \leq \underline{\theta} \leq \theta^* \leq \bar{\theta}. D_L = 0$ (as $\theta' \leq \underline{\theta}$ and $\theta'' \leq \underline{\theta}$) and then $D_H = \bar{\theta} - \underline{\theta}$. The restrictions on price domains follow:

$$P_H - \underline{\theta}\Delta \leq P_L \; or \; P_H \leq P_L + \underline{\theta}\Delta \tag{12}$$

A.7) $\theta' \leq \underline{\theta} \leq \theta'' \leq \theta^* \leq \bar{\theta}$. $D_L = 0$ because all uninformed consumers purchase H given that $\theta' \leq \underline{\theta}$, and informed consumers buy H as well : $D_H = \bar{\theta} - \underline{\theta}$. Consumers' misperceptions are then affecting D_L. The restrictions on price domains follow. From $\underline{\theta} \leq \theta'' \leq \theta^*$ we get:

$$P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta \; or \; conversely \; P_L + \underline{\theta}\Delta \leq P_H \leq P_L + \theta^*\Delta \tag{13}$$

and from $\theta' \leq \underline{\theta}$ we get.

$$P_H - \underline{\theta}\Delta_E \leq P_L \; or \; P_H \leq P_L + \underline{\theta}\Delta_E \tag{14}$$

A.8) $\underline{\theta} \leq \theta^* \leq \theta' \leq \bar{\theta} \leq \theta''$. $D_L = \bar{\theta} - \underline{\theta}$, as $\theta'' \geq \bar{\theta}$ and $\theta' \geq \theta^*$ (thus θ'in inactive in shaping market demands) and $D_H = 0$. From $\theta^* \leq \theta' \leq \bar{\theta}$ we get:

$$P_H - \bar{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta_E \; or \; conversely \; P_L + \theta^*\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta_E \tag{15}$$

and from $\bar{\theta} \leq \theta''$ we get:

$$P_L + \bar{\theta}\Delta \leq P_H \; or \; P_L \leq P_H - \bar{\theta}\Delta \tag{16}$$

A.9) $\underline{\theta} \leq \theta^* \leq \bar{\theta} \leq \theta' \leq \theta''$. $D_L = \bar{\theta} - \underline{\theta}$ as $\theta' \geq \bar{\theta}$ and $\theta'' \geq \bar{\theta}$.and then $D_H = 0$.

$$P_L + \bar{\theta}\Delta_E \leq P_H \; or \; P_L \leq P_L - \bar{\theta}\Delta_E \tag{17}$$

A.10) $\theta' \leq \underline{\theta} \leq \theta^* \leq \bar{\theta} \leq \theta''.D_L$ comes only from all informed consumers:$D_L = \bar{\theta} - \theta^*$ while all uninformed consumers purchase H: $D_H = \theta^* - \underline{\theta}$. From $\theta' \leq \underline{\theta}$ we get:

$$P_H - \underline{\theta}\Delta_E \leq P_L \; or \; P_H \leq P_L + \underline{\theta}\Delta_E \tag{18}$$

and from $\bar{\theta} \leq \theta''$ we get:

$$P_L + \bar{\theta}\Delta \leq P_H \; or \; P_L \leq P_H - \bar{\theta}\Delta \tag{19}$$

Appendix III

Demand functions in case A.a and A.d

A.a

By assuming the following restriction:

$$P_H - \underline{\theta}\Delta_E \geq P_H - \theta^*\Delta \geq P_H - \theta^*\Delta_E \geq P_H - \bar{\theta}\Delta \tag{20}$$

$$P_L + \bar{\theta}\Delta \geq P_L + \theta^*\Delta_E \geq P_L + \theta^*\Delta \geq P_L + \underline{\theta}\Delta_E$$

We obtain:

$$1 \leq \frac{\Delta_E}{\Delta} \leq min\left\{\frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*}\right\}$$

Given these restrictions we can specify price domains and demand function as follows:

$$D_L(P_L, P_H) = \begin{cases} \theta' - \underline{\theta} & if\ P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta' - \underline{\theta} + \theta'' - \theta^* & if\ P_H - \theta^*\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta'' - \underline{\theta} & if\ P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E \\ \bar{\theta} - \underline{\theta} & if\ 0 \leq P_L \leq P_H - \bar{\theta}\Delta \end{cases}$$

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta'' & if\ P_L + \theta^*\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta \\ \bar{\theta} - \theta'' + \theta^* - \theta' & if\ P_L + \theta^*\Delta \leq P_H \leq P_L + \theta^*\Delta_E \\ \bar{\theta} - \theta' & if\ P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \\ \bar{\theta} - \underline{\theta} & if\ 0 \leq P_H \leq P_L + \underline{\theta}\Delta_E \end{cases}$$

A.d

By assuming the following restriction:

$$P_H - \theta^*\Delta \geq P_H - \underline{\theta}\Delta_E \geq P_H - \bar{\theta}\Delta \geq P_H - \theta^*\Delta_E \tag{21}$$

$$P_L + \theta^*\Delta_E \geq P_L + \bar{\theta}\Delta \geq P_L + \underline{\theta}\Delta_E \geq P_L + \theta^*\Delta$$

from this restriction in this case we need $\underline{\theta} > 0$ to obtain:

$$max\left\{\frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}}\right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$$

Given these restrictions we can specify price domains and demand functions as follows:

$$D_L\left(P_L, P_H\right) = \begin{cases} \theta'' - \theta^* & if\ P_H - \underline{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta' - \underline{\theta} + \theta'' - \theta^* & if\ P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ 1 - \theta^* + \theta' & if\ P_H - \theta^*\Delta_E \leq P_L \leq P_H - \bar{\theta}\Delta \\ \bar{\theta} - \underline{\theta} & 0 \leq P_L \leq P_H - \theta^*\Delta_E \end{cases} \quad (22)$$

$$D_H\left(P_L, P_H\right) = \begin{cases} \theta^* - \theta' & if\ P_L + \bar{\theta}\Delta \leq P_H \leq P_L + \theta^*\Delta_E \\ \bar{\theta} - \theta'' + \theta^* - \theta' & if\ P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta \\ 1 - \theta'' + \theta^* & if\ P_L + \theta * \Delta \leq P_H \leq P_L + \underline{\theta}\Delta_E \\ \bar{\theta} - \underline{\theta} & if\ 0 \leq P_H \leq P_L + \theta * \Delta \end{cases} \quad (23)$$

Appendix IV

A.a.1

We consider the following price domain

$$P_H - \theta^*\Delta \leq P_L^* \leq P_H - \underline{\theta}\Delta_E$$

$$P_L + \underline{\theta}\Delta_E \leq P_H^* \leq P_L + \theta^*\Delta$$

To get profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\bar{\theta} - \theta'\right) - \alpha q_H^2$$

Then we obtain equilibrium prices:

$$P_L^* = \frac{\Delta_E\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta_E\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

And equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E\left(\bar{\theta}-2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E\left(2\bar{\theta}-\underline{\theta}\right)^2}{9} - \alpha q_H^2$$

Considering the price domain we can get the following restriction on θ^*:

$$\theta^* \geq \frac{\Delta_E\left(2\underline{\theta}+1\right)}{3\Delta}$$

Then we consider the quality choice. By profit maximization we get:

$$\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta}-\bar{\theta}^2-4\underline{\theta}^2}{9} - 2\alpha q_L$$

$$q_L = \frac{4\bar{\theta}\underline{\theta}-\bar{\theta}^2-4\underline{\theta}^2}{18\alpha} \quad and \quad \frac{4\bar{\theta}\underline{\theta}-\bar{\theta}^2-4\underline{\theta}^2}{18\alpha} \leq 0$$

$$\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_H \leq 0$$

And considering the price domain we get the following restriction on q_H

$$q_H \geq q_0 + \frac{\Delta_E\left(2\underline{\theta}+1\right)}{3\theta^*}$$

The results are equivalent to case A.b.1, already discussed in section 5.1

A.a.2

We consider the following price domain

$$P_H - \theta^*\Delta_E \leq P_L^* \leq P_H - \theta^*\Delta$$

$$P_L + \theta^*\Delta \leq P_H^* \leq P_L + \theta^*\Delta_E$$

To get profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

Then we obtain equilibrium prices:

$$P_L^* = \frac{\Delta_E\Delta\left(1-\left(\underline{\theta}+\theta^*\right)\right)}{3\left(\Delta_E+\Delta\right)} \qquad P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta}+\theta^*+2\right)}{3\left(\Delta_E+\Delta\right)}$$

Considering the price domain we can get the following restriction on θ^*:

$$\frac{\Delta\left(1+2\underline{\theta}\right)}{3\Delta_E+\Delta} \leq \theta^* \leq \frac{\Delta_E\left(1+2\underline{\theta}\right)}{\Delta_E+3\Delta}$$

And equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \Delta \left(\underline{\theta} + \theta^* + 2\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_H^2$$

Then we consider the quality choice. By profit maximization we get:

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{\gamma \left(q_E^2 + q_H^2 + 2q_L^2 - 2q_H q_L - 2q_E q_L\right)}{\left(9q_E + 9q_H - 18q_L\right)^2} - 2\alpha q_L \qquad and \quad \gamma = \left[1 - (\underline{\theta} + \theta^*)\right]^2$$

$$\frac{\partial \Pi_L}{\partial q_L} \leq 0 \quad if \; q_E \geq q_H \geq q_L \, , \, \gamma \geq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -\frac{2\gamma \left(q_E - q_H\right)^2}{81\left(q_E + q_H - 2q_L\right)^3} - 2\alpha \leq 0$$

for Π_H

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\varphi \left(q_E - q_L\right)^2}{9\left(q_E + q_H - 2q_0\right)^2} - 2\alpha q_H \qquad \varphi = \left(2 + \underline{\theta} + \theta^*\right)^2$$

$$\frac{\partial^2 \Pi_H}{\partial q_H^2} = -\frac{2\varphi \left(q_E - q_L\right)^2}{9\left(q_E + q_H - 2q_L\right)^3} - 2\alpha$$

$$q_H^* = -\tfrac{1}{3*2^{2/3}\alpha} \left(\Phi\right)^{1/3} + \tfrac{-324\alpha^2 q_E^2 + 1296\alpha^2 q_E q_0 - 1296\alpha^2 q_0^2}{(\Phi)^{1/3}} - \tfrac{2(\alpha q_0 - 2\alpha q_0)}{3\alpha}$$

$$\begin{aligned}
\Phi \;=\; & -3\varphi^2\alpha\left(q_E - q_0\right) + \sqrt{3}\sqrt{8\alpha^5 q_E^3 \varphi\left(q_E - q_0\right)^2 - 48\alpha^5 q_E^2 q_0 \varphi\left(q_E - q_0\right)^2} + \\
+ \; & \sqrt{3}\sqrt{-64\alpha^5 q_0^3 \varphi\left(q_E - q_0\right)^2 + 96\alpha^5 q_E q_0^2 \varphi\left(q_E - q_0\right)^2 + 3\alpha^4 \varphi\left(q_E - q_0\right)^4} \\
+ \; & \sqrt{3}\sqrt{+96\alpha^5 q_E q_0^2 \varphi\left(q_E - q_0\right)^2 + 3\alpha^4 \varphi\left(q_E - q_0\right)^4} + \left(-4\alpha^3 q_E^3 + 24\alpha^3 q_E^2 q_0 - 48\alpha^3 q_E q_0^3 + 32\alpha^3 q_0^3\right)
\end{aligned}$$

The previous expression is the only real solution of $\frac{\partial \Pi_H}{\partial q_H}$, we have to check if q_H^* is consistent with the restriction on the price domain:

$$q_H \leq q_0 + \frac{\left(q_E - q_0\right)\left(1 + 2\underline{\theta}\right) - \theta^*\left(q_E - q_0\right)}{3\theta^*}$$

$$q_H \leq q_0 + \frac{3\left(q_E - q_0\right)\theta^*}{1 + 2\underline{\theta} - \theta^*}$$

The results are equivalent to case A.b.2 and A.c.2 (see section 5.1.2 and 5.2.2, and Appendix VI)

A.a.3

We consider the following price domain:

$$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \theta^* \Delta_E$$

$$P_L + \theta^* \Delta_E \leq P_H^* \leq P_L + \bar{\theta}\Delta$$

To get profit functions:

$$\Pi_L(P_L, P_H) = P_L\left(\theta'' - \underline{\theta}\right) - \alpha q_L^2 \qquad \Pi_H(P_L, P_H) = P_H\left(\bar{\theta} - \theta''\right) - \alpha q_H^2$$

Then we obtain equilibrium prices:

$$P_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)}{3} \qquad P_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)}{3}$$

And equilibrium profits:

$$\Pi_L^* = \frac{\Delta\left(\bar{\theta} - 2\underline{\theta}\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2\bar{\theta} - \underline{\theta}\right)^2}{9} - \alpha q_H^2$$

Considering the price domain we can get the following restriction on θ^*:

$$\theta^* \leq \frac{\Delta\left(2\underline{\theta} + 1\right)}{3\Delta_E}$$

Then we consider the quality choice. By profit maximization we get:

$$\frac{\partial \Pi_L}{\partial q_L} = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{9} - 2\alpha q_L$$

$$q_L = \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{18\alpha} \quad and \quad \frac{4\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 4\underline{\theta}^2}{18\alpha} \leq 0$$

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{9} - 2\alpha q_H$$

$$q_H^* = \frac{\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta}}{18\alpha}$$

from the restriction on the price domain we get:

$$q_H \geq q_0 + \frac{3\Delta_E \theta^*}{1 + 2\underline{\theta}}$$

The results are equivalent to case A.c.3 already discussed in section 5

A.d.1

We consider the following price domain:

$$P_H - \underline{\theta}\Delta_E \leq P_L^* \leq P_H - \theta^*\Delta$$

$$P_L + \theta^*\Delta \leq P_H^* \leq P_L + \underline{\theta}\Delta_E$$

To get profit functions:

$$\Pi_L\left(P_L, P_H\right) = P_L\left(\theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H\left(P_L, P_H\right) = P_H\left((\bar{\theta} - \underline{\theta}) - \theta'' + \theta^*\right) - \alpha q_H^2$$

Then we obtain equilibrium prices:

$$P_L^* = \frac{\Delta\left(1(\bar{\theta} - \underline{\theta}) - \theta^*\right)}{3} \qquad P_H^* = \frac{\Delta\left(2(\bar{\theta} - \underline{\theta}) + \theta*\right)}{3}$$

And equilibrium profits:

$$\Pi_L^* = \frac{\Delta\left(1 - \theta^*\right)^2}{9} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta\left(2 + \theta^*\right)^2}{9} - \alpha q_H^2$$

Considering the price domain we can get the following restriction on θ^*:

$$\theta^* \leq min\left\{1\,,\,\frac{3\underline{\theta}\Delta_E}{2\Delta} - \frac{\bar{\theta} - \underline{\theta}}{2}\right\}$$

Then we consider the quality choice. By profit maximization we get:

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{{\theta^*}^2 - 2\theta^* + 1}{9} - 2\alpha q_L \leq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_0^2} = -2\alpha$$

$$\frac{\partial \Pi_H}{\partial q_{\mathrm{H}}} = \frac{{\theta^*}^2 + 4\theta^* + 4}{9} - 2\alpha q_H$$

$$q_H = \frac{{\theta^*}^2 + 4\theta^* + 4}{18\alpha}$$

from restriction

$$q_H \leq q_0 + \frac{3\Delta_E\underline{\theta}}{2\theta^* + 1}$$

The results are equivalent to case A.c.1 already discussed in in section 5

A.d.2

We consider the following price domain:

$$P_H - \bar{\theta}\Delta \leq P_L^* \leq P_H - \underline{\theta}\Delta_E$$

$$P_L + \underline{\theta}\Delta_E \leq P_H^* \leq P_L + \bar{\theta}\Delta$$

To get profit functions:

$$\Pi_L(P_L, P_H) = P_L\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H(P_L, P_H) = P_H\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

Then we obtain equilibrium prices:

$$P_L^* = \frac{\Delta_E\Delta\left(1 - (\underline{\theta} + \theta^*)\right)}{3\left(\Delta_E + \Delta\right)} \qquad P_H^* = \frac{\Delta_E\Delta\left(\underline{\theta} + \theta^* + 2\right)}{3\left(\Delta_E + \Delta\right)}$$

Considering the price domain we can get the following restriction on θ^*:

$$\frac{\underline{\theta} - 1}{2} + \frac{3\Delta_E\underline{\theta}}{2\Delta} \leq \theta^* \leq \frac{\underline{\theta} + 2}{2} + \frac{3\Delta\bar{\theta}}{2\Delta_E}$$

And equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E\Delta\left(1 - (\underline{\theta} + \theta^*)\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E\Delta\left(\underline{\theta} + \theta^* + 2\right)^2}{9\left(\Delta_E + \Delta\right)} - \alpha q_H^2$$

Then we consider the quality choice. By profit maximization we get:

for Π_L

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{\gamma\left(q_E^2 + q_H^2 + 2q_L^2 - 2q_H q_L - 2q_E q_0\right)}{\left(9q_E + 9q_H - 18q_L\right)^2} - 2\alpha q_L \qquad and \quad \gamma = \left[1 - (\underline{\theta} + \theta^*)\right]^2$$

$$\frac{\partial \Pi_L}{\partial q_L} \leq 0 \quad if\ q_E \geq q_H \geq q_L\ ,\ \gamma \geq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_0^2} = -\frac{2\gamma\left(q_E - q_H\right)^2}{81\left(q_E + q_H - 2q_0\right)^3} - 2\alpha \leq 0$$

for Π_H

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\varphi\left(q_E - q_0\right)^2}{9\left(q_E + q_H - 2q_0\right)^2} - 2\alpha q_H \qquad \varphi = \left(2 + \underline{\theta} + \theta^*\right)^2$$

$$\frac{\partial^2 \Pi_H}{\partial q_H^2} = -\frac{2\varphi\left(q_E - q_0\right)^2}{9\left(q_E + q_H - 2q_0\right)^3} - 2\alpha$$

$q_H^* = -\frac{1}{3*2^{2/3}\alpha}(\Phi)^{1/3} + \frac{-324\alpha^2 q_E^2+1296\alpha^2 q_E q_0-1296\alpha^2 q_0^2}{(\Phi)^{1/3}} - \frac{2(\alpha q_0-2\alpha q_0)}{3\alpha}$

$$\begin{aligned}\Phi &= -3\varphi^2\alpha(q_E-q_0)+\sqrt{3}\sqrt{8\alpha^5 q_E^3\varphi(q_E-q_0)^2-48\alpha^5 q_E^2 q_0\varphi(q_E-q_0)^2}+\\ &+ \sqrt{3}\sqrt{-64\alpha^5 q_0^3\varphi(q_E-q_0)^2+96\alpha^5 q_E q_0^2\varphi(q_E-q_0)^2+3\alpha^4\varphi(q_E-q_0)^4}\\ &+ \sqrt{3}\sqrt{+96\alpha^5 q_E q_0^2\varphi(q_E-q_0)^2+3\alpha^4\varphi(q_E-q_0)^4}+\left(-4\alpha^3 q_E^3+24\alpha^3 q_E^2 q_0-48\alpha^3 q_E q_0^3+32\alpha^3 q_0^3\right)\end{aligned}$$

this is the only real solution of $\frac{\partial \Pi_H}{\partial q_H}$, we have to check if the q_H^* is consistent with the restriction on price domain:

$$q_0+\frac{2\Delta_E\theta^*-\Delta_E(\underline{\theta}+2)}{3\bar{\theta}} \le q_H \le q_0-\frac{3(q_E-q_0)\theta}{\underline{\theta}-1-2\theta^*}$$

$\underline{\theta}-1-2\theta^*<0$ because $\underline{\theta}\le\theta^*$ by definition.

The results are equivalent to case A.b.2 and A.c.2 (see sections 5.1.2 and 5.2.2, and Appendix VI)

A.d.3

We consider the following price domain:

$$P_H-\theta^*\Delta_E \le P_L^* \le P_H-\bar{\theta}\Delta$$

$$P_L+\bar{\theta}\Delta \le P_H^* \le P_L+\theta^*\Delta_E$$

To get profit functions:

$$\Pi_L(P_L,P_H)=P_L\left((\bar{\theta}-\underline{\theta})-\theta^*+\theta'\right)-\alpha q_L^2 \qquad \Pi_H(P_L,P_H)=P_H\left((\theta^*-\theta')\right)-\alpha q_H^2$$

Then we obtain equilibrium prices:

$$P_L^*=\frac{\Delta_E(2-\theta^*)}{3} \qquad P_H^*=\frac{\Delta_E(1+\theta^*)}{3}$$

And equilibrium profits:

$$\Pi_L^*=\frac{\Delta_E(2-\theta^*)^2}{9}-\alpha q_L^2 \qquad \Pi_H^*=\frac{\Delta_E(1+\theta^*)^2}{9}-\alpha q_H^2$$

Considering the price domain we can get the following restriction on θ^*:

$$\theta^*\ge\frac{1}{2}+\frac{3\Delta\bar{\theta}}{2\Delta_E}$$

Then we consider the quality choice. By profit maximization we get:

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{{\theta^*}^2 - 4\theta^* + 4}{9} - 2\alpha q_L \leq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -2\alpha \leq 0$$

$$\frac{\partial \Pi_H}{\partial q_H} = -2\alpha q_L$$

$$\frac{\partial \Pi_E}{\partial q_E} = \frac{{\theta^*}^2 + 2\theta^* + 1}{9} \geq 0$$

from restriction

$$q_H \leq q_0 + \frac{\Delta_E (2\theta^* - 1)}{3\bar{\theta}}$$

The results are equivalent to case A.b.3 already discussed in section 5

Appendix V

Proof of equilibrium existence in case A.a.2, A.b.2, A.c.2 and A.d.2. Starting from the follow profit function:

$$\Pi_L (P_L, P_H) = P_L \left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H (P_L, P_H) = P_H \left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

leading to the following equilibrium prices:

$$P_L^* = \frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)}{3 (\Delta_E + \Delta)} \qquad P_H^* = \frac{\Delta_E \Delta \left(\underline{\theta} + \theta^* + 2\right)}{3 (\Delta_E + \Delta)}$$

By substitution we can find equilibrium profit functions as follows:

$$\Pi_L^* = \frac{\Delta_E \Delta \left(1 - (\underline{\theta} + \theta^*)\right)^2}{9 (\Delta_E + \Delta)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \Delta \left(\underline{\theta} + \theta^* + 2\right)^2}{9 (\Delta_E + \Delta)} - \alpha q_H^2$$

Turning then to the quality selection stage, by profit maximization in qualities we get:

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{\gamma \left(q_E^2 + q_H^2 + 2q_L^2 - 2q_H q_0 - 2q_E q_L\right)}{\left(9q_E + 9q_H - 18q_L\right)^2} - 2\alpha q_L \qquad and \quad \gamma = \left[1 - (\underline{\theta} + \theta^*)\right]^2 \tag{24}$$

$$\frac{\partial \Pi_L}{\partial q_L} \leq 0 \quad if : q_E \geq q_H \geq q_0 \, , \, \gamma \geq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -\frac{2\gamma \left(q_E - q_H\right)^2}{81 \left(q_E + q_H - 2q_0\right)^3} - 2\alpha \leq 0 \tag{25}$$

solving equation (24) for q_L we find 3 solutions, a real one and two complex solutions, so we discard non real solutions. Now we have to understand if real solution is positive or negative. First of all our derivative (eq 24) is negative for $q_L = 0$ and also $\lim_{q_L \to \infty} \frac{\partial \Pi_L}{\partial q_L} = \infty$. Using the second derivative equation (25) we can say that the first derivative is always decreasing then under that condition the only real solution must be a negative solution. Therefore the low quality firm is lead to produce the MQS, i.e $q_L^* = q^{\circ}$ as a corner solution.

Concerning the high quality firm we get:

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\varphi \left(q_E - q_0\right)^2}{9\left(q_E + q_H - 2q_0\right)^2} - 2\alpha q_H \geq 0 \qquad \varphi = \left(2 + \underline{\theta} + \theta^*\right)^2 \tag{26}$$

$$\frac{\partial^2 \Pi_H}{\partial q_H^2} = -\frac{2\varphi \left(q_E - q_0\right)^2}{9\left(q_E + q_H - 2q_0\right)^3} - 2\alpha \tag{27}$$

from equation (26) we obtain three solutions, one real solution and two complex solutions, discarding the complex one we obtain the following real solution:

$q_H^* = -\frac{1}{3*2^{2/3}\alpha}\left(\Phi\right)^{1/3} + \frac{-324\alpha^2 q_E^2 + 1296\alpha^2 q_E q_0 - 1296\alpha^2 q_0^2}{\left(\Phi\right)^{1/3}} - \frac{2(\alpha q_0 - 2\alpha q_0)}{3\alpha}$

$$\begin{aligned}
\Phi &= -3\varphi^2\alpha\left(q_E - q_0\right) + \sqrt{3}\sqrt{8\alpha^5 q_E^3\varphi\left(q_E - q_0\right)^2 - 48\alpha^5 q_E^2 q_0\varphi\left(q_E - q_0\right)^2} + \\
&+ \sqrt{3}\sqrt{-64\alpha^5 q_0^3\varphi\left(q_E - q_0\right)^2 + 96\alpha^5 q_E q_0^2\varphi\left(q_E - q_0\right)^2 + 3\alpha^4\varphi\left(q_E - q_0\right)^4} \\
&+ \sqrt{3}\sqrt{+96\alpha^5 q_E q_0^2\varphi\left(q_E - q_0\right)^2 + 3\alpha^4\varphi\left(q_E - q_0\right)^4} + \left(-4\alpha^3 q_E^3 + 24\alpha^3 q_E^2 q_0 - 48\alpha^3 q_E q_0^3 + 32\alpha^3 q_0^3\right)
\end{aligned}$$

now we have to prove that the above solution is positive, the proof follows:
$\frac{\partial \Pi_H}{\partial q_H}|_{q_H=0} > 0$ and $\lim_{q_H \to \infty} \frac{\partial \Pi_H}{\partial q_H} = -\infty$. From these results and $\frac{\partial^2 \Pi_H}{\partial q_H^2} < 0$ we can say that the real solution is unique and positive. As usual then we have to check restrictions on q_H for each price domain (A.a.2, A.b.2, A.c.2, A.c.2)

Appendix VI

Case A.b.2, this sub-case is defined by the following price domains:

$$P_H - \theta^*\Delta_E \leq P_L^* \leq P_H - \underline{\theta}\Delta_E$$

$$P_L + \underline{\theta}\Delta_E \leq P_H^* \leq P_L + \theta^*\Delta_E$$

Considering the related demand segments we get the following profit functions:

$$\Pi_L(P_L, P_H) = P_L\left(\theta' - \underline{\theta} + \theta'' - \theta^*\right) - \alpha q_L^2 \qquad \Pi_H(P_L, P_H) = P_H\left(\bar{\theta} - \theta'' + \theta^* - \theta'\right) - \alpha q_H^2$$

To get equilibrium prices:

$$P_L^* = \frac{\Delta_E \Delta\left(1 - (\underline{\theta} + \theta^*)\right)}{3(\Delta_E + \Delta)} \qquad P_H^* = \frac{\Delta_E \Delta\left(\underline{\theta} + \theta^* + 2\right)}{3(\Delta_E + \Delta)}$$

profit functions:

$$\Pi_L^* = \frac{\Delta_E \Delta\left(1 - (\underline{\theta} + \theta^*)\right)^2}{9(\Delta_E + \Delta)} - \alpha q_L^2 \qquad \Pi_H^* = \frac{\Delta_E \Delta\left(\underline{\theta} + \theta^* + 2\right)^2}{9(\Delta_E + \Delta)} - \alpha q_H^2$$

By checking that equilibrium prices belong to the price domains we get a further restriction on θ^*

$$\theta^* \geq max\left\{\frac{\Delta(1 + 2\underline{\theta})}{3\Delta_E + \Delta}, \frac{\underline{\theta}(3\Delta_E + \Delta)}{2\Delta} - \frac{\Delta}{2\Delta}\right\}$$

Then we consider the quality stage. By profit maximization we get for Π_L

$$\frac{\partial \Pi_L}{\partial q_L} = -\frac{\gamma\left(q_E^2 + q_H^2 + 2q_L^2 - 2q_H q_L - 2q_E q_L\right)}{(9q_E + 9q_H - 18q_L)^2} - 2\alpha q_L \qquad and \quad \gamma = \left[1 - (\underline{\theta} + \theta^*)\right]^2$$

$$\frac{\partial \Pi_L}{\partial q_L} \leq 0 \quad if: q_E \geq q_H \geq q_L\,, \gamma \geq 0$$

$$\frac{\partial^2 \Pi_L}{\partial q_L^2} = -\frac{2\gamma(q_E - q_H)^2}{81(q_E + q_H - 2q_L)^3} - 2\alpha \leq 0$$

and for Π_H

$$\frac{\partial \Pi_H}{\partial q_H} = \frac{\varphi(q_E - q_0)^2}{9(q_E + q_H - 2q_0)^2} - \alpha q_H^2 \qquad \varphi = (2 + \underline{\theta} + \theta^*)^2$$

$$\frac{\partial^2 \Pi_H}{\partial q_H^2} = -\frac{2\varphi(q_E - q_0)^2}{9(q_E + q_H - 2q_0)^3} - 2\alpha$$

Still considering the price domains together with equilibrium prices, we can find lower and upper bounds for q_H:

$$q_0 + \frac{3(q_E - q_0)\underline{\theta}}{1 + 2\theta^* - \underline{\theta}} \leq q_H \leq q_0 \frac{3(q_E - q_0)\theta^*}{1 + 2\underline{\theta} - \theta^*}$$

Figure

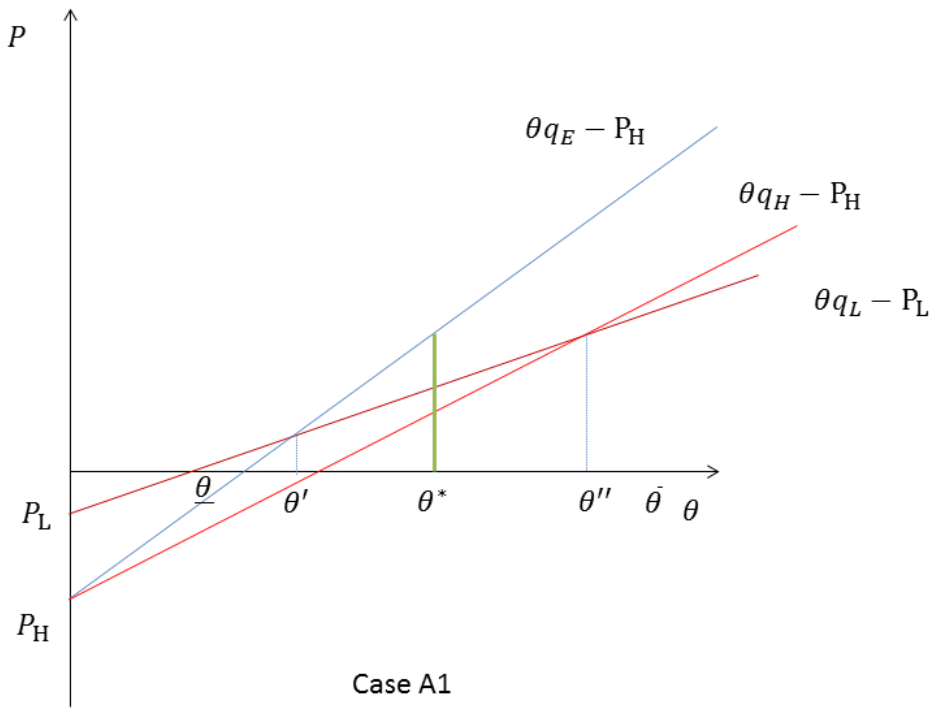

Figure 1: Position of utility functions for case A1

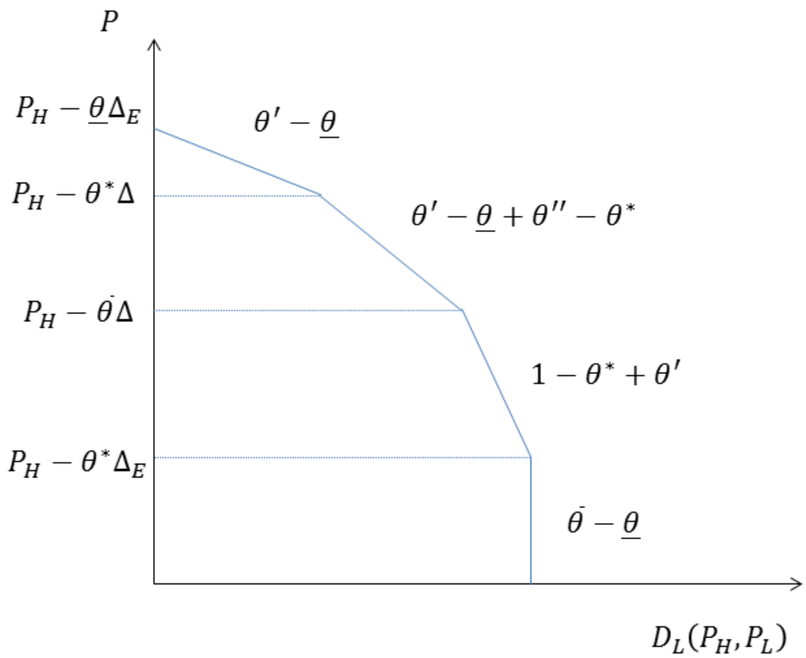

Figure 2: Demand function for low quality in case A.b

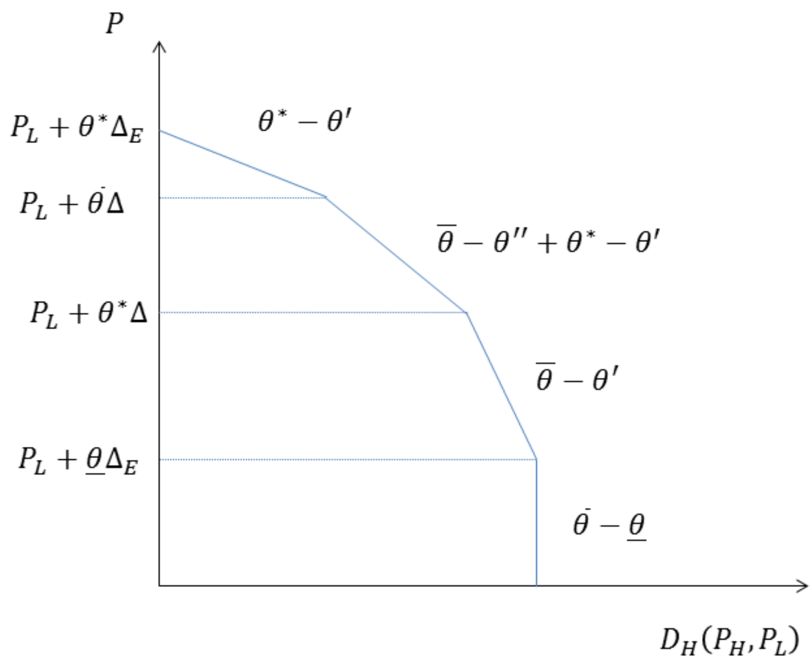

Figure 3: Demand function for high quality in case A.b