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Abstract

We study monopolistic competition equilibria with free entry under symmetric Generalized Additively Separable preferences, whose demand systems feature a single aggregator of prices or quantities. They include Gorman-Pollak preferences (which nest directly and indirectly additive preferences, a homothetic family and other preferences) and implicit CES preferences. With heterogeneous firms our extension of the Melitz model produces a variety of pricing and selection effects, and allows us to solve the social planner problem. We illustrate the inefficiency of the market equilibrium for a new specification of generalized translated power preferences, and show its optimality for the entire class of implicit CES preferences.

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In this work we analyze monopolistic competition under a type of preferences that generalizes the additivity assumed by Dixit and Stiglitz (1977). We show that these preferences deliver richer pricing and selection effects than in the existing variations of the Melitz (2003) model of monopolistic competition with heterogeneous firms, and we extend the optimality result proved by Dhingra and Morrow (2019) for CES preferences to an entire class of preferences.

The classic Dixit-Stiglitz model of monopolistic competition was based on symmetric CES (Constant Elasticity of Substitution) preferences, whose demands depend on a common price index and are isoelastic with respect to own prices (Dixit and Stiglitz, 1977: Section I). The peculiar properties of this setting are well known. In particular, with homogeneous firms markups are common and constant, changes in market size create pure gains from variety (Krugman, 1980) and the market equilibrium is optimal (Dixit and Stiglitz, 1977). With heterogeneous firms, markups remain common and constant, changes in market size do not exert selection effects on the set of active firms (Melitz, 2003) and the equilibrium is still optimal (Dhingra and Morrow, 2019).

The original contribution of Dixit and Stigliz (1977: Section II) also explored the more general class of directly additive preferences whose direct utility can be written as:

$$U = \int_{\Omega} u(x(\omega))d\omega, \tag{1}$$

where Ω is the set of consumed goods and the consumption $x(\omega)$ of variety $\omega \in \Omega$ has subutility u. With these preferences the elasticity of substitution between a good and the others, which determines demand elasticity, depends only on its consumption level. Under monopolistic competition among homogeneous firms with free entry the markup is independent from income, and it changes with market size and marginal cost depending on the shape of the marginal subutility of consumption, while the equilibrium is in general inefficient. The analysis has been extended to heterogeneous firms in Zhelobodko *et al.* (2012), Bertoletti and Epifani (2014) and Dhingra and Morrow (2017, 2019) emphasizing the selection effects which are generated by the market size and depend on the shape of the elasticity of substitution, and the general inefficiency of the market equilibrium.[2]

Recently, we have studied monopolistic competition for demand functions derived from indirectly additive preferences (Bertoletti and Etro, 2017a), namely when indirect utility can be written as:

$$V = \int_{\Omega} v(s(\omega))d\omega, \tag{2}$$

where $s(\omega) \equiv p(\omega)/E$ is the price of variety ω normalized by income E, with subutility v. Demand elasticity depends only on its own normalized price: under monopolistic competition with homogeneous firms, the markup is independent from market size and changes with income and marginal cost depending on the shape of the marginal subutility of prices, while changes in market size

[2] Further applications are in Kuhn and Vives (1999) and more recently Simonovska (2015), Mrázová and Neary (2018) and Arkolakis *et al.* (2019).

generate pure gains from variety, and the equilibrium is in general inefficient. The analysis has also been extended to heterogeneous firms emphasizing the absence of selection effects induced by market size and the general inefficiency of the equilibrium.[3]

Additive preferences belong to a more general type of preferences whose demand system features a common aggregator of prices or quantities. These were introduced by Gorman (1970, 1987) and Pollak (1972) under the name of Generalized Additively Separable (GAS) preferences, and their use for monopolistic competition is the subject of this work. Following Bertoletti and Etro (2017b), we call *Gorman-Pollak preferences* (henceforth GP preferences) a generalization of (1) and (2) that delivers a demand elasticity depending on the product of the own price (or consumption level) with a common aggregator. Following Hanoch (1975), we define the second class of GAS preferences as *implicit CES preferences*: they feature an elasticity of substitution that is common across commodities, but can change through indifference curves, and therefore with the utility level.[4] We study monopolistic competition under symmetric versions of these preferences.

We start by analyzing the comparative statics of the monopolistic competition equilibrium with homogeneous firms. Under GP preferences, the comparative statics for prices and number of firms rely on two elasticities, one depending on the effective price and driving demand elasticity, and another depending on the common aggregator. Broadly speaking, the advantage of GP preferences for applied work is that they are more flexible on the impact of market size, income and costs, allowing a better match to empirical features of markups and entry. For the case of implicit CES preferences, the equilibrium with homogeneous firms implies that an increase in income or market size reduces the markup and increases the number of firms less than proportionally if and only if the elasticity of substitution is increasing in utility. With respect to optimal choices, we find that while GP preferences can deliver either excess or suboptimal entry, under implicit CES preferences the decentralized market equilibrium is optimal, which generalizes the classic result by Dixit and Stiglitz (1977).

We then explore the case of heterogeneous firms. For the GP preferences we provide an extension of the Melitz (2003) model with and without fixed costs of production, showing that pricing and selection effects can be much richer than in standard models. Most important, we characterize the optimal markup, the optimal measure of firms created and the optimal set of consumed goods that would be chosen by a social planner, extending the results obtained by

[3] Further applications are in Boucekkine *et al.* (2017) and Bertoletti *et al.* (2018).

[4] To avoid ambiguities, in this work we adopt a taxonomy for the *kingdom* of well-behaved preferences. GAS preferences are a *type* of preferences characterized by a demand system with a common aggregator. A subset of this type can be a super-class, as for the Gorman-Pollak preferences. This includes different classes of preferences: a *class* is identified by a functional property, as direct additivity or indirect additivity. A subset of a class can be a *family* of preferences, as the homothetic family of GAS preferences, which belongs to the homothetic class. Asymmetric CES preferences can represent a *genre* which includes more specifications. The symmetric CES preferences are a *specification* of preferences since they are represented by a specific utility function up to the choice of a parameter from a range of possible values.

Dhingra and Morrow (2019) for directly additive preferences and by Bertoletti *et al.* (2018) for indirectly additive preferences. Assuming null fixed costs of production and a Pareto distribution of unit costs across firms, as in Arkolakis *et al.* (2019) and Bertoletti *et al.* (2018), we make substantial progress in deriving both decentralized and optimal allocations. Closed form solutions can be actually obtained for a novel specification of "generalized translated power preferences" which nests directly additive, indirectly additive and homothetic cases, as well as demand functions that can be linear, perfectly rigid or perfectly elastic, and it is suitable for quantitative explorations. In this case we find that the equilibrium generates the optimal number of firms, but typically too many goods are consumed and in excessive quantity for those with high cost and in suboptimal quantity for those with low cost. Instead, for the case of implicit CES preferences, our extension of the Melitz (2003) model with arbitrary cost distribution and positive fixed cost shows that opening up to costless trade (i.e., increasing the market size) generates selection effects and reduces markups as long as the demand elasticity is increasing in utility. Moreover, we confirm the optimality of the decentralized equilibrium for this entire class, generalizing a result established by Dhingra and Morrow (2019) only for explicit CES preferences. Overall, our results suggest the possibility of studying gains from trade or macroeconomic dynamics in a framework that is much less restrictive on the demand side than the ones usually adopted.

This work contributes to a recent literature that has examined monopolistic competition models beyond the classic Dixit-Stiglitz one. Some paper have made progress with general symmetric preferences, under a discrete number of homogeneous goods (Bertoletti and Etro, 2016) or a continuum of heterogeneous firms (Parenti *et al.*, 2017). However, it is by exploiting the unique properties of the GAS preferences that we are able to derive quite general results concerning both the market equilibrium of free entry and the nature of optimal allocations with both homogeneous and heterogeneous firms. Arkolakis *et al.* (2019) have explored demand systems nesting those generated by directly additive preferences and by some homothetic ones, though their focus is on quantifying the gains from trade liberalization in a multicountry model. GAS preferences overlap with theirs only for the directly additive class, and with those of Bertoletti *et al.* (2018) for the indirectly additive class, and could be usufully employed for similar quantitative applications. We should remark that this literature and the present work are limited to symmetric preferences: in a companion paper we have investigated the general case of asymmetric preferences (Bertoletti and Etro, 2017b).

The remaining of this work is organized as follows. The next section studies GP preferences focusing first on homogeneous firms and then on heterogeneous ones. The following one studies implicit CES preferences again under both homogeneous and heterogeneous firms. In each case we also discuss optimality. We then conclude.

1 Gorman-Pollak preferences

We consider monopolistic competition in a market with a population of L identical consumers with income/expenditure E. Suppose that their preferences can be represented by the following direct and indirect utility functions (Gorman, 1970, 1987, and Pollak, 1972):

$$U = \int_\Omega u(\xi x(\omega))d\omega - \phi(\xi) \quad \text{and } V = \int_\Omega v(\rho s(\omega))d\omega - \theta(\rho), \tag{3}$$

where $x(\omega)$ and $s(\omega)$ are consumption and (normalized) price of variety $\omega \in \Omega$, with respectively increasing and concave subutility u and decreasing and convex subutility v, and the aggregators ξ and ρ satisfy:

$$\phi'(\xi) = \int_\Omega u'(\xi x(\omega))x(\omega)d\omega \quad \text{and} \quad \theta'(\rho) = \int_\Omega v'(\rho s(\omega))s(\omega)d\omega, \tag{4}$$

for an increasing function $\phi(\xi)$ and a decreasing function $\theta(\rho)$. Before illustrating these preferences, it seems useful to anticipate that they are *directly additive* as in (1) when $\theta(\rho) = -\rho$, they are *indirectly additive* as in (2) when $\phi(\xi) = \xi$ and they are *homothetic* when $\theta(\rho) = -\log\rho$ and $\phi(\xi) = \log\xi$.

The role of ξ and ρ is to cancel out any direct cross effect on utility, as in the case of additive preferences, which is key to obtain demand systems depending only on one aggregator. Intuitively, ξ can be seen as generating the benefit of increasing the *effective* quantity of good ω to $\xi x(\omega)$ at the utility cost $\phi(\xi)$, which is equivalent to the possibility of reducing the inconvenience of consumption $\theta(\rho)$ at the cost of increasing the *effective* price of good ω to $\rho s(\omega)$. The demand system can then be easily computed from the Hotelling-Wold and Roy identities as:

$$s(\iota) = \frac{u'(\xi x(\iota))}{\int_\Omega u'(\xi x(\omega))x(\omega)d\omega} \quad \text{and} \quad x(\iota) = \frac{v'(\rho s(\iota))}{\int_\Omega v'(\rho s(\omega))s(\omega)d\omega}, \tag{5}$$

for $\iota \in \Omega$. By using (4), (3) confirms that preferences are of the GAS type in the sense that the demand systems depend only on a common aggregator. It also holds that $\rho = \phi'$, $\xi = -\theta'$, that $\rho\xi$ is equal to the marginal utility of income times the expenditure level E, and that the following relations link the direct and dual expressions of utility:

$$u'(z) = v'^{-1}(-z) \quad \text{and} \quad v'(z) = -u'^{-1}(z),$$

$$\theta'\left(\phi'(z)\right) = -z \quad \text{and} \quad \phi'\left(-\theta'(z)\right) = z.$$

Obviously, the functional forms (3)-(4) have to satisfy further regularity conditions for preferences to be well-behaved (sufficient ones have been explored in Fally, 2018).

Under monopolistic competition, each firm chooses its price or quantity taking as given the aggregators. To characterize pricing, let us define the following elasticities of the marginal subutilities:

$$\epsilon(z) \equiv -\frac{u''(z)z}{u'(z)} \quad \text{and} \quad \varepsilon(z) \equiv -\frac{v''(z)z}{v'(z)}.$$

Variable profit for a firm with marginal cost c can be expressed in general as:

$$\pi = \left[\frac{u'(\xi x)E}{\phi'(\xi)} - c\right] xL = \frac{(sE - c)\,v'(\rho s)L}{\theta'(\rho)}. \tag{6}$$

This clarifies that the relevant demand elasticities are given by $\epsilon(\xi x)$ and $\varepsilon(\rho s)$, which are assumed respectively smaller and larger than unity, implying the following conditions for monopolistic competition pricing:

$$p = \frac{c}{1 - \epsilon(\xi x)} = \frac{\varepsilon(\rho s)\,c}{\varepsilon(\rho s) - 1}. \tag{7}$$

To verify that the GP preferences are directly additive when $\theta(\rho) = -\rho$ notice that in such a case $\xi = -\theta'(\rho) = 1$, which delivers the same demand system as (1). To verify that they are indirectly additive when $\phi(\xi) = \xi$ notice that it must be the case that $\rho = \phi'(\xi) = 1$, which delivers the same demand system as (2). Finally, to verify that the GP preferences nest a homothetic family of preferences when $\phi(\xi) = \log \xi$ and $\theta(\rho) = -\log \rho$ notice that (4) implies that the aggregators must then be homogeneous of degree -1, so that from (5) demand ratios are homogeneous of degree 0:[5] on the demand system of this homothetic family, which includes also a restricted version of translog preferences, see Matsuyama and Ushchev (2017).

An obvious specification nested in the GP preferences is the CES case, whose demand system is given by:

$$s(\iota) = \frac{x(\iota)^{-\epsilon}}{\int_\Omega x(\omega)^{1-\epsilon} d\omega} \quad \text{and} \quad x(\iota) = \frac{s(\iota)^{-\varepsilon}}{\int_\Omega s(\omega)^{1-\varepsilon} d\omega},$$

where $\varepsilon = 1/\epsilon > 1$ is constant. This specification emerges immediately whenever $u(z) = \frac{z^{1-\epsilon}}{1-\epsilon}$ and $v(z) = \frac{z^{1-\varepsilon}}{\varepsilon - 1}$ and thus, from (4), $\xi = \left[\int x(\omega)^{1-\epsilon} d\omega\right]^{\frac{1}{\epsilon - 1}}$ and $\rho = \left[\int s(\omega)^{1-\varepsilon} d\omega\right]^{\frac{1}{\varepsilon - 1}}$.

As a useful unexplored specification, consider the following extension of the "translated power" preferences, which we will refer to as "generalized translated power preferences":

$$U = \int_\Omega \left(a\xi x(\omega) - \frac{(\xi x(\omega))^{\frac{1+\gamma}{\gamma}}}{1 + \frac{1}{\gamma}} \right) d\omega - \frac{\xi^{1-\frac{1}{\beta}}}{1 - \frac{1}{\beta}} \text{ and } V = \int_\Omega \frac{(a - \rho s(\omega))^{1+\gamma}}{1+\gamma} d\omega + \frac{\rho^{1-\beta}}{1-\beta}, \tag{8}$$

with $a > 0$, $\beta, \gamma \geq 0$. Demand systems are given by:

$$s(\iota) = \frac{a - [\xi x(\iota)]^{\frac{1}{\gamma}}}{\int_\Omega \left[a - (\xi x(\omega))^{\frac{1}{\gamma}}\right] d\omega} \quad \text{and} \quad x(\iota) = \frac{[a - \rho s(\iota)]^{\gamma}}{\int_\Omega (a - \rho s(\omega))^{\gamma} d\omega},$$

[5] Preferences are also homothetic (and indeed CES) whenever u and v are homogeneous.

nesting the cases of direct additivity for $\beta = 0$, homotheticity for $\beta = 1$, and indirect additivity for $\beta \to \infty$, as well as demand functions that are perfectly rigid for $\gamma \to 0$, linear for $\gamma = 1$ and perfectly elastic for $\gamma \to \infty$. While the indirectly additive case has been considered by Bertoletti *et al.* (2018), we are not aware of applications of the more general specification.

1.1 Monopolistic competition with homogeneous firms

We now consider the monopolistic competition equilibrium with n homogenous firms to study its comparative statics and compare it with the optimal market structure. While results can be derived starting from the primal or the dual version of the preferences, it will be convenient to focus on the indirect utility in (3).

If each firm has a constant marginal cost c, the symmetric equilibrium price p and the aggregator ρ satisfy the following conditions:

$$p = \frac{\varepsilon(\rho s) c}{\varepsilon(\rho s) - 1} \quad \text{where } \theta'(\rho) = n v'(\rho s) s, \tag{9}$$

which depend on the number of firms n, on the marginal cost c and on income E. The second-order condition for profit maximization requires that $2\varepsilon(\rho s) > \zeta(\rho s)$, where $\zeta(z) = -z v'''(z)/v''(z)$. For given n, the definition of the aggregator ρ in the second expression of (9) implies that it is negatively related to the (common) normalized price s if:

$$\varepsilon(\rho s) > \beta(\rho) \equiv -\frac{\theta''(\rho)\rho}{\theta'(\rho)}$$

(and $\varepsilon > 1$), where $\beta(\rho)$ is a standard measure of curvature of the function $\theta(\rho)$ - which is clearly a constant in the specification (8).[6] The comparative statics of the equilibrium price, however, depends on the shape of ε as well as on the behavior of the aggregator ρ.

With a fixed cost of production $F > 0$, the free-entry equilibrium implies the following expressions for the price, the number of firms and the individual consumption:

$$p = \frac{\varepsilon(\rho s) c}{\varepsilon(\rho s) - 1}, \; n = \frac{EL}{\varepsilon(\rho s) F} \quad \text{and} \quad x = \frac{[\varepsilon(\rho s) - 1] F}{cL}, \tag{10}$$

which are not independent, since obviously $pnx = E$, and where the aggregator satisfies the expression in (9).

The analysis simplifies if we assume homotheticity, in which case by (9) the equilibrium value of ρs and the number of firms are positively related, namely $d\{\rho s\}/dn > 0$ (always under the assumption $\varepsilon > 1$). Accordingly, the marginal cost is neutral on markups, while an increase of EL increases the number of firms less than proportionally and decreases markups if and only if $\varepsilon' > 0$. As

[6] In general, we have $\frac{\partial \ln \rho}{\partial \ln s} = \frac{1-\varepsilon(\rho s)}{\varepsilon(\rho s)-\beta(\rho)}$ and thus $\frac{\partial \ln\{\rho s\}}{\partial \ln s} = \frac{1-\beta(\rho)}{\varepsilon(\rho s)-\beta(\rho)}$.

an example consider the parametrization $\gamma = \beta = 1$ of our previous specification, delivering the following indirect utility:

$$V = \int_{\Omega} \frac{(a - \rho s(\omega))^2}{2} d\omega + \ln \rho. \tag{11}$$

In this case we obtain the elasticity $\varepsilon = \rho s/(a - \rho s)$ implying the price:

$$p = \frac{c + aE/\rho}{2}.$$

For a given aggregator, this price exhibits incomplete pass-through of the marginal cost and pricing to market (a markup increasing in income). However, the aggregator, which is linear with respect to income, must satisfy $1 = n\,(a - \rho s)\,\rho s$, therefore we can solve for:

$$\rho s = \frac{a}{2} + \sqrt{\frac{a^2}{4} - 1/n},$$

that requires a number of firms large enough ($\sqrt{n} > 2/a$). Under this assumption we obtain the equilibrium price rule:

$$p = \frac{c}{2}\left(1 + \frac{a}{2\sqrt{\frac{a^2}{4} - \frac{1}{n}}}\right), \tag{12}$$

which is decreasing in the number of firms, linear in the marginal cost and independent from income, as one should expect under homotheticity.

For the general case, the GP preferences offer a variety of possible comparative statics results. To fix ideas, we present here the following result:

Proposition 1. *Under Gorman-Pollak preferences the free-entry equilibrium of monopolistic competition with homogeneous firms implies that, if* $\beta(\rho) < \varepsilon(z)$*, an increase of maket size increases the number of firms less than proportionally and decreases markups if* $\varepsilon'(z) > 0$*, in which case: a) pass-through of marginal cost changes is incomplete if* $\beta(\rho) < 1$*, complete if* $\beta(\rho) = 1$ *(homothetic preferences) and more than complete is* $\beta(\rho) > 1$*; and b) a rise of income decreases markups and raises the number of firm less than proportionally if* $\beta(\rho) > 0$.[7]

Proof. We can combine the equilibrium relations to obtain the following equation for $z = \rho s$:

$$f \equiv \theta'\left(\frac{z\,[\varepsilon(z) - 1]\,E}{\varepsilon(z)\,c}\right) - \frac{v'(z)\,cL}{[\varepsilon(z) - 1]\,F} = 0.$$

[7] In the directly additive case $\beta(\rho) = 0$ and, as is well known, income does not affect pricing; on the contrary, when $\beta(\rho) > 0$ a rise of income raises the equilibrium effective price ρs (whenever $\beta(\rho) < \varepsilon(z)$).

This gives:

$$\begin{aligned}\frac{\partial f}{\partial z} &= \frac{\theta'' E}{c}\left(\frac{\varepsilon\left[\varepsilon-1\right]+z\varepsilon'\varepsilon-z\varepsilon'\left[\varepsilon-1\right]}{\varepsilon^{2}}\right)-\frac{cL}{F}\frac{v''\left[\varepsilon-1\right]-\varepsilon' v'}{\left[\varepsilon-1\right]^{2}}\\ &= \frac{\left(2\varepsilon-\zeta\right)\theta'\left(\rho\right)\left(\varepsilon-1\right)\left(\varepsilon-\beta\right)}{z},\end{aligned}$$

which is negative if $\varepsilon > \beta$. Since:

$$\frac{\partial f}{\partial L}=-\frac{v' c}{\left(\varepsilon-1\right)F}>0,\quad \frac{\partial f}{\partial E}=\theta''\frac{\rho}{E}\quad \text{and}\quad \frac{\partial f}{\partial c}=\frac{\theta'}{c}\left(\beta-1\right),$$

we get the comparative statics:

$$sign\left\{\frac{\partial \ln p}{\partial L}\right\}=-sign\left\{\varepsilon'(z)[\varepsilon(z)-\beta(\rho)]\right\}=sign\left\{\frac{\partial \ln n}{\partial \ln L}-1\right\},$$

$$sign\left\{\frac{\partial \ln p}{\partial \ln E}\right\}=-sign\left\{\varepsilon'(z)[\varepsilon(z)-\beta(\rho)]\right\}sign(\{\beta\left(\rho\right)\}=sign\left\{\frac{\partial \ln n}{\partial \ln E}-1\right\},$$

and

$$sign\left\{\frac{\partial \ln p}{\partial \ln c}-1\right\}=sign\left\{\varepsilon'(z)[\varepsilon(z)-\beta(\rho)]\right\}sign\left\{\beta\left(\rho\right)-1\right\}=sign\left\{\frac{\partial \ln n}{\partial \ln c}\right\},$$

which immediately implies the results of Proposition 1 under the assumpion that $\varepsilon(z) > \beta(\rho)$. □

With this characterization of the market equilibrium of monopolistic competition, we can now derive the optimal allocation of resources to verify whether the equilibrium generates excess or insufficient entry of firms. For this purpose, it is convenient to define the elasticity of the indirect subutility:

$$\eta\left(z\right)\equiv-\frac{v'(z)z}{v(z)}>0, \tag{13}$$

which measures of incremental social benefit created by an additional good. The optimal allocation problem is given by:

$$\max_{n,s}\left\{W=nv\left(\rho s\right)-\theta\left(\rho\right)\right\}$$

$$\begin{aligned} s.t.\quad \left(EL-nF\right)s &\geq cL,\\ nv'\left(\rho s\right)s &= \theta'\left(\rho\right),\end{aligned}$$

where the first constraint is the resource constraint and the second one defines the aggregator under symmetry. The solution can be easily obtained from the two first-order conditions, and it is summarized as follows:

PROPOSITION 2. *Under Gorman-Pollak preferences with homogeneous firms, the optimal allocation satisfies:*

$$p^* = \left[1 + \frac{1}{\eta(\rho^* s^*)}\right] c, \tag{14}$$

$$n^* = \frac{EL}{[1 + \eta(\rho^* s^*)] F}, \tag{15}$$

where $\theta'(\rho^*) = n^* v'(\rho^* s^*) s^*$.

In general the equilibrium is not optimal, except for the CES case for which $\eta = \varepsilon - 1$. By immediate comparison, we have $p > p^*$ with excess entry in equilibrium if $\eta(\rho^* s^*) > \varepsilon(\rho s) - 1$. In the case of GP preferences the $sign\{\eta'(z)\}$ $= sign\{\eta(z) - 1 - \varepsilon(z)\}$ is not sufficient to compare the market equilibrium to the social optimum except for the cases of indirect additivity. Nevertheless, the result above allows one to determine the form of inefficiency case by case. For instance, in our linear homothetic example, we have $\eta(\rho s) = 2\varepsilon(\rho s)$, which provides $p = \frac{a+\rho s}{2\rho s} c$ and therefore the optimal price:

$$p^* = \frac{c}{2}\left(1 + \frac{a}{\frac{a}{2} + \sqrt{\frac{a^2}{4} - \frac{1}{n}}}\right). \tag{16}$$

This is always below the equilibrium price, implying that the model delivers excess entry in equilibrium.

1.2 Monopolistic competition with heterogeneous firms

Consider now the extension of the celebrated Melitz (2003) model to GP preferences. We assume that there is a common (and sunk) entry cost $F_e > 0$ and a fixed cost of production $F \geq 0$. After entry, firms draw a marginal cost from a distribution $G(c)$ on the support $[0, \bar{c}]$,[8] and then decide whether to produce and pay the fixed cost of production or not. As before, it will be convenient to focus on the dual representation of preferences in (3).

In equilibrium, only the most efficient firms with marginal costs which belong to the interval $[0, \hat{c}]$ are *ex-post* active, where the cut-off $\hat{c}$ identifies the marginal firm that obtains zero profits if active. When there are no fixed costs of production ($F = 0$), this is the firm whose unit cost is given by aE/ρ, where a is the normalized, effective choke-off price such that $v(z) = 0$ for $z \geq a$ (if $a \to \infty$, as with CES preferences, all firms would be active). When there are positive fixed costs of production ($F > 0$), the variable profits of the marginal firm are equal to them in a market equilibrium.

[8] In this paper $\bar{c}$ is supposed to be large enough to never become binding (this requires positive fixed costs, i.e., $F > 0$, in the case of preferences without finite choke prices).

Accordingly, given $\widehat{c}$ and the measure of entrant firms N, we can rewrite the utility as follows:

$$V = N \int_0^{\widehat{c}} v\left(\rho s\left(c\right)\right) dG\left(c\right) - \theta\left(\rho\right), \tag{17}$$

where the aggregator satisfies:

$$\theta'\left(\rho\right) = N \int_0^{\widehat{c}} v'\left(\rho s\left(c\right)\right) s\left(c\right) dG\left(c\right), \tag{18}$$

and $s\left(c\right) = p(c)/E$ is the normalized price of a c-firm. The variable profits of such a firm are:

$$\pi(c) = \left(p(c) - c\right) x\left(c\right) L,$$

and the equilibrium pricing rule $p(c)$ satisfies:

$$p(c) = \frac{\varepsilon\left(\rho s\left(c\right)\right) c}{\varepsilon\left(\rho s\left(c\right)\right) - 1}, \tag{19}$$

with $x\left(c\right) = \frac{v'(s(c)\rho)}{\theta'(\rho)}$ denoting the individual demand for a c-variety. We assume that the pricing rule is uniquely defined by (19), which is certainly the case if $\varepsilon(z)$ is monotonic. Since $\pi'(c) < 0$, the cut-off $\widehat{c}$ satisfies the zero profit condition:

$$\left(p(\widehat{c}) - \widehat{c}\right) v'\left(\rho s\left(\widehat{c}\right)\right) L = \theta'\left(\rho\right) F, \tag{20}$$

which in the limit case of $F = 0$ identifies the cut-off

$$\widehat{c} = \frac{aE}{\rho} \tag{21}$$

as the minimum price that drives demand to zero.

The measure of firms that will be *ex-post* active, whose goods are actually consumed, is given by $n = G(\widehat{c})N$. The measure of the entrant firms, instead, is determined by the free-entry condition of zero expected profit:

$$\mathbb{E}\left\{\pi(c)\right\} = \int_0^{\widehat{c}} \left(p(c) - c\right) x\left(c\right) L dG(c) - G(\widehat{c})F = F_e. \tag{22}$$

Combining (20) and (22) we can also see that the cut-off $\widehat{c}$ has to satisfy (assuming $F > 0$):

$$\int_0^{\widehat{c}} \frac{\pi(c)}{\pi(\widehat{c})} dG(c) = \frac{F_e}{F} + G(\widehat{c}), \tag{23}$$

which emphasizes that equilibrium firm selection depends on the ratio between average and marginal profits as well as on the ratio between entry and fixed costs. Rewriting (22) by using (19) and the budget constraint we obtain the mass of created firms:

$$N = \frac{EL}{\overline{\varepsilon}\left[F_e + FG(\widehat{c})\right]}, \tag{24}$$

where

$$\bar{\varepsilon} = \left[\int_0^{\widehat{c}} \frac{1}{\varepsilon(\rho s(c))} \frac{x(c)\, s(c)}{\int_0^{\widehat{c}} x(c)\, s(c)\, dG(c)} dG(c) \right]^{-1} \tag{25}$$

is the harmonic average of demand elasticities weighted by the market shares. We assume that a unique equilibrium exists. This is the case under CES preferences (with $F > 0$), which implies that $\bar{\varepsilon} = \varepsilon$ is a constant and that both cut-off and markups are independent from income and population: as is well-known from Melitz (2003), opening up to costless trade (i.e., a larger market size) does not induce any selection effects in this case.

The number of goods created and consumed are determined by intuitive conditions (which below will be compared to the optimality conditions). The average elasticity of demand $\bar{\varepsilon}$ drives average profitability, and therefore determines the measure of firms created in equilibrium, while the marginal cost cut-off $\widehat{c}$ depends on the ratio between average and marginal profitability, and markups change with the marginal cost of each good depending on the elasticity of demand $\varepsilon(z)$. The selection effects of market size, income, fixed costs and entry cost on $\hat{c}$ and the consequent impact on prices can be obtained in principle by differentiating (23). In the following proposition, we summarize the equilibrium and derive properties that apply to special classes of GP preferences:

Proposition 3. *Under Gorman-Pollak preferences and heterogeneous firms the equilibrium of monopolistic competition with free entry is given by a pricing rule $p(c)$ satisfying (19), an aggregator ρ satisfying (18), a measure of firms N satisfying (24)-(25) and a cut-off $\hat{c}$ satisfying (23) with positive fixed costs, and (21) without fixed costs. Changes in income E are neutral on the selection of firms and on pricing under direct additivity, changes in market size L are neutral on the selection of firms and on pricing under indirect additivity, and changes in EL drive selection effects and pricing under homotheticity.*

Proof. Defining the equilibrium value of the normalized aggregator as:

$$\tilde{\rho} = \frac{\rho}{E},$$

the market equilibrium $\{p(c), \tilde{\rho}, N, \widehat{c}\}$ can be rewritten through the following equations:

$$\theta'(\tilde{\rho}E)\, E = N \int_0^{\widehat{c}} v'(\tilde{\rho}p(c))\, p(c)\, dG(c), \tag{26}$$

$$p(c) = \frac{\varepsilon(\tilde{\rho}p(c))\, c}{\varepsilon(\tilde{\rho}p(c)) - 1} \tag{27}$$

$$(p(\widehat{c}) - \widehat{c})\, v'(\tilde{\rho}p(\widehat{c}))\, L = \theta'(\tilde{\rho}E)\, F. \tag{28}$$

$$N = \frac{EL}{\bar{\varepsilon}\,[F_e + FG(\widehat{c})]}, \tag{29}$$

where

$$\bar{\varepsilon} = \left[\int_0^{\widehat{c}} \frac{1}{\varepsilon(\tilde{\rho}p(c))} \frac{v'(\tilde{\rho}p(c))\, p(c)}{\int_0^{\widehat{c}} v'(\tilde{\rho}p(c))\, p(c)\, dG(c)} dG(c) \right]^{-1}. \tag{30}$$

Under our assumptions, (27) implies that the pricing rule is uniquelly determined by $\tilde{\rho}$, and this in turn implies, by (30), that $\overline{\varepsilon}$ only depends on $\widehat{c}$ and $\tilde{\rho}$. Combining (26) and (29) we reduce the system to

$$\theta'(\tilde{\rho}E)\,\overline{\varepsilon}\,[F_e + FG(\widehat{c})] = L\int_0^{\widehat{c}} v'(\tilde{\rho}p(c))\,p(c)\,dG(c)\,, \tag{31}$$

$$(p(\widehat{c}) - \widehat{c})\,v'(\tilde{\rho}p(\widehat{c}))\,L = \theta'(\tilde{\rho}E)\,F. \tag{32}$$

Suppose that $F > 0$. When preferences are directly additive $\theta'(\rho) = -1$ and, therefore, conditions (31) and (32) determine $\tilde{\rho}$ and $\widehat{c}$ independently from income E. Thus neither pricing nor the selection of firms are affected by income E (and N is linear with respect to E). When preferences are indirectly additive $\rho = 1$ and $\tilde{\rho} = 1/E$. Then, by (27) pricing only depends on income and (31) and (32) reduce to a single equation which determine the threshold $\widehat{c}$ as a function of E which does not depend on market size L (and N is linear with respect to L). Finally, when preferences are homothetic $\theta'(\tilde{\rho}E) = -1/\tilde{\rho}E$ and, accordingly, conditions (31) and (32) determine $\tilde{\rho}$ and $\widehat{c}$ as a function of EL.

Suppose now that $F = 0$: then by (32) $\widehat{c} = a/\tilde{\rho}$ and (31) determines $\tilde{\rho}$, which is independent from E under direct additivity, and depends only on EL when preferences are homothetic. Finally, under indirect additivity of preferences $\tilde{\rho} = 1/E$ and $\widehat{c} = aE$, which is again independent from L. $\square$

To obtain more detailed results on the effects occurring under GP preferences, we now consider more restrictive conditions on technology.

1.2.1 Pareto distribution without fixed costs

We now focus on the case of a Pareto distribution of the unit costs, i.e.:

$$G(c) = \left(\frac{c}{\bar{c}}\right)^{\kappa} \tag{33}$$

with shape parameter $\kappa > 1$, assuming that there are no fixed costs of production after entry. These assumptions are the same as those used by Arkolakis *et al.* (2019) under direct additivity and by Bertoletti *et al.* (2018) under indirect additivity, therefore we extend their settings to the entire type of GP preferences.

In this case the cut-off $\widehat{c}$ is determined by the choke price at which demand is null:

$$\widehat{c} = \frac{aE}{\rho}. \tag{34}$$

The price rule (19) still applies, but it can be rewritten as:

$$p(c) = \frac{\varepsilon(p(c)\,a/\widehat{c})c}{\varepsilon(p(c)\,a/\widehat{c}) - 1}, \tag{35}$$

which depends on $\widehat{c}$ and therefore on ρ in a simple way. Let us assume $\varepsilon'(z) > 0$, implying that selection effects reducing $\widehat{c}$ are going to reduce markups. Let us also define b by:

$$\varepsilon(b) \equiv 1,$$

namely, b is the effective normalized price $p(0)\,a/\widehat{c}$ set by the most efficient firm with $c = 0$ (assuming that it is well-defined by the first-order condition for profit maximization). The distribution of the effective normalized price can be computed as

$$\begin{aligned} F_{\rho s}(z) &= \Pr\{\rho s(c) \leq z, c \leq \widehat{c}\} \\ &= \Pr\left\{c \leq \frac{\varepsilon(z)-1}{\varepsilon(z)}\frac{zE}{\rho}; c \leq \frac{aE}{\rho}\right\} \\ &= \frac{G(h(z)E/\rho)}{G(aE/\rho)}, \end{aligned}$$

where $h(z) = z\,[1 - 1/\varepsilon(z)]$, with $h' > 0$. $F_{\rho s}$ is the equilibrium distribution of the normalized effective prices of the active firms on the support $[b, a]$. In general it would depend on the expenditure level and it might also depend, through ρ, on the market size, but under the assumption of a Pareto distribution, it reads as:

$$F_{\rho s}(z) = \left(\frac{h(z)}{a}\right)^{\kappa}, \tag{36}$$

which depends neither on the market size nor on the expenditure level. The intuition is that the inframarginal price adjustments due to variations in the threshold $\widehat{c}$, in turn due to changes in E or in L/F_e, are exactly compensated by the process of entry/exit in terms of the effective normalized prices. This has far reaching implications.

First, let us define the constant:

$$\Psi \equiv -\int_a^b v'(t)\,t dF_{\rho s}(t) > 0$$

Since we can rewrite the average demand elasticity as:

$$\overline{\varepsilon} = \Psi\left[\int_b^a \frac{-v'(t)\,t}{\varepsilon(t)} dF_{\rho s}(t)\right]^{-1},$$

we obtain that this is a constant, and in particular it is independent from market size and income as well as from the entry costs. The consequence is that the measure of entrants N is linear with respect to EL/F_e. Second, using (34), (24), we can rewrite the definition of the aggregator (18) as:

$$\theta'(\rho) = \frac{-\Psi n}{\rho},$$

which implies the following equation for $\widehat{c}$:

$$\theta'\left(\frac{aE}{\widehat{c}}\right) = \frac{-\Psi L}{\overline{\varepsilon} a F_e}\frac{\widehat{c}^{\kappa+1}}{\overline{c}^{\kappa}}. \tag{37}$$

This formula shows that a rich array of selection effects of E and L/F_e arises depending on the nature of preferences and technological conditions, which in turn affect pricing as mentioned above (through (35)) and the measure of consumed variety $n = N\left(\frac{\widehat{c}}{\bar{c}}\right)^{\kappa}$.

In particular, by differentiating last expression we immediately get:

$$\frac{\partial \ln \widehat{c}}{\partial \ln E} = \frac{\beta(\rho)}{\beta(\rho) - \kappa - 1} \quad \text{and} \quad \frac{\partial \ln \widehat{c}}{\partial \ln L} = \frac{1}{\beta(\rho) - \kappa - 1},$$

which depend on the curvature of $\theta(\rho)$ and on the Pareto shape parameter κ. In particular, selection effects (which reduce prices given our assumption that $\varepsilon' > 0$) are caused by a rise of market size if and only if $\beta < \kappa + 1$, and also by a rise of income if and only if $0 < \beta < \kappa + 1$.[9] In addition, they affect the measure of consumed varieties as follows:

$$\frac{\partial \ln n}{\partial \ln E} = \frac{(\beta(\rho) - 1)(\kappa + 1)}{\beta(\rho) - \kappa - 1} \quad \text{and} \quad \frac{\partial \ln n}{\partial \ln L} = \frac{\beta(\rho) - 1}{\beta(\rho) - \kappa - 1}.$$

We can illustrate these results by considering the main classes of preferences. Consider the case of direct additivity, in which $\theta = -\rho$ and therefore $\beta = 1$. Then we can compute the measure of consumed varieties:

$$n = \frac{\rho}{\Psi},$$

and the equilibrium cut-off:

$$\widehat{c} = \left[\frac{\bar{c}^{\kappa} a \bar{\varepsilon} F_e}{\Psi L}\right]^{\frac{1}{\kappa+1}}. \tag{38}$$

While the expenditure level does not change the set of active firm, an increase in L/F_e exerts a selection effect. Changes in income remain neutral on prices, while the impact of market size changes depend on the shape of demand elasticity.

In the indirectly additive case in which $\phi = \xi$ and $\rho = 1$, we compute from (37):

$$n = \frac{E^{\kappa+1} L}{\bar{\varepsilon} F_e}\left(\frac{a}{\bar{c}}\right)^{\kappa}$$

for the measure of consumed goods, and:

$$\widehat{c} = aE \tag{39}$$

for the marginal cost cut-off, in line with Bertoletti *et al.* (2018). Population does not affect the cost threshold, which is however linearly increasing in income, while the measure of consumed goods is proportional to population, and increases more than proportionally with income. Accordingly, prices are neutral with respect to market size while they change with income depending on the shape of the demand elasticity.

Consider finally the homothetic case in which $\theta = -\ln \rho$, and therefore $\beta = 1$. Then, we have:

$$n = \Psi^{-1},$$

[9] Of course we get anti-selection effects and price rises if $\beta > \kappa + 1$.

which is independent from both income and population. An increase of EL/F_e instead, exerts a selection effect on the set of the active firms:

$$\widehat{c} = \left[\frac{\overline{c}^{\kappa} \overline{\varepsilon} F_e}{\Psi EL} \right]^{\frac{1}{\kappa}}, \tag{40}$$

leaving unchanged the measure of consumed goods. When the total income in the market EL changes, consumers keep consuming the same number of goods while adjusting their consumption levels. Similar results emerge with other specifications of homothetic preferences which have been examined elsewhere (see Feenstra, 2018 and Arkolakis *et al.*, 2019), suggesting the generality of these properties under homotheticity.

The analysis above should clarify a variety of results obtained by the literature considering alternative versions of additive or homothetic preferences. Moving beyond these classes of preferences, a wide range of comparative statics results emerges again. To fix ideas, we state the following result which derives immediately from the conditions above:

PROPOSITION 4. *Under Gorman-Pollak preferences, a Pareto distribution of marginal costs without fixed costs of production, and $\varepsilon'(z) > 0$ the equilibrium of monopolistic competition with free entry implies that increases in market size or income are (weakly) associated with selection effects of the more efficient firms, increases in the measure of consumed varieties and price reductions whenever the curvature of $\theta(\rho)$ satisfies $\beta(\rho) \in [0, \kappa + 1)$.*

To conclude this section, let us define the average incremental surplus of consumed goods as:

$$\Xi = \int_b^a v(t) \, dF_{\rho s}(t),$$

Under GP preferences we can write equilibrium welfare as:

$$V = \Xi n - \theta(\rho). \tag{41}$$

Accordingly, there are only two welfare channels by which gains from opening up to costless trade (i.e., a change in market size $dL > 0$), as well as other shocks, can materialize, either by changing the number of consumed varieties, or by affecting the price aggregator. In addition, the second effect is entirely captured by $\theta'(\rho)$. In particular:

$$dV = \Xi dn - \theta'(\rho) \, d\rho.$$

With directly additive preferences an increase of L generates both an increase of n and a welfare improving rise of ρ associated to the alledged selection effect. Instead, with indirectly additive preferences the whole welfare improvement comes from the increase in the measure of consumed varieties n. When preferences are homothetic we know that $dn/dL = 0$, since the linear impact on N is exactly offset by the selection effect of a reduction of $\widehat{c}$, which is associated to a welfare improving increase of ρ. In general, the gains from globalization derive in part from an increase in the measure of the consumed varieties and in part from a reduction of the inconvenience of consumption made possible by the selection of cheaper goods. While an extension to costly trade with multiple countries complicates the analysis, the impact of reductions in transport costs on welfare are going to operate through these same channels.

1.2.2 Optimality

We can now consider the social planner problem, generalizing results obtained by Dhingra and Morrow (2019) for the case of directly additive preferences, and by Bertoletti *et al.* (2018) for the case of indirectly additive preferences (in the absence of fixed costs of production). It is well known that a condition for optimality is that the markup must be common across commodities to insure that the marginal rate of substitution between any two goods equals the ratio of marginal costs (the social, marginal rate of transformation).[10] Therefore, we set a price $p = mc/E$, where m is the common markup. Then, we can write the social planner problem as follows:

$$\max_{N,\widehat{c},m} \left\{ V = N \int_0^{\widehat{c}} v\left(\rho \frac{mc}{E}\right) dG(c) - \theta(\rho) \right\}$$

$$s.t. \quad N \int_0^{\widehat{c}} cx(c) L dG(c) = EL - N\left[F_e + FG(\widehat{c})\right],$$

$$\theta'(\rho) = N \int_0^{\widehat{c}} v'\left(\rho \frac{mc}{E}\right) \frac{mc}{E} dG(c),$$

$$x(c) = \frac{v'\left(\rho \frac{mc}{E}\right)}{N \int_0^{\widehat{c}} v'\left(\rho \frac{mc}{E}\right) \frac{mc}{E} dG(c)},$$

where the three constrains are respectively the resource contraint, the definition of the aggregator and the individual demand associated with GP preferences. Combining the constraints we obtain the following expression for the markup:

$$m = \frac{EL}{EL - N\left[F_e + FG(\widehat{c})\right]}. \tag{42}$$

Using this, the social planner problem reduces to:

$$\max_{N,\widehat{c}} \left\{ N \int_0^{\widehat{c}} v\left(\frac{\rho c L}{EL - N\left[F_e + FG(\widehat{c})\right]}\right) dG(c) - \theta(\rho) \right\},$$

where the aggregator must satisfy its definition above, but its changes do not affect the objective function.

[10] A proof ot this would be similar to the one in Bertoletti *et al.* (2018, Appendix A) for the case of indirectly additive preferences.

If there are positive fixed costs of production (namely, if $F > 0$), the two first-order conditions can be solved for:

$$N = \frac{EL}{[\bar{\eta}(m,\rho,\widehat{c}) + 1]\,[F_e + FG(\widehat{c})]}, \tag{43}$$

and:

$$\int_0^{\widehat{c}} \frac{v\left(\frac{\rho mc}{E}\right)}{v\left(\frac{\rho m\widehat{c}}{E}\right)} dG(c) = \frac{F_e}{F} + G(\widehat{c}), \tag{44}$$

where $\bar{\eta}$ is an "average" of the elasticities $\eta(z)$ given by (13):

$$\bar{\eta}(m,\rho,\widehat{c}) = \int_0^{\widehat{c}} \eta\left(\frac{\rho mc}{E}\right) \frac{v(\frac{\rho mc}{E})}{\int_0^{\widehat{c}} v\left(\frac{\rho mc}{E}\right) dG(c)} dG(c), \tag{45}$$

whose weights are the corresponding shares of the incremental social benefit. Using (42) and (43) we can also rewrite the otptimal markup as:

$$m = 1 + \frac{1}{\bar{\eta}(m,\rho,\widehat{c})}.$$

When there are no fixed costs of production (namely, if $F = 0$) the social planner problem simplifies to:

$$\max_{N,\widehat{c}} \left\{ N \int_0^{\widehat{c}} v\left(\frac{cL\rho}{EL - NF_e}\right) dG(c) - \theta(\rho) \right\},$$

and in this case it is always optimal to consume any good that provides positive subutility, so that the optimal cut-off must satisfy:

$$\widehat{c} = \frac{a\,(EL - NF_e)}{\rho L}.$$

Given this, the planner problem simplifies further to:

$$\max_{N} \left\{ N \int_0^{\frac{a(EL-NF_e)}{\rho L}} v\left(\frac{cL}{EL - NF_e}\rho\right) dG(c) - \theta(\rho) \right\},$$

whose first-order condition gives:

$$N = \frac{EL}{[\bar{\eta}(m,\rho,\widehat{c}) + 1]\,F_e}. \tag{46}$$

This is consistent with the result under positive fixed cost, and implies the same markup expression as there, i.e., $m = 1 + 1/\bar{\eta}(m,\rho,\widehat{c})$. Replacing in the earlier expression for the cut-off, we finally have:

$$\widehat{c} = \frac{aE\bar{\eta}(m,\rho,\widehat{c})}{\rho\,[\bar{\eta}(m,\rho,\widehat{c}) + 1]} \tag{47}$$

We summarize these results in the following Proposition:

PROPOSITION 5. *Under Gorman-Pollak preferences and heterogeneous firms, the solution of the social planner problem is given by a pricing rule $p(c) = m^* c$, an aggregator ρ^*, a measure of firms N^* and a cut-off $\hat{c}^*$ satisfying:*

$$m^* = 1 + \frac{1}{\bar{\eta}^*},$$

$$N^* = \frac{EL}{(\bar{\eta}^* + 1)\left[F_e + FG(\widehat{c}^*)\right]},$$

and

$$\int_0^{\widehat{c}^*} \frac{v\left(\frac{m^*\rho^*}{E} c\right)}{v\left(\frac{m^*\rho^*}{E} \widehat{c}^*\right)} dG(c) = \frac{F_e}{F} + G(\widehat{c}^*)$$

with positive fixed costs, and $\widehat{c} = \frac{aE\bar{\eta}^*}{\rho(\bar{\eta}^*+1)}$ without fixed costs, where $\theta'(\rho^*) = \frac{m^* N^*}{E} \int_0^{\widehat{c}^*} v'\left(\frac{m^* c}{E}\rho^*\right) cdG(c)$ and $\bar{\eta}^* = \bar{\eta}(m^*, \rho^*, \widehat{c}^*)$ is the average subutility elasticity.

Intuitively, $\bar{\eta}^*$ governs the average incremental surplus created by goods, and therefore it determines the measure of goods to be introduced in the economy before knowing the marginal cost at which they can be produced, while the optimal threshold for actual production $\widehat{c}^*$ depends on the ratio between the average incremental surplus and the one generated by the marginal firm. The comparison with the market equilibrium is rather simple: in the latter case product creation depends on the average demand elasticity (which determines the expected profits) and firm selection depends on the ratio between average and marginal actual profitability.

Again, further results can be obtained from additional assumptions on preferences and technology. In particular, CES preferences imply that $\eta = \varepsilon - 1$ is constant and:

$$\frac{\pi(c)}{\pi(c')} = \frac{cx(c)}{c'x(c')} = \left(\frac{c}{c'}\right)^{1-\varepsilon} = \frac{v(\rho mc)}{v(\rho mc')},$$

for any c and c' of active firms ($c' > c$), which in turn confirms that the equilibrium of the Melitz model is efficient, as already known from Dhingra and Morrow (2019).

Further progress for the general case can be made after noticing that, integration by parts delivers:

$$\int_0^{\widehat{c}} v'\left(\rho s(c)\right) \rho s(c) dG(c) = -\int_0^{\widehat{c}} v\left(s(c)\right)\left[g(c) + cg'(c)\right] dc,$$

which allows us to rewrite the average elasticity as:

$$\bar{\eta}(m, \rho, \widehat{c}) = \frac{\int_0^{\widehat{c}} v\left(\rho \frac{mc}{E}\right)\left[g(c) + cg'(c)\right] dc}{\int_0^{\widehat{c}} v\left(\rho \frac{mc}{E}\right) dG(c)}, \tag{48}$$

where the role of cost distribution in shaping the optimal markup emerges more clearly. In particular, under the assumption of a Pareto distribution we immediately obtain $\bar{\eta} = \kappa$ independently from the nature of preferences. This delivers the following result:

PROPOSITION 6. *Under Gorman-Pollak preferences with heterogeneous firms and a Pareto distribution of marginal costs, the optimal allocation requires:*

$$m^* = 1 + \frac{1}{\kappa},$$

$$N^* = \frac{EL}{(\kappa+1)[F_e + F(\widehat{c}^*/\bar{c})^\kappa]},$$

and

$$\int_0^{\widehat{c}^*} \frac{v\left(\frac{\rho^* c(\kappa+1)}{E\kappa}\right)}{v\left(\frac{\rho^* \widehat{c}^*(\kappa+1)}{E\kappa}\right)} d\left(\frac{c}{\bar{c}}\right)^\kappa = \frac{F_e}{F} + (\widehat{c}^*/\bar{c})^\kappa$$

with positive fixed costs, and $\widehat{c}^* = \frac{aE\kappa}{(\kappa+1)\rho^*}$ *without fixed costs, where* $\theta'(\rho^*) = \frac{(\kappa+1)N^*}{E\kappa} \int_0^{\widehat{c}^*} v'\left(\frac{\rho^* c(\kappa+1)}{E\kappa}\right) c d(c/\bar{c})^\kappa$.

Remarkably, the optimal markup depends only on the shape parameter of the Pareto distribution and decreases with it. The optimal number of firms decreases with the same parameter directly, but also with the optimal cut-off (when there are fixed costs) which depends on the shape parameter. Broadly speaking, the equilibrium is inefficient because low-cost firms choose too high prices and high-cost firms choose too low prices, but we are unable to draw unambiguous comparisons between equilibrium and optimal measures of goods created and consumed without additional assumptions on the nature of preferences.

1.2.3 Generalized translated power preferences

Closed form solutions for the model with a Pareto distribution of unit costs can be easily obtained for our specification of preferences (8), for which we reproduce the indirect utility function:

$$V = \int_\Omega \frac{(a - \rho s(\omega))^{1+\gamma}}{1+\gamma} d\omega + \frac{\rho^{1-\beta}}{1-\beta}. \tag{49}$$

As mentioned above, this specification becomes directly additive for $\beta = 0$, homothetic for $\beta = 1$, and indirectly additive for $\beta \to \infty$, and its demand functions are perfectly rigid for $\gamma \to 0$, linear for $\gamma = 1$ and perfectly elastic for $\gamma \to \infty$.

Given the individual demand $x = [a - \rho s]^\gamma \rho^\beta$, it is easy to verify that a firm with marginal cost c adopts the pricing rule:

$$p(c) = \frac{\gamma c + \widehat{c}}{\gamma + 1} \quad \text{with} \quad \widehat{c} = \frac{aE}{\rho}, \tag{50}$$

which resembles the one obtained under indirect additivity by Bertoletti *et al.* (2018), except for the presence of the aggregator ρ, that drives the properties of this model. We can compute the expected profits as follows:

$$\begin{aligned}\mathbb{E}\left\{\pi(c)\right\} &= \int_0^{\hat{c}} \pi(c)dG(c) = \\ &= \frac{\kappa\gamma^\gamma\rho^{\beta+\gamma}EL}{\overline{c}^\kappa}\left(\frac{1}{(1+\gamma)E}\right)^{1+\gamma}\int_0^{\hat{c}}(\hat{c}-c)^{\gamma+1}c^{\kappa-1}dc.\end{aligned}$$

The latter expression can be integrated by substitution (using $t = c/\hat{c}$) to get:

$$\begin{aligned}\mathbb{E}\left\{\pi(c)\right\} = \frac{\kappa\gamma^\gamma\hat{c}^{\gamma+\kappa+1}\rho^{\beta+\gamma}EL}{\overline{c}^\kappa}\left(\frac{1}{(1+\gamma)E}\right)^{1+\gamma}\int_0^1 t^{\kappa-1}(1-t)^{1+\gamma}dt \\ = \frac{\kappa\gamma^\gamma a^{\gamma+\kappa+1}E^{\kappa+1}L}{\overline{c}^\kappa(1+\gamma)^{1+\gamma}\rho^{\kappa-\beta+1}}B(\kappa,\gamma+2),\end{aligned}$$

where $B(z,h) = \int_0^1 t^{z-1}(1-t)^{h-1}dt$ is the Euler Beta function.[11] By using the free entry condition we can solve for the aggregator as:

$$\rho = \left[\frac{\kappa\gamma^\gamma a^{\gamma+\kappa+1}E^{\kappa+1}L}{\overline{c}^\kappa(1+\gamma)^{\gamma+1}F_e}B(\kappa,\gamma+2)\right]^{\frac{1}{\kappa-\beta+1}}.$$

This allows us to obtain the cut-off:

$$\hat{c} = \left[\frac{\overline{c}^\kappa(1+\gamma)^{\gamma+1}F_e}{\kappa\gamma^\gamma a^{\beta+\gamma}B(\kappa,\gamma+2)E^\beta L}\right]^{\frac{1}{\kappa-\beta+1}}, \qquad (51)$$

as a function of both income E and per capita entry cost F_e/L. Using the definition of the aggregator we can compute:

$$1 = N\int_0^{\hat{c}}\left[a - \frac{\rho}{E}p(c)\right]^\gamma\frac{\rho^\beta}{E}p(c)dG(c)$$

or

$$\overline{c}^\kappa(1+\gamma)^{\gamma+1}E^{\gamma+1} = \kappa\gamma^\gamma\widehat{c}^{\gamma+1}\rho^{\beta+\gamma}N\int_0^{\hat{c}}\left(1-\frac{c}{\widehat{c}}\right)^\gamma\left(c^{\kappa-1}+\gamma\frac{c^\kappa}{\widehat{c}}\right)dc.$$

Integrating by substitution we obtain:

$$\overline{c}^\kappa(1+\gamma)^{\gamma+1}E^{\gamma+1} = \kappa\gamma^\gamma\widehat{c}^{\kappa+\gamma+1}\rho^{\beta+\gamma}N\left[\int_0^1 t^{\kappa-1}(1-t)^\gamma dt + \gamma\int_0^1 t^\kappa(1-t)^\gamma dt\right],$$

[11] It holds that $B(z+1,h) = zB(z,h)/(z+h)$ and $B(z,h+1) = hB(z,h)/(z+h)$.

which can be solved for the measure of entrant firms as:

$$\begin{aligned} N &= \frac{\bar{c}^{\kappa}(1+\gamma)^{\gamma}(\kappa+\gamma+1)\rho^{\kappa-\beta+1}}{\kappa\gamma^{\gamma}a^{\kappa+\gamma+1}(\kappa+1)B(\kappa,\gamma+1)E^{\kappa}} \\ &= \frac{EL}{(\kappa+1)F_e} \end{aligned} \tag{52}$$

after susbtituting for ρ and using the properties of the Beta function. We obtain therefore an average demand elasticity $\bar{\varepsilon} = \kappa + 1$ which depends only on the Pareto parameter and implies that the equilibrium measure of created goods is the same as the optimal one. It is then easy to compute the measure of consumed goods $n = NG(\widehat{c})$ as:

$$n = \frac{\bar{c}^{\frac{\kappa(\beta-1)}{\kappa-\beta+1}}(\gamma+1)^{\frac{(\gamma+1)\kappa}{\kappa-\beta+1}}\left(\frac{L}{F_e}\right)^{\frac{1-\beta}{\kappa-\beta+1}}E^{\frac{(\kappa+1)(1-\beta)}{\kappa-\beta+1}}}{(\kappa+1)\left[\kappa\gamma^{\gamma}a^{\beta+\gamma}B(\kappa,\gamma+2)\right]^{\frac{\kappa}{\kappa-\beta+1}}}.$$

This is constant whenever preferences are homothetic ($\beta = 1$), linear with respect to E when preferences are directly additive ($\beta = 0$) and more than proportional in income when they are indirectly additive ($\beta \to \infty$). Equilibrium welfare reads as:

$$V = \frac{\kappa\gamma^{\gamma+1}a^{\gamma+1}B(\kappa,\gamma+2)}{(1+\gamma)^{2+\gamma}}n + \frac{\rho^{1-\beta}}{1-\beta}. \tag{53}$$

Moving to the social planner problem, we can refer to the earlier results under a Pareto distribution and compute the optimal value of the aggregator as:

$$\rho^* = \left[\frac{\kappa^{\kappa+2}a^{\kappa+\gamma+1}LE^{\kappa+1}}{(\gamma+1)\bar{c}^{\kappa}(1+\kappa)^{\kappa+1}F_e}B(\kappa,\gamma+2)\right]^{\frac{1}{\kappa+1-\beta}},$$

and the optimal the cut-off:

$$\widehat{c}^* = \left[\frac{(\gamma+1)\bar{c}^{\kappa}(1+\kappa)^{\beta}F_e}{\kappa^{\beta+1}a^{\gamma+\beta}B(\kappa,\gamma+2)LE^{\beta}}\right]^{\frac{1}{\kappa+1-\beta}}, \tag{54}$$

which differs from the equilibrium one in general, implying an inefficient measure of consumed goods. Notice that too many goods are consumed under both direct and indirect additivity (for $\beta = 0$ and $\beta \to \infty$), and that the same applies under homotheticity ($\beta = 1$) if and only if:

$$\left(1+\frac{1}{\gamma}\right)^{\gamma} > 1 + \frac{1}{\kappa},$$

a condition which holds if demand is close to linear or κ is sufficiently large.

2 Implicit CES preferences

In this section we consider monopolistic competition based on demand systems derived from implicit CES preferences. These represent a particular class of GAS preferences which also belong to the implicitly additive type studied by Hanoch (1975).[12] They generalize the CES case by having an elasticity of substitution that is common across goods but can possibly change across indifference curves. As far as we know, they have never been employed to analyze monopolistic competition, though, as we will show, they preserve some of the convenient properties of the explicit CES preferences (namely, those with $\varepsilon'(V) = 0$) while providing more flexible implications for the comparative statics of markup.

We focus on the following version of implicit CES:

$$U = \left[\int_\Omega x(\omega)^{1-\epsilon(U)} d\omega\right]^{\frac{1}{1-\epsilon(U)}} \quad \text{and} \quad V = \left[\int_\Omega s(\omega)^{1-\varepsilon(V)} d\omega\right]^{\frac{1}{\varepsilon(V)-1}}, \tag{55}$$

where, differently from the explicit CES case, $\varepsilon(z) = 1/\epsilon(z) > 1$ is a function of the utility level. As long as this is not constant, preferences are non-homothetic since the relative demands change according to the utility level. Obviously, $\epsilon(U)$ has to satisfy some regularity conditions to ensure that utility is well-defined (see Fally, 2018).

The demand system can be easily derived as follows:

$$s(\iota) = \frac{x(\iota)^{-\epsilon(U)}}{U^{1-\epsilon(U)}} \quad \text{and} \quad x(\iota) = \frac{s(\iota)^{-\varepsilon(V)}}{V^{\varepsilon(V)-1}},$$

which shows that it belongs to the GAS type. A firm producing with marginal cost c maximizes its variable profits:

$$\pi = \left[\frac{x^{-\epsilon(U)}E}{U^{1-\epsilon(U)}} - c\right] xL = \frac{(sE-c)}{V^{\varepsilon(V)-1}} s^{-\varepsilon(V)} L,$$

taking utility as given under monopolistic competition. Its profit-maximizing price satisfies:

$$p = \frac{c}{1-\epsilon(U)} = \frac{\varepsilon(V)\, c}{\varepsilon(V)-1}, \tag{56}$$

therefore the markup is the same for all firms and changes with the utility index.

2.1 Monopolistic competition with homogeneous firms

We now consider the free entry equilibrium with n homogenous firms and compare it to the optimal market structure. If each firm has marginal cost c and

[12] General implicit additivity requires either a direct or an indirect utility that is implicitly defined by an additive specification, and delivers demand systems depending on up to two aggregators, one of which is the utility itself (they include a homothetic family popularized by Kimball, 1995): see Bertoletti and Etro (2017b) for details.

pays a positive fixed cost of production F, it is standard to verify that the free entry equilibrium implies:

$$p = \frac{\varepsilon(V)\,c}{\varepsilon(V)-1} \quad \text{and } n = \frac{EL}{\varepsilon(V)\,F}, \tag{57}$$

where, using the implicit definition of indirect utility, the equilibrium utility level V satisfies:

$$V = \frac{\varepsilon(V)-1}{c}\left(\frac{E}{\varepsilon(V)}\right)^{\frac{\varepsilon(V)}{\varepsilon(V)-1}}\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon(V)-1}},$$

which we assume to have a unique solution. Whenever $\varepsilon'(V) > 0$ (or equivalently $\epsilon'(U) < 0$), marginal cost changes are incompletely passed to prices, and an increase of income or of market size increases utility, reduces markups and raises less than proportionally the number of firms. This delivers the following conclusion:

PROPOSITION 7. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of homogeneous firms implies that an increase in utility (due to higher income or market size) is associated with a markup reduction and a less than proportional increase of the number of firms if and only if* $\varepsilon'(V) > 0$.

Also in this case it is interesting to evaluate the optimal allocation of resources. This solves the problem:

$$\max_{n,s}\{V\}$$

$$s.t.\;\; V = \frac{n^{\frac{1}{\varepsilon(V)-1}}}{s},$$
$$(EL - nF)s \geq cL,$$

where the first constraint is the definition of utility after imposing symmetry and the second is the resource constraint. It is easy to verify that the first-order conditions for the solution can be rewritten as:

$$p^* = \frac{\varepsilon(V^*)\,c}{\varepsilon(V^*)-1} \text{ and } n^* = \frac{EL}{\varepsilon(V^*)\,F}, \tag{58}$$

where the utility satisfies $V^* = (E/p^*)n^{*\frac{1}{\varepsilon(V^*)-1}}$, implying the same system of equations as in the equilibrium. Thus, we can conclude with:

PROPOSITION 8. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of homogeneous firms is optimal.*

This extends to the class of implicit CES preferences a result which is well-known since Spence (1976) and Dixit and Stiglitz (1977) to hold for the case of explicit CES preferences.

2.2 Monopolistic competition with heterogeneous firms

We now consider the free entry equilibrium with firms differing in marginal costs as in Melitz (2003), and in our above analysis of GP preferences. Since implicit CES preferences lack a finite choke price, without fixed costs of production all the goods would be demanded. Therefore, it is convenient to focus on the relevant case in which firms face a positive fixed cost to produce, as in the original Melitz model.

For a given utility V, which here is the relevant aggregator, a c-firm using price p faces variable profits given by :

$$\pi = \frac{(p-c)}{V^{\varepsilon(V)-1}}\left(\frac{p}{E}\right)^{-\varepsilon(V)} L.$$

Its optimal price $p(c)$ satisfies:

$$p(c) = \frac{\varepsilon(V)c}{\varepsilon(V)-1}, \tag{59}$$

which implies a common markup across firms, which decreases with the utility level if and only if $\varepsilon' > 0$.

Let us write individual demand and variable profits for a c-firm as:

$$x(c) = \frac{(p(c)/E)^{-\varepsilon(V)}}{V^{\varepsilon(V)-1}},$$

and

$$\pi(c) = \frac{[p(c)-c]}{V^{\varepsilon(V)-1}}\left(\frac{p(c)}{E}\right)^{-\varepsilon(V)} L.$$

The monotonicity of $\pi(c)$ with respect to c allows us to determine the threshold $\hat{c}$:

$$\pi(\hat{c}) = F. \tag{60}$$

Free entry requires:

$$\int_0^{\hat{c}} [\pi(c) - F] dG(c) = F_e. \tag{61}$$

The equilibrium measure of entrant firms can be derived from the budget constraint by using (59) and (61) as:

$$N = \frac{EL}{\varepsilon(V)[FG(\hat{c}) + F_e]}, \tag{62}$$

and the equilibrium level V computed by the utility expression:

$$V = \left[N E^{\varepsilon(V)-1} \int_0^{\hat{c}} p(c)^{1-\varepsilon(V)} dG(c)\right]^{\frac{1}{\varepsilon(V)-1}},$$

where the measure of firms and the cut-off satisfy the equilibrium conditons above. We assume that there is a unique equilibrium, as it is the case in the Melitz model.

Combining the equilibrium conditions (60) and (61), one can obtain the following relation between the threshold $\hat{c}$ and the equilibrium utility level V:

$$\int_0^{\hat{c}} \left(\frac{c}{\hat{c}}\right)^{1-\varepsilon(V)} dG(c) = \frac{F_e}{F} + G(\widehat{c}), \tag{63}$$

which is crucial to analyze selection effects. It is easy to verify that with explicit CES preferences there are none: in particular, an increase in utility associated to an increase in EL does not affect $\hat{c}$, while increasing proportionally the measure of consumed goods (as is well known, it takes costly trade to induce selection effects in the Melitz model). Consider now the case of variable elasticity: as long as utility increases, there must be a reduction (increase) in the cut-off $\widehat{c}$ if $\varepsilon(V)$ is increasing (decreasing) in utility. We immediately obtain:

PROPOSITION 9. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of heterogeneous firms implies that an increase in utility (due to higher income or market size) is associated with a markup reduction and a selection of the more efficient firms if* $\varepsilon'(V) > 0$.

Of course, the opposite result (a markup rise and an anti-selection effect) is associated to the case of $\varepsilon'(V) < 0$.

One can also analyze the social planner problem as we have done previously for the GP preferences. Again, the optimal markup must be constant across goods, say m. Then, the problem can be written as:

$$\max_{N,\widehat{c},m} \{V\}$$

$$s.t.\ \ V = \left[N \int_0^{\widehat{c}} \left(\frac{mc}{E}\right)^{1-\varepsilon(V)} dG(c)\right]^{\frac{1}{\varepsilon(V)-1}},$$

$$N \int_0^{\widehat{c}} cx(c)LdG(c) = EL - N\left[F_e + FG(\widehat{c})\right],$$

$$x(c) = \frac{\left(\frac{mc}{E}\right)^{-\varepsilon(V)}}{V^{\varepsilon(V)-1}}.$$

By combining the resource constraint, the demand function and the implicit definition of V we obtain that the markup must satisfy $m = \frac{EL}{EL-N[F_e+FG(\widehat{c})]}$, and the problem reduces to:

$$\max_{N,\widehat{c}} \left\{ V = \frac{EL - N\left[F_e + FG(\widehat{c})\right]}{L} \left[N \int_0^{\widehat{c}} c^{1-\varepsilon(V)} dG(c)\right]^{\frac{1}{\varepsilon(V)-1}} \right\}.$$

The first-order condition with respect to N gives:

$$N = \frac{EL}{\varepsilon(V)[FG(\hat{c}) + F_e]},$$

which implies the markup:

$$m = \frac{\varepsilon(V)}{\varepsilon(V) - 1}. \tag{64}$$

This reduces the above problem to:

$$\max_{\widehat{c}} \left\{ V = [\varepsilon(V) - 1] \left[\left(\frac{E}{\varepsilon(V)} \right)^{\varepsilon(V)} L \int_0^{\widehat{c}} \frac{c^{1-\varepsilon(V)}}{FG(\hat{c}) + F_e} dG(c) \right]^{\frac{1}{\varepsilon(V)-1}} \right\},$$

whose first-order condition with respect to $\widehat{c}$ satisfies:

$$\widehat{c}^{1-\varepsilon(V)} \left[FG(\hat{c}) + F_e \right] = \int_0^{\widehat{c}} c^{1-\varepsilon(V)} dG(c) F, \tag{65}$$

which is equivalent to the equilibrium condition (63). It follows that the optimal values $(N^*, \widehat{c}^*, m^*, V^*)$ must correspond to the ones of the unique equilibrium. Summing up we have:

Proposition 10. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of heterogeneous firms is optimal.*

Dhingra and Morrow (2019) have proved optimality of the equilibrium in the Melitz model with CES preferences and heterogeneous firms. Their result extends to the entire class of implicit CES preferences.

We conclude this section noticing that preferences as these can be exploited for a variety of applications. For instance, the heterogeneous firms model can be extended to endogenous quality differentiation, assuming that quality can be increased at a cost depending on an idiosyncratic parameter drawn after entry: such a model is consistent with either positive or negative correlation between quality and cost efficiency. Moreover, an increasing demand elasticity would imply that the markups decrease with the utility level and product quality increases (decreases) with the utility level if the elasticity of cost to quality is decreasing (increasing) in the cost parameter. Hopefully, one could estimate such a model to fit empirical patterns. Also the case of costly trade can be addressed in a standard fashion, and it generates new selection effects: in particular, opening up to trade can reduce markups, and also change the endogenous distribution of qualities across firms.

Finally, one could introduce implicit CES preferences in a flexible price macroeconomic model: while optimality would be lost due to changes in markup across periods, the propagation of shocks would be affected by this same variability, and indeed amplified when the demand elasticity is increasing.[13]

[13] See Cavallari and Etro (2017) and Etro (2019).

3 Conclusion

In this and previous contributions, the literature has now completed the exploration of monopolistic competition equilibria for the GAS type of preferences. Comparative static results in equilibrium display a wide array of possibilities when preferences range between direct and indirect additivity going through the homothetic family included in the GP preferences. Concerning optimality, GAS type of preferences can deliver excess or suboptimal entry, but market structures with both homogeneous and heterogeneous firms are optimal under implicit CES preferences.

Future research may consider more general preferences than the GAS, featuring more than one aggregator (as in case of implicit additivity). However, we believe that the flexible preferences discussed here should soon find their way to the applied literature, especially to study trade and the gains from its liberalization in multicountry models where markups differ among firms and across destination markets.

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