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The Redistributive Effects of a Money-Financed Fiscal Stimulus

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The Redistributive Effects of a Money-Financed Fiscal Stimulus

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Abstract

This paper analyzes the redistributive channel of a money financed fiscal stimulus (MFFS). It shows that the way in which this regime is implemented is crucial to determine its redistributive effects and consequently its effectiveness. In normal times, the most effective regime is a MFFS with no additional intervention by the Central Bank to stabilize the real public debt using inflation, whereas a MFFS accompanied by real debt stabilization - through the adjustment of seigniorage - is the most effective one in a ZLB scenario. In a TANK model this regime is so effective to avoid the recessionary effects implied by the ZLB. This result does not hold in a RANK model, where the redistributive channel is absent. Remarkably, contrary to the common wisdom a MFFS is followed by a moderate increase of inflation, which is only temporarily higher than the target.

KEYWORDS: Money-Financed fiscal stimulus, seignorage, government spending, redistribution, borrower-saver, fiscal multipliers, welfare, RANK versus TANK.
JEL CODES: E32, E52, E62

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1 Introduction

This paper contributes to the literature on the redistributive effects of monetary and fiscal policies. In particular, it analyzes the redistributive channel of a money-financed fiscal stimulus (MFFS - henceforth) compared to that of a debt financed fiscal stimulus (DFFS) in a borrowers-savers framework. It looks at the strength of these alternative fiscal financing regimes and investigates how their redistributive effects influence the effectiveness of the stimulus itself.

By considering a Two-Agents New-Keynesian model - henceforth, TANK model - in a Borrower-Saver framework, this paper argues that redistribution is a key channel through which the fiscal financing regimes affect the macroeconomic aggregates. Usually, the debate about money financing is burdened by the deep fear of hyperinflation. The idea that a future inflation tax can reduce the ability of money to stimulate the aggregate demand has been central in the economic debate that followed the recent crisis. However, in a borrowers-savers model, the inflation tax can be used to redistribute from one household to the other one further amplifying or reducing the effect of the stimulus. For this reason, in this paper we consider two types of MFFS: i) a MFFS in which the increase in government purchases are entirely financed through money creation without any additional intervention by the Central Bank (our benchmark MFFS, hereinafter); ii) a MFFS in which, though the increase in government spending is financed through money, the seigniorage is adjusted in each period to keep unchanged the real value of public debt using the inflation, as in Gali (2017) (alternative MFFS, hereinafter). In this respect, this paper shows that, the way in which a MFFS is implemented is crucial to determine the redistributive effects of the stimulus and consequently its effectiveness. It also results crucial to determine whether the stimulus is welfare improving with respect to a DFFS or not. Overall, we show that consumption and output multipliers are larger in a MFFS than in a DFFS. This occurs because the stimulus redistributes more from savers to borrowers, that is to the households with the higher marginal propensity to consume. However, we show that the effectiveness of a MFFS is state dependent. In normal times the most effective regime is the benchmark MFFS, whereas it is the alternative MFFS that becomes the most effective financing scheme in a ZLB scenario. In a TANK model this regime is so effective to avoid the recessionary effects of the ZLB. Remarkably, this result does not hold in a RANK model, where the redistributive channel is absent. Further, contrary to the common wisdom we show that a MFFS is followed by a moderate increase of inflation which is only temporarily higher than the target.

In this paper, Borrowers and Savers are modeled as in Bilbiie, Monacelli and Perotti (2013 - BMP henceforth). The two agents differ in their degree of impatience, they are both intertemporal maximizers, so that borrowing and lending take place in equilibrium, and financial markets are imperfect. Borrowers face a suitable defined borrowing limit, and it is important to highlight that, differently from the standard rule-of-thumb framework, the distribution of debt/saving across agents is endogenous. In this context, the paper studies the dynamics of the model in response to an exogenous increase in government pur-
chases under the two types of MFFS. The results obtained under a MFFS are compared with those implied by a DFFS, with the central bank implementing a standard inflation targeting interest rate rule. To better understand the role played by the redistribution channel, the results obtained in the TANK model are then compared with the ones characterizing a representative agent model (RANK henceforth).

In details, the contribution of the paper can be summarized in five main results.

First, in normal times, all the regimes imply a redistributive effect from savers to borrowers. However, thanks to the accommodative monetary policy, borrowers gains are much larger under the benchmark MFFS than under the alternative one and the DFFS. This is due to the fact that borrowers, who are the households with the higher marginal propensity to consume, increase their consumption by a larger amount than savers. The consumption ratio between borrowers and savers is indeed five times higher under the benchmark MFFS than under a DFFS. As a consequence, the aggregate demand increases by a larger amount and the benchmark MFFS is followed by a stronger expansionary effect on output than a DFFS. Further, thanks to the redistribution channel the benchmark MFFS strongly amplifies the impact of the fiscal intervention with respect to a RANK model, increasing the effectiveness of the MFFS. Remarkably, the amplification is obtained at the cost of a mild increase of inflation, which is only temporarily higher than the target. The alternative MFFS instead only mildly amplifies the effect of the policy with respect to a DFFS. In this case, the transmission mechanism is indeed less effective since the redistributive channel of public bonds and that of inflation are muted to stabilize the real value of the debt.

Second, we compute consumption and output fiscal multipliers associated to the three different financing regimes and we find that, under our benchmark MFFS fiscal multipliers are largely higher than one in normal times. Interestingly, these multipliers increase exponentially as the share of borrowers increases. The reason is the following. Given that borrower’s consumption reacts more than saver’s consumption to an increase in public spending, the higher is the share of borrowers the higher is the contribution of the redistribution channel to the fiscal multipliers under both financing regimes. Also, we find that fiscal multipliers are an increasing function of the borrowing limit. By relaxing the borrowing constraint, the amount of private debt obtained by borrowers increases and the implied wealth effects become larger, so that borrower’s consumption increases by a larger amount. This reflects on higher fiscal multipliers. On the contrary fiscal multipliers associated to the alternative MFFS are close to the ones implied by a DFFS. They become instead lower than those implied by a DFFS for sufficiently high values of the share of borrowers, larger than 40%, that is largely higher than what found in the data.

Third, in terms of consumption equivalent welfare, we find that borrowers are better off under our benchmark MFFS than under a DFFS, whereas savers are worse off. Fourth, our benchmark MFFS is preferable in terms of aggregate welfare only in the TANK model. In the RANK model it is instead welfare
detrimental.

Overall, we can state that in normal times our benchmark MFFS results more effective than the other two regimes, it implies larger multipliers and is welfare improving. Our benchmark MFFS is preferable in a TANK model since the expansionary effects of government spending are larger, thanks to very high fiscal multipliers.

The key economic drivers of these results are the following. Under the benchmark MFFS, government spending shocks are entirely financed through money, with the real debt free to change. In this case, the decline in the real rate brought about by the injection of new liquidity implies a consumption crowding-
in for both agents. Inflation increases more than under a DFFS and the decline in the real interest rate is much bigger. Though a consumption crowding-in characterizes also the standard RANK model, in a TANK model the redistribution channel affects significantly the dynamics of aggregate variables. Indeed, the increase in inflation erodes the real value of debt, with savers losing and borrowers gaining. The marginal value of one unit of debt decreases inducing borrowers, which represent the households with the higher marginal propensity to consume, to increase their demand. Borrowers’ consumption increases more than savers’ consumption. As a result, the aggregate demand increases more than in a RANK model and so does the aggregate output. In normal times, a DFFS is less inflationary than the benchmark MFFS, implying a lower erosion of the real value of debt and thus a less expansionary effect on output. On the contrary, when the MFFS is accompanied by a seigniorage adjusted in every period to keep real value of the debt unchanged as in Gali (2017), the redistribution channels due to higher inflation and lower real debt are not effective and the policy results much less expansionary in normal times.

Fifth, this paper shows the alternative MFFS becomes the most effective financing regime in a ZLB scenario. It redistributes from savers to borrowers and, by sustaining higher level of inflation than in normal times, is able to avoid the economy to enter into a recession. The intuition for this last result is rather simple. As soon as an adverse demand shock pushes the economy into a ZLB scenario, the real GDP falls down bringing about an increase of the real value of public debt, both under a DFFS and under the benchmark MFFS. On the contrary, under the alternative MFFS the seigniorage is adjusted to keep unchanged the real value of public debt by generating higher inflation than in the benchmark MFFS. The stronger inflationary effect of the alternative MFFS pushes the real interest rate down more than under the benchmark one and the policy redistributes from savers to borrowers, so that borrowers consumption increases and so does aggregate consumption. As a result, the policy pushes up the aggregate demand and results so effective to avoid the economy to enter into a recession. On the contrary, under the DFFS and the benchmark MFFS, the real value of debt increases so that savers who are the owners of the public debt are better off, while borrowers are worse off. The substitution effect, due to the decrease of the real interest rate under the benchmark MFFS, is more than compensated by the income effect. Both consumption of borrowers and that of savers decreases on impact and the policy results less effective than in normal
times. Last but not the least, it is rather important to notice that the strong effectiveness of the alternative MFFS does not hold in a RANK model, where the redistributive channel is absent. Again, this suggests that the redistributive channel is very important and that it cannot be neglected by policy makers.

The Technical Appendix of this paper considers an economy characterized by a non-competitive labor market. In this case wage-setting decisions are made by labor type specific unions. Each union pools the labor income of agents, leading borrowers and savers to work for the same amount of time. This implies that the redistribution channel does not affects differently labor supply decisions of the two agents, which is in line with the evidence that wealthy households do not work an amount of hours lower than that of poorer households. We show that our results are even reinforced under the alternative labor market.

The last Great Recession has opened a wide spectrum of policy tools. A more moderate example of money creation has already been implemented by central banks, as quantitative easing operations. The latter has been accompanied by expansive fiscal policies at least in the US. On this last issue, Giavazzi and Tabellini (2014) argue that measures as quantitative easing should always take place together with fiscal easing. In a recent paper, Gali (2017) compares the effectiveness of a MFFS defined as in ii) and of a DFFS, in a standard Representative Agent New Keynesian model (RANK), with and without binding zero lower bound (ZLB). By considering a simple RANK model with perfect financial markets, the paper voluntary ignores the possible redistributive effects of the two alternative regimes associated to the fiscal stimulus. However, in an economy where a fraction of agents is financially constrained, the way of financing fiscal stimulus matters, since it can redistribute away from some agents to others. The redistribution channel originated from a stimulus may then interact with the stimulus itself by reducing or increasing its effectiveness, significantly affecting the dynamics of aggregate variables and the aggregate welfare. Our paper goes in this direction by investigating the redistributive effects of money financed spending policies and the role played by the redistribution channel in affecting aggregate dynamics in normal times as well as in bad times.

The remainder of this paper is organized as follows. Section 2 briefly presents the related literature, Section 3 spells out the model economy, while Section 4 analyzes the effect of a money-financed fiscal stimulus in a NK-DSGE model with savers and borrowers. Section 5 summarizes the main findings and concludes.

2 Related Literature

Our paper is closely related to the literature of heterogeneous households models. The latest generation of models with heterogeneous agents and financial frictions, the so called Heterogeneous Agent New Keynesian models (HANK henceforth) are characterized by multi-agents and thus they are particularly suitable to study redistributive issues, however at the cost of being computational more complex. In these models the effects of any aggregate shock will be amplified or dampened depending on the way the shock affects the distribution
of income and wealth across households. Among many others, Oh and Reis (2012), McKay and Reis (2016) were the first contributions to this literature. More recent contributions are Guerrieri and Lorenzoni (2017), Auclert (2017), Kaplan et al. (2017), Ravn and Sterk (2017), Farhi and Werning (2017), Kaplan and Violante (2018) and Gali and Debertoli (2017), among many others. Guerrieri and Lorenzoni (2017) focus for example on the effects of credit crunches. Auclert (2017) considers a Bewley-Hugget-Aiyagari model calibrated on the U.S. economy and provides a careful analysis of the redistributive effects of monetary policy, showing that redistribution works through three main channels. These channels may amplify the effects of monetary policy on aggregate consumption. Similarly, Kaplan et al. (2017) argue that the aggregate effects of monetary policy shocks will depend on the type of the fiscal policy in response to them. Kaplan and Violante (2018) present an updated survey of the HANK literature and analyze the role of households’ heterogeneity for the response of the macroeconomic to aggregate shocks. They find that monetary shocks are weaker and fiscal shocks are stronger in HANK than in RANK, so that they conclude that degree of equivalence between HANK and RANK models crucially depends on the shock being analyzed. They also recognize that the development of HANK models is still in its infancy, mostly because of the computational complexity in dealing with the equilibrium distribution as a state variable, which limits the development of more structured models. Importantly, none of these papers compares the effects of a MFFS to a DFFS.

A very recent paper of Debertoli and Gali (2017) shows that TANK models can provide a good and tractable approximation of the HANK models. They show that a TANK model approximates well, both quantitatively and qualitatively the dynamics of an HANK model in response to aggregate shocks. Also in this case they do not investigate the effects of a MFFS. The Borrower-Saver framework used in our paper goes in the same direction of showing that the presence of borrowers in a TANK setup is sufficient to introduce a strong redistribution channel generating non negligible effects on the dynamics of the aggregate variables. Importantly, differently from previous papers our paper considers the effects of a MFFS both in normal times and in bad times.

Our paper is also related to the recent DSGE literature which has shown a renewed interest in monetary and fiscal policy interactions under a ZLB scenario. For example, Woodford (2011) shows analytically that in a simple NK model fiscal multipliers are larger than one when monetary policy is constrained by the ZLB.\footnote{Eggerson and Woodford (2003) previously study the optimal monetary policy at the ZLB.} Christiano, Eichenbaum and Rebelo (2011), Eggertsson and Woodford (2011) also point to the existence of very large government spending multipliers when the ZLB is binding. Gali (2017) analyzes the effects of an alternative and not conventional monetary policy to recover the economy: a fiscal stimulus, in the forms of both a temporary increase in government purchases and a tax cut, financed entirely through money creation both in normal times and at the ZLB. He finds that when the ZLB is not binding, a MFFS has much larger multipliers than a DFFS. That difference in effectiveness persists, but is much
smaller, under a binding ZLB. All these papers on monetary and fiscal policy interaction at the ZLB however consider a representative agent economy and thus they do not investigate the possible redistributive effects of these policies which are instead the main objective of our paper. Differently from these papers, we show that how the redistribution channel interacts with the stimulus is state dependent and crucial to determine the effectiveness of the stimulus itself.

Finally, a number of recent empirical papers substantiate our interest in studying the redistributive effects of the two stimuli. In particular, Doepke and Schneider (2006) show that inflationary episodes can cause significant revaluations of assets and redistributive effects from wealthy, middle age, and old households towards the government (the main debtor) and poor, young households. Notice that, while wealthy and middle age households are usually net savers, poor and young households are instead net borrowers in the US economy. Similar evidence is documented by Adam and Zhu (2014) for European countries and Canada. An additional paper motivating our analysis is Coibion et al. (2012), who, relying on the CEX survey, find that monetary expansions reduce inequality, as measured by Gini coefficients, suggesting a redistribution away from wealthier individuals (savers). In Sterk and Tenreyro (2016), monetary policy expansions cause a redistribution of income from retirees, who rely more heavily on their nominal wealth as source of finance for consumption, to working agents and future tax payers. The consumption of goods by working agents increases relative to that of retired agents following a monetary expansion.

With these evidence in mind, in the remainder of the paper, we present the Borrower-Saver setup of our model and we study the strength of the two regimes and how their redistributive effects influence the effectiveness of the policy itself.

3 The Model

The model considered is a closed economy composed by four agents: households, firms, the fiscal authority and the monetary authority.

3.1 Households

All households have preferences defined over private consumption, $C_{t,t}$, real balances, $m_{t,t} = M_{t,t}/P_t$, and labor services, $N_{t,t}$, according to the following separable period utility function,

$$\ln(C_{t,t}) - \chi \left( \frac{\Pi_t - \frac{m_{t,t}}{C_{t,t}}}{1 + \sigma} \right)^{1+\sigma} - \frac{N_{t,t}^{1+\varphi}}{1 + \varphi}, \text{ with } \chi > 0, \sigma > 0 \text{ and } \varphi \geq 0$$

In a very similar framework, English et al (2017) find that money-financed fiscal programs, if communicated successfully and seen as credible by the public, could provide significant stimulus. Conversely, such a program would be ineffective in providing stimulus if the public doubted the central bank’s commitment to a such extreme.
where \( m_{s,t} = \frac{M_{s,t}}{\pi t} \), \( \pi \) is a satiation level of money and \( \overline{C}_{s,t} \) is the aggregate level of consumption for each type of household \((t = b, s, \text{respectively borrowers and savers})\).  

**Savers**

Savers’ problem becomes:

\[
U_0 = E_0 \sum_{t=0}^{\infty} \beta_t^t \left[ \ln (C_{s,t}) - \chi \left( \frac{m_{s,t}}{C_{s,t}} \right)^{1+\sigma} - \frac{N_{s,t}^{1+\varphi}}{1+\varphi} \right],  \tag{2}
\]

s.t.

the budget constraint

\[
P_t C_{s,t} + B_{s,t}^H + A^n_{s,t} + M_{s,t} + \Omega_{s,t} P_t V_t \leq (1+i_{t-1}) B_{s,t-1}^H + (1+i_{t-1}) A^n_{s,t}
\]

\[
M_{s,t-1} + \Omega_{s,t-1} P_t (V_{t-1} + \Gamma_t)
\]

\[
+ W_t N_{s,t} - P_t T_{s,t},  \tag{3}
\]

where \( W_t \) is the nominal wage, \( A^n_{s,t-1} \) is the nominal value at beginning of period \( t \) of total private assets held in period \( t \), a portfolio of one-period bonds issued in \( t-1 \) on which the household receives the nominal interest \( i_{t-1} \). \( V_{t-1} \) is the real market value at time \( t \) of shares in intermediate good firms, \( \Gamma_t \) are real dividend payoffs of these shares, \( \Omega_{s,t} \) are share holdings, \( T_{s,t} \) is the lump-sum tax, \( B_{s,t}^H \) are the savers’ holdings of nominally riskless one-period government bonds (paying an interest \( i_t \)). The nominal debt \( B_{t}^H \) pays one unit in nominal terms in period \( t+1 \).

Given prices, policies and transfers \( \{P_t(z), W_t, I_t, G_t, T_{s,t}, V_t, \Gamma_t, T_t\}_{t \geq 0} \), the saver chooses the set of processes \( \{C_{s,t}(z), C_{s,t}, N_{s,t}, M_{s,t}, A_{s,t}, B_{s,t}^H, \Omega_{s,t}\}_{t \geq 0} \) so as to maximize (2) subject to (3), the usual \( C_{s,t} \geq 0, N_{s,t} \geq 0, M_{s,t} \geq 0 \) and the no-Ponzi game conditions.

After defining the aggregate price level as \( P_t = \left[ \int_0^1 P_t(z)^{1-t} \, dz \right]^{1-t} \), as well as real debt as, \( b_t^H \equiv B_t/P_t \), optimality is characterized by the following first-order conditions for savers:

\[
\beta_s E_t \left\{ \frac{C_{s,t} (1+i_t)}{C_{s,t+1}^{1+\pi_{t+1}}} \right\} = 1,  \tag{4}
\]

\[
\beta_s E_t \left\{ \frac{C_{s,t} V_{t+1} + \Gamma_{t+1}}{V_t} \right\} = 1,  \tag{5}
\]

\[
N_{s,t} C_{s,t} = w_t, \tag{6}
\]

\(^3\)Our assumption of the utility function for real balances is borrowed from English et al. (2017).
Equation (4) is the standard Euler equation for bond holdings, while equation (5) is the Euler equation for share holdings. Equation (6) is the savers’ labor supply, whereas equation (7) is their demand for real balances. Notice that, under log-utility of consumption, real money balances vary directly with consumption with a unitary coefficient. Also notice that, for \( i_t > 0 \), \( \frac{\mu_t}{1 + \mu_t} > 0 \), so that the opportunity cost of holding money always guarantees that demand for money will be \( \frac{m_{s,t}}{C_{s,t}} < \bar{x} \). When \( i_t = 0 \), then the RHS of the money demand will be zero, implying that the money is equal to its satiation level \( \bar{x} \).

**Borrowers**

In each period \( t \geq 0 \) and under all contingencies, the rest of households on the \([0, \lambda]\) interval is impatient (and will borrow in equilibrium, hence we index them by \( b \) for borrowers). They face the following budget constraint in nominal terms:

\[
P_t C_{b,t} + P_t A_{b,t}^n + M_{b,t} \leq (1 + i_{t-1}) A_{b,t-1}^n + M_{b,t-1} + W_t N_{b,t} - P_t T_{b,t},
\]

the borrowing constraint (on borrowing in real terms) at all times \( t \):

\[
-A_{b,t} \leq \bar{D},
\]

the usual \( C_{b,t} \geq 0, N_{b,t} \geq 0 \), and

\[
M_{b,t} \geq 0.
\]

The constraint (9), as we will show, becomes particularly important when the borrowing constraint is binding.

Given prices, policies and transfers \( \{P_t(z), W_t, \phi_t, i_t, G_t, T_{b,t}, T_t\}_{t \geq 0} \), the borrower chooses the set of processes \( \{C_{b,t}(z), C_{b,t}, N_{b,t}, M_{b,t}, A_{b,t}^n\}_{t \geq 0} \), so as to maximize (2) subject to (8), \( C_{b,t} \geq 0, N_{b,t} \geq 0 \) and (9). Optimality is characterized by the first-order conditions:

\[
C_{b,t}^{-1} = \beta_b E_t \left( \frac{1 + i_t}{\pi_{t+1}} C_{b,t+1}^{-1} \right) + \phi_t, \tag{10}
\]

\[
N_{b,t} C_{b,t} = w_t, \tag{11}
\]

\[
M_{b,t} = 0 \tag{12}
\]

implying that

\[
\chi \bar{x} = \left( \frac{i_t}{1 + i_t} \right) - \phi_t M^b \tag{13}
\]

Notice that, in (10), \( \phi_t \) takes a positive value whenever the constraint is binding. Indeed, because of BMP assumptions on the relative size of the discount factors,
the borrowing constraint will always bind. As shown in the technical Appendix, this implies that borrowers are net borrowers and that their demand is $M_{b,t} = 0$, and equation (13) is the equation that determines the value of $\phi^M_{t}$. Notice that at the ZLB, with $i_t = 0$, $\phi^M_t = 1 - \chi$ is constant.

### 3.2 Firms

The economy is characterized by an infinite number of firms indexed by $z$ on the unit interval $[0, 1]$. Each firm produces a differentiated variety with a constant return to scale technology,

$$Y_t(z) = N_t(z), \quad (14)$$

where $N_t(z)$ denotes the quantity of labor hired by firm $z$ in period $t$. Following Rotemberg (1982), we assume that firms face quadratic price adjustment costs $P_t(z) \left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2$ and $\theta \geq 0$. Nominal profits read as:

$$E_t \left\{ \sum_{i=0}^{\infty} Q_{t,t+i} \left[ P_{t+1}(z) Y_{t+1}(z) - W_{t+i} N_{t+i}(z) \right] - P_{t+i-2} \left( P_{t+i}(z) - 1 \right) \right\}, \quad (15)$$

where $Q_{t,t+i}$ is the discount factor in period $t$ for nominal profits $i$ periods ahead.

Assuming that firms discount at the same rate as savers implies $Q_{t,t+i} = \beta_s^{t} \frac{C_{s,t}}{C_{s,t+i} \pi_{t+i}}$, each firm faces the following demand function:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^d, \quad (16)$$

where $Y_t^d$ is aggregate demand and it is taken as given by any firm $z$. Cost minimization, taking the wage as given, implies that the real marginal cost is $w_t$. Firms choose processes $\{P_t(z), N_t(z), Y_t(z)\}_{t \geq 0}$ so as to maximize (15) subject to (14) and (16), taking as given aggregate prices and quantities $\{P_t, W_t, Y_t^d\}_{t \geq 0}$. Let the real marginal cost be denoted by

$$mc_t = w_t \quad (17)$$

Then, at a symmetric equilibrium where $P_t(z) = P_t$ for all $z \in [0, 1]$, profit maximization and the definition of the discount factor imply:

$$\pi_t (\pi_t - 1) = \beta E_t \left[ \frac{C_{s,t}}{C_{s,t+1}} \pi_{t+1} (\pi_{t+1} - 1) \right] + \frac{\epsilon N_t}{\gamma} \left( mc_t - \frac{\epsilon - 1}{\epsilon} \right), \quad (18)$$

where (18) is the standard Phillips curve according to which current inflation depends positively on future inflation and current marginal cost. The aggregate real profits are:

$$\Gamma_t = (1 - mc_t) Y_t - \frac{\gamma}{2} (\pi_t - 1)^2. \quad (19)$$
3.3 Fiscal Authority and Monetary Authority

The fiscal authority provides the public good \( G_t(z) \) for any \( z \in [0,1] \) and we aggregate them according to:

\[
G_t = \left[ \int_0^1 G_t(z) \frac{dz}{\bar{z}} \right]^{\frac{1}{\bar{z} - 1}}, \tag{20}
\]

so that total government expenditures in nominal terms is \( P_t G_t \) and the public demand of any variety is:

\[
G_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} G_t. \tag{21}
\]

Expenditures are financed by levying a lump-sum tax or by issuing one-period, risk-free, non state contingent nominal bonds. Hence, the fiscal authority’s period budget constraint is given by

\[
P_t G_t + B_{t-1}(1 + i_{t-1}) = (1 - \lambda) T_{s,t} + \lambda T_{b,t} + P_t S^G_t + B_t, \tag{22}
\]

where \( G_t \) and \( T_{i,t} \) denote government purchases and lump-sum taxes in nominal terms, \( B_t \) is the stock of one-period nominally riskless government debt issued in period \( t \) yielding a nominal return \( i_t \), and \( S^G_t \) denotes a real transfer from the central bank to the fiscal authority. Equivalently, and after letting \( b_t = B_t/P_t \) we can write:

\[
G_t + b_{t-1} \frac{(1 + i_{t-1})}{\pi_t} = (1 - \lambda) \tau_{s,t} + \lambda \tau_{b,t} + S^G_t + b_t. \tag{23}
\]

where \( \tau_{i,t} \) denotes lump-sum taxes (in real terms).

The central bank’s budget constraint is given by

\[
B^M_t + P_t S^G_t = B^M_{t-1}(1 + i_{t-1}) + \Delta M_t,
\]

where \( B^M_t \) denotes the central bank’s holdings of government debt at the end of period \( t \), and \( M_t \) is the quantity of money in circulation\(^4\). Equivalently, in real terms

\[
b^M_t + S^G_t = b^M_{t-1} \frac{(1 + i_{t-1})}{\pi_t} + \frac{\Delta M_t}{P_t}, \tag{24}
\]

where \( b^M_t \equiv B^M_t/P_t \) and \( \frac{\Delta M_t}{P_t} \) is the amount of seigniorage generated in period \( t \).

The amount of government debt held by households (expressed in real terms), and denoted by \( b^H_t \equiv B^H_t/P_t \), is given by

\[^4\text{The balance sheet of the central bank is given by} \]

\[
B^M_t = M_t.
\]
In what follows we often refer to $b_t^H$ as net government debt, for short. Combining (23), (24) and (25) one can derive the government’s consolidated budget constraint

$$G_t + b_t^H \left( \frac{1 + i_{t-1}}{\pi_t} \right) = (1 - \lambda) \tau_{s,t} + \lambda \tau_{b,t} + b_t^H + \frac{\Delta M_t}{P_t} \tag{26}$$

which may also be interpreted as a difference equation describing the evolution of net government debt over time. Below, following Galì (2017), we consider equilibria near a steady state with zero inflation, no trend growth, and constant government debt $b^H$, government purchases $G$, and taxes $\tau^i$. On the other hand, constancy of real balances requires that $\Delta M = 0$ in the steady state. It follows from (26) that

$$\tau_b = \frac{G + ib^H - (1 - \lambda) \tau_s}{\lambda}, \tag{27}$$

and

$$\tau_s = \frac{G + ib^H - \lambda \tau_b}{(1 - \lambda)}. \tag{28}$$

Note that (24) implies

$$S^G = ib^M, \tag{29}$$

i.e. in that steady state the central bank’s transfer to the fiscal authority equals the interest revenue generated by its holdings of government debt. In particular, the level of seigniorage (expressed as a fraction of steady state output) can be approximated as

$$\left( \frac{\Delta M_t}{P_t} \right) \left( \frac{1}{Y} \right) = \left( \frac{\Delta M_t}{M_{t-1}} \right) \left( \frac{M_{t-1}}{P_{t-1}} \right) \left( \frac{P_{t-1}}{P_t} \right) \left( \frac{1}{Y} \right) \approx \frac{1}{V} \Delta m_t \tag{30}$$

where $m_t = \log M_t$ and $V \equiv \frac{PY}{M}$ is the steady state income velocity of money. Equation (30) implies that the level of seigniorage is proportional to money growth up to a first order approximation.

Also, as in Galì (2016) $b_t^H = \frac{(b_t^H - b^H)}{Y}, \hat{g}_t = \frac{(G_t - G)}{Y}$ and $\hat{\tau}_{s,t} = \frac{\tau_{s,t} - \tau_s}{Y}$ are the deviations of net government debt, government purchases and taxes from their steady state values, expressed as a fraction of steady state output. Finally, assume that the fiscal authority implements the following feedback rule

---

\footnote{The constancy of the net government debt in the steady state implicitly assumes a tax rule designed to stabilize that variable about some target $b^H$.}
This tax rule is general enough to allow taxes on each agent to react to stabilize government debt ($\Phi_B = 0.02$ is the debt feedback coefficient). The plan prescribes a tax path that depends on public debt. We think that this is an interesting case, even though it is a simple one, because an endogenous tax response to public debt roughly agrees with the intentions declared by most of public debt-targeting governments.

Importantly, in our benchmark model the definition of a MFFS is different from the one used in Gali (2017), since the temporary increase in government purchases is entirely financed through money, and the fiscal authority does not fully stabilize public debt. The adoption of this definition is particularly relevant in a TANK model, since public debt reduces in real terms implying an additional redistributive effect, which amplifies the effects of the stimulus.

3.4 Money-Financed vs. Debt-Financed Fiscal Stimulus

We now define deviations of government purchases from that "normal" level $G$, as. $\bar{G}_t = G_t - G$. We refer to those deviations as "fiscal stimulus". Further, we assume that such fiscal stimulus, expressed as a fraction of steady state output and denoted by $\hat{g}_t \equiv \frac{(G_t - G)}{Y_t}$, follows the following exogenous AR(1) process

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g,$$  \hspace{1cm} (32)

where $\rho_g \in [0, 1]$ indexes the persistence of the government spending shock. Our baseline policy experiment consists of an increase in government purchases financed entirely through seigniorage. Formally,

$$\frac{\Delta M_t}{P_t} = \hat{G}_t,$$  \hspace{1cm} (33)

or, equivalently, using (30),

$$\Delta m_t = V \hat{g}_t,$$  \hspace{1cm} (34a)

i.e., the growth rate of the money supply is proportional to the fiscal stimulus, inheriting the latter’s exogeneity.

As an alternative to the benchmark MFFS, we consider an alternative MFFS (Gali, 2016) in which seigniorage is adjusted every period in order to keep real debt unchanged. In terms of the notation, it requires to replace equation (34a) with:

$$\hat{b}_t^H = 0$$  \hspace{1cm} (35)

As an alternative to the two types of MFFS, we analyze the effects of a DFFS in which the central bank follows a simple interest rate rule of the following type

$$\hat{\tau}_{t, t} = \Phi_B \hat{b}_t^H.$$  \hspace{1cm} (31)
\[
\log \left( \frac{1 + i_t}{1 + \hat{\pi}_t} \right) = \phi \hat{\pi}_t, \tag{36}
\]

where \( \hat{\pi}_t = \log \frac{\pi_t}{\pi} \) and \( \phi > 1 \) determines the strength of the central bank’s response of inflation deviations from the zero long-term target. Notice that, in contrast with the money-financing regime, \( \Delta m_t \) is no longer determined by \( \hat{g}_t \). The interest rate rule requires that the central bank injects or withdraws money from circulation by means of open market operations (in exchange for government debt) in order to accommodate whatever money is demanded by households at the targeted interest rate.

As discussed below, an interest rate rule like (36) gives the central bank a tight control over inflation in response to a fiscal stimulus, through its choice of coefficient \( \phi \). Yet, that tighter control comes at the price of a smaller impact of the fiscal stimulus on economic activity (i.e. a smaller "fiscal multiplier").

### 3.5 Equilibrium

The equilibrium allocation \( Y_t = C_t + G_t + \frac{\phi}{2}(\pi_t - 1)^2 \) is based on additional markets clearing conditions,

\[
C_t = \lambda C_{b,t} + (1 - \lambda)C_{s,t}; \tag{37}
\]

\[
M_t = \lambda M_{b,t} + (1 - \lambda)M_{s,t}; \tag{38}
\]

\[
N_t = \lambda N_{b,t} + (1 - \lambda)N_{s,t}; \tag{39}
\]

respectively, aggregate consumption, money market clearing condition and labor market clearing condition.

### 4 Model Dynamics and Welfare: Normal Times

This section is divided in four parts. First, it reports the quarterly calibration used. Second, it shows the IRFs to government spending shocks comparing the model dynamics implied by the benchmark MFFS with those implied by the other two financing schemes, that are the alternative MFFS and the DFFS. Then, it computes the implied fiscal multipliers and the welfare implication of the three regimes.

The Technical Appendix of this paper describes a TANK model under the assumption of an alternative non-competitive labor market. In this case, wage-setting decisions are made by labor type specific unions. Each union pools the labor income of agents, leading borrowers and savers to work for the same amount of time. This implies that the redistribution channel does not affects differently labor supply decision of the two consumers type, which is in line with the evidence that wealthy households do not work an amount of hours.
lower than that of poorer households. We show (Figures 9 – 13) that our results are robust and even reinforced under this alternative labor market.

4.1 Calibration

We solve the model by taking a first order approximation around the steady state. Before showing our results, we briefly describe the baseline calibration of the parameters. That calibration is summarized in the top panel of Table (1). We assume the following settings for the household related parameters in line to those of BMP (2013): discount factors of borrowers and savers are set respectively $\beta_b = 0.95$ and $\beta_s = 0.99$. Analogously, as in BMP, we set the borrowing constraint $D = 0.5$. Parameter $\lambda$, denoting the share of impatient agents, is set to 0.25.

The remaining parameters are kept at their baseline values, as in Gali (2017). We assume the elasticity of substitution among goods $\epsilon = 6$ and the curvature of labor disutility $\varphi = 1$. The model’s main frictions are given by price stickiness and market power in goods market. We assume a baseline setting of $\alpha = 0.75$, an average price duration of four quarters, a value consistent with much of the empirical micro and macro evidence. The focus on direct financing of the fiscal stimulus through money creation by the central bank calls for choosing the monetary base (M0) as that empirical counterpart. Average (quarterly) M0 income velocity in the U.S. over the 1960-2015 period is 3.6. The corresponding value for the euro area over the period 1999-2015 is 2.7. We take a middle ground and set $V = 3$ as the steady state inverse velocity in the baseline calibration. Further, we assume the following setting for the parameters related to money demand in utility function. The weight of real balances in utility function is set $\chi = 0.018$, in line with Annicchiarico et al. (2012). The specification of money demand implies a unitary long-run elasticity with respect to consumption. We impose a short run interest rate semi-elasticity of money demand equal to 2.5 (when expressed at an annual rate), in line with English et al. (2017). Finally, the money satiation level $\bar{x}$ is calibrated so that $\bar{x} > \frac{m}{C_s}$.

We calibrate the fiscal parameters so that the value of the tax adjustment parameter, $\Phi_B$, is set so that one-twentieth of the deviation from target in the debt ratio is corrected over four periods (i.e. one year), in the absence of further deficits. This requires $\Phi_B$ equal to 0.02. That calibration can be seen as a rough approximation to the fiscal adjustment speed required for euro area countries, as established by the so called "fiscal compact" adopted in 2012. In addition we assume the following fiscal policy settings: $\gamma = 0.2$ (steady state share of government purchases in output), $b^H = 2.4$ (corresponding to a 60 percent ratio of debt to annual output) and, for the persistence parameter $\rho_g$, we choose 0.5 as a baseline setting.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrower’s discount factor $\beta_b$</td>
<td>0.95</td>
</tr>
<tr>
<td>Saver’s discount factor $\beta_s$</td>
<td>0.99</td>
</tr>
<tr>
<td>Borrowing Constraint $D$</td>
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</tr>
<tr>
<td>Share of impatient agents $\lambda$</td>
<td>0.25</td>
</tr>
<tr>
<td>Weight of money in utility function $\chi$</td>
<td>0.018</td>
</tr>
<tr>
<td>Money satiation level $\bar{x}$</td>
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</tr>
<tr>
<td>short run interest rate semi-elasticity of money demand $\sigma$</td>
<td>2.5</td>
</tr>
<tr>
<td>Velocity (quarterly) $V$</td>
<td>3</td>
</tr>
<tr>
<td>Government spending share $\gamma$</td>
<td>1/5</td>
</tr>
<tr>
<td>Fiscal stimulus persistence $\rho_g$</td>
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</tr>
<tr>
<td>Steady state debt ratio (quarterly) $b^H$</td>
<td>2.4</td>
</tr>
<tr>
<td>Elasticity of substitution (goods) $\epsilon$</td>
<td>6</td>
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<tr>
<td>Elasticity of money demand to output $\eta$</td>
<td>7</td>
</tr>
<tr>
<td>Index of price rigidities $\alpha$</td>
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</tr>
<tr>
<td>Debt feedback coefficient $\Phi_B$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 1: Baseline Calibration

4.2 Money-Financed vs. Debt-Financed Fiscal Stimulus

Figures 1 and 2 compare the impulse response functions (IRFs) implied by our TANK model under the two types of MFFS (red solid line and pink dotted line) with those obtained under a DFFS accompanied by a monetary policy described by the simple interest rate Taylor rule (blue dotted line). In particular, the red solid line indicates a MFFS defined as an increase in government purchases entirely financed through the emission of new liquidity by the Central bank, with the real value of debt free to change consequently and tax changes corresponding to real debt changes according to a fiscal rule (31). We label this stimulus as our benchmark MFFS. The pink dotted line indicates instead a MFFS in which, though the increase in government purchases is entirely financed through money, the seigniorage is adjusted in each period to keep real debt unchanged. In other words, it avoids that inflation erodes the real value of the debt. We label this stimulus as our alternative MFFS. Figure 1 displays the IRFs of selected aggregate variables to a one percent increase in government purchases, while Figure 2 shows the redistributive effects of the alternative financing regimes. To quantify the importance of the redistribution channel in the transmission of the fiscal stimulus to the aggregate economy, Figure 3 compares the aggregate responses of output and consumption of our TANK model with those obtained in a RANK model\(^7\).

As shown in Figure 1, the initial shock consists in a one percent increase of government expenditure. First of all, notice that under the benchmark MFFS,\(^7\)

\(^7\)We consider an economy with a representative household which is equivalent to the saver in our benchmark TANK model.
the creation of new money strongly reduces the real value of public debt due to the increase in inflation that the injection of new liquidity provokes. On the contrary, by construction, under the alternative MFFS the real value of public debt remains unchanged. Notice that the benchmark MFFS implies an higher inflationary effect. The lower increase results from the behavior of the monetary authority which starts reducing money creation from the second period on in order to to keep real value of debt unchanged. As a consequence, the real interest rate decreases less under the alternative MFFS. The dynamics of inflation, together with that of the real interest rate and the real debt, is key to understand the stronger expansionary effect of the benchmark MFFS with respect to the alternative one and the role played by the redistribution channel. In both models, the decline in real rates brought about by the increase in inflation due to the injection of new liquidity implies a consumption crowding-in for both agents, though for the reasons stated above the crowding-in effect is much stronger under the benchmark MFFS. Finally, the gradualism in the price response, implied by staggered price setting amplifies the transmission mechanism of a MFFS. Remarkably and contrary to the common wisdom, in both cases a MFFS is followed by a moderate increase of inflation which is only temporarily higher than the target.

The blue dotted line shows the case in which the fiscal stimulus is financed through the issuance of public debt. In this case the Central Bank withdraws money from circulation in order to satisfy the reduction of the money demand due to higher targeted interest rate. The presence of an inflation targeting interest rate rule - which accompanies a DFFS - implies that the nominal interest rate increases more than one to one with inflation, leading to an increase in the real interest rate, that triggers a smaller real expansion than in a MFFS.

- Figure 1 about here -

Figure 2 underlines the redistributive effects of the fiscal policy under the different financing regimes. We can observe how the redistributive effects influence the effectiveness of the three financing schemes, particularly of our benchmark MFFS. First of all notice that, under the benchmark MFFS, the resulting increase in inflation erodes the real value of debt, with savers losing and borrowers gaining. The marginal value of one unit of debt decreases, inducing borrowers to consume more. Overall, borrowers’ consumption increases more than savers’ consumption. Remarkably, under the benchmark MFFS, the consumption ratio between the consumption of borrowers and that of savers is three times higher than under the alternative MFFS and the DFFS which, in turn, are more similar among them. This underlines the key role of inflation in affecting the redistribution channel which strongly amplifies the effects of the benchmark MFFS on aggregate consumption. As shown in Figure 3, the redistribution channel plays a key role in amplifying the effect of the stimulus with respect to a RANK

---

As in Galì (2016), our model implies an upward response of the nominal interest rate which suggests that the existence of a zero lower bound on that variable (whether currently binding or not) should not be an impediment to the implementation.
model. Indeed, the difference in consumption is of the order of 2 percentage points under the benchmark MFFS, while it is almost 0.5 percentage points under the alternative MFFS and under the DFFS. Differently from a MFFS, a DFFS implies a standard crowding-out effect on saver’s consumption. Also in this case the stimulus implies a redistribution from savers to borrowers (measured in terms of consumption ratio), so that such the redistribution channel amplifies also the effects of the DFFS on aggregate output. However, the lower consumption ratio implies a lower effect on aggregate consumption, and thus on aggregate output, than under our benchmark MFFS. The response of output is instead only slightly higher under the alternative MFFS than under the DFFS, both in the RANK and in the TANK model.

- Figure 2 about here -

Overall, our analysis confirms that the effects a MFFS, which leaves the real value of debt free to change in a NK monetary economy should be preferable to a DFFS and to a MFFS that keep the real value of debt unchanged, due to the much stronger redistributive effects implied by the baseline MFFS. Last but not the least, private debt results in a strong reduction. While the impact on inflation is very limited, a DFFS, accompanied by a simple interest rate rule, has instead the disadvantage of a smaller impact on aggregate activity, at the cost of higher debt.

- Figure 3 about here -

Our results on the dynamic effects of the two regimes in normal times can be summarized as follows.

Result 1: In normal times, all the regimes considered imply a redistributive effect from savers to borrowers. The redistribution is much larger under the benchmark MFFS, so that policy results highly effective and expansionary, though being only slightly inflationary. Last but not the least under the benchmark MFFS the real debt decreases. The redistribution channel significantly increases the effectiveness of the fiscal stimulus under all financing regimes with respect to a RANK model, however it is particularly strong under the benchmark MFFS.

4.3 Multipliers

In this section, we firstly compute the fiscal multipliers of output and consumption associated to an increase in government purchases, under the three alternative fiscal financing regimes. We will compute the multipliers changing \( \lambda \), corresponding to changing the fraction of borrowers in the economy. Then, in order to understand the role played by the financial constraint, we evaluate the same multipliers under different values of the borrowing limit, \( D \).

To differentiate between the immediate impact of a change in fiscal spending and its long-run implications for the economy, we compute both the instantaneous and the cumulative fiscal multiplier, following Uhlig (2010).
The Instantaneous Fiscal Multiplier (ICF, hereinafter) measures, in each period, the percentage deviation of a generic variable $X_t$ from its steady state in response to a change in government purchases that, on impact, amounts to one percent of the SS value of output. That is:

$$IFM(b) = \frac{x_t}{g_T}, \forall t \geq T$$

where $x_t = \frac{X_t - \overline{X}}{\overline{X}}$, $\overline{g_T} = \frac{G_T - \overline{G}}{\overline{G}}$ with $t$ being the time index for the periods following the initial fiscal shock in period $T$. $\overline{C}$ and $\overline{Y}$ are, respectively, the steady state values of government spending and output. In particular, we will consider the impact multipliers associated to $t = T$, and we refer to them as Impact Multipliers of $\hat{x}$.

As stressed by Uhlig (2010), policy makers cannot solely rely on the instantaneous multiplier since it can be misleading as it ignores the cumulated impact of the initial fiscal policy measure on the economy over time. Thus, in order to capture the cumulative impact on the variable of interest of the fiscal shock, we consider also the cumulative fiscal multiplier in analogy to Uhlig (2010).

The Cumulative Fiscal Multiplier (CFM, hereinafter) identifies, in each period, the discounted cumulative change of a variable $X_t$ measured in terms of percentage deviation from its steady state relative to the discounted cumulative change of government spending from its steady state value. That is

$$CFM(b) = \frac{\sum_{s=T}^{t} \frac{R^{-(s-t)}}{\overline{R}} x_s}{\sum_{s=T}^{t} \frac{R^{-(s-t)}}{\overline{R}} g_s}, \forall t \geq T$$

with $R$ being the steady state of the nominal interest rate used as discount rate.

Figure 4 shows the IFM and CFM for consumption and output, under the three financing schemes, as $\lambda$ changes from 0.1 to 0.45. As expected, consumption and output multipliers (both the impact and the cumulative ones) are much higher under the benchmark MFFS, being the monetary authority more accommodative. In this case, the multipliers of consumption and output are always greater than one. Further, they increase exponentially as the share of borrowers, $\lambda$, increases, particularly the impact multipliers. As expected under the alternative MFFS the multipliers are much lower, but they also increase with $\lambda$. Under a DFFS, the multipliers of output are always above one and they also increase with $\lambda$. The multipliers of consumption take a value, instead, larger than one only for values of $\lambda \geq 0.35$ for the IFM(C), and for $\lambda \geq 0.37$ in the case of CFM(C). Remarkably, notice that the multipliers associated to a DFFS become largely higher than that implied by a MFFS with debt stabilization for values of $\lambda$ larger than 0.4. As the share of borrowers increases the inflationary effect of both the stimulus becomes higher than under our baseline calibration with $\lambda = 0.25$. This, in turn, implies that from the second period on, the monetary authority has to increase the real rate by a much larger amount under the alternative MFFS than under a DFFS (which follows a standard Taylor rule).
to stabilize the real debt. As a consequence, from the second period two on the real interest rate becomes positive and higher than under a DFFS. This implies a lower effect on both savers and borrowers consumption.

-Figure 4 about here-

Finally, Figure 5 shows the CFM multipliers for three different values of the borrowing limit, that are $D = 0.1; D = 0.5; D = 1$, under the alternative financing schemes. Notice that, in all cases, fiscal multipliers increase as $D$ increases. By relaxing the borrowing constraint, the borrower’s consumption can increase more, and it generates higher fiscal multipliers.

-Figure 5 about here-

Our results on fiscal multipliers can be summarized as follows.

Result 2: Under the benchmark MFFS, consumption and output multipliers are largely higher than one. The benchmark MFFS implies higher fiscal multipliers than the DFFS and the alternative MFFS. Under all the financing schemes, fiscal multipliers are an increasing function of the share of borrowers $\lambda$, and of the borrowing limit, $D$. However, multipliers associated to DFFS become largely higher than that implied by the alternative MFFS for values of $\lambda$ larger than 0.4 and for sufficiently high value of $D$.

To further understand the contribution of the borrowing limit, we now compute the cumulative multipliers of output and consumption under the three regimes for $D$ increasing from 0.1 to 7 and for $\lambda$ respectively equal to 0.3 and 0.4. Figure 8 shows the results.

Notice that, overall, a DFFS implies multipliers higher than the other regimes only for values of $\lambda$ and $D$ larger than what reported by the empirical evidence.

4.4 Welfare

To assess the normative implications of the two stimula, we numerically evaluate the welfare. To do it, we solve the model using a second-order approximation of the structural equations, for each regime. We compute the individual welfare for savers and borrowers, respectively, as follows:

\begin{align*}
W_{s,t} &= (1 - \beta_s) E_t \sum_{k=0}^{\infty} \beta_s^k \left( \ln (C_{s,k}) - \frac{N_{s,k}^{1+\varphi}}{1 + \varphi} + \chi V (m_{s,t}) \right) \quad (42) \\
W_{b,t} &= (1 - \beta_b) E_t \sum_{k=0}^{\infty} \beta_b^k \left( \ln (C_{b,k}) - \frac{N_{b,k}^{1+\varphi}}{1 + \varphi} + \chi V (m_{b,t}) \right) \quad (43)
\end{align*}

Notice that we assume that real balances have a negligible weight in utility relative to consumption or employment, so that they do not affect welfare results. As in Mendicino and Pescatori (2007), Rubio and Carrasco-Gallego

\footnote{We do not want that welfare results on MFFS are driven by the presence of real balances in the utility function. In respect of it, our assumption is conservative.}
(2014), Forlati and Lambertini (2014) and Notarpietro et al. (2015), among others, it implies that each type of agent receives the same level of utility from a constant consumption stream. The social welfare is, then, a weighted sum of the individual welfare of each different kind of household

\[ W_t = \omega_s W_{s,t} + \omega_b W_{b,t} \]  

(44)

where \( \omega_s = 1 - \lambda \) and \( \omega_b = \lambda \). Given that the utility function is not cardinal, a welfare measure based on the value function is not revealing. For this reason, we convert the welfare measures in consumption equivalent units, defined as the constant fraction of consumption under a MFFS, that households should give away in each period to equate the value function under a DFFS.\(^{10}\) Then, whenever a MFFS implies welfare gains, households would be willing to pay in consumption units for the policy to be implemented. The derivation of the welfare equivalent units for savers and borrowers implies that:

\[
CE_s = \exp \left[ (1 - \beta_s) \left( W_{s,t}^{MFFS} - W_{s,t}^{DFFS} \right) \right] - 1
\]

(45)

\[
CE_b = \exp \left[ (1 - \beta_b) \left( W_{b,t}^{MFFS} - W_{b,t}^{DFFS} \right) \right] - 1
\]

(46)

are respectively consumption equivalent measures of savers and borrowers, where the superscripts in the welfare values denote the benchmark MFFS (and the alternative MFFS) and the DFFS. Positive values of consumption equivalent welfare imply that the agent is better under a MFFS than a DFFS, whereas negative values of consumption equivalent imply that the agent is worse off under a MFFS than under a DFFS. The consumption equivalent derived from the social welfare is instead given by,

\[
CE = \exp \left( W_t^{MFFS} - W_t^{DFFS} \right) - 1
\]

(47)

thus, a positive value of \( CE \) implies that a MFFS is preferable in terms of aggregate welfare than a DFFS, whereas a negative value of \( CE \) means that aggregate welfare is higher under a DFFS than under a MFFS.

Table 2 shows the values of the consumption equivalent welfare implied by the benchmark MFFS with respect to a DFFS. In particular, it shows the two agents consumption equivalent measures \( CE_b \) and \( CE_s \) and that of aggregate consumption equivalent welfare, \( CE \), in percentage terms. We compute these measures under the three different values of the borrowing constraint parameter, \( D \), that is \( D = 0.1; 0.5; 1 \).

<table>
<thead>
<tr>
<th>Borrowing Limit</th>
<th>( CE_b )</th>
<th>( CE_s )</th>
<th>( CE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D = 0.1 )</td>
<td>2.7862</td>
<td>-0.4460</td>
<td>0.3524</td>
</tr>
<tr>
<td>( D = 0.5 )</td>
<td>2.8979</td>
<td>-0.4635</td>
<td>0.3664</td>
</tr>
<tr>
<td>( D = 1 )</td>
<td>3.0162</td>
<td>-0.4858</td>
<td>0.3784</td>
</tr>
</tbody>
</table>

\(^{10}\)Notice that the MFFS and the DFFS are characterized by the same steady state and therefore the two stimuli are comparable in terms of consumption equivalent units.
Notice that, in all the three cases considered, borrowers get welfare gains, in terms of consumption equivalent measures, when the fiscal stimulus is accompanied by money injection. On the contrary, savers incur in welfare losses. This means that borrowers are better off under our benchmark MFFS than under a DFFS, while savers are worse off. Borrowers consumption equivalent is around 2.9%, whereas that of savers is negative and close to -0.46% in the benchmark model with $D = 0.5$. Consumption equivalent units derived from the social welfare are positive, implying welfare gains from the MFFS of about 0.37% in terms of consumption equivalent measure. Remarkably, notice that when the borrowing limit passes from $D = 0.1$ to the baseline calibration of $D = 0.5$ and to $D = 1$, meaning that borrowers are less financially constrained, both the welfare gains for borrowers and the welfare losses for savers increase and also aggregate welfare gets higher\textsuperscript{11}.

Our results on welfare can be summarized as follows.

**Result 3:** In terms of consumption equivalent welfare, borrowers are better off while savers are worse off under a MFFS than under a DFFS. The aggregate welfare is higher under the benchmark MFFS than under the DFFS.

To better understand the role played by the redistribution channel, we now compare the consumption equivalent welfare implied by our baseline TANK model with those implied by a RANK model characterized by the same stimulus. Table 3 reports the results obtained using our benchmark MFFS (first column) and those of a MFFS accompanied by a fiscal authority that, as in Galí (2017), fully stabilizes real debt, that where $b^H_t = 0$ (second column). In both cases the value of the borrowing limit is set to 0.5 as in our baseline calibration. Remarkably, notice that our benchmark MFFS is preferable in terms of aggregate welfare only in the TANK model. In the RANK model a MFFS is welfare detrimental, unless the monetary contemporary stabilize the real debt by increasing the real interest rate from the second period on.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark MFFS, $D = 0.5$</th>
<th>MFFS with $b^H_t = 0$, $D = 0.5$</th>
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</thead>
<tbody>
<tr>
<td>$CE_{TANK}$</td>
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<td>0.0463</td>
</tr>
<tr>
<td>$CE_b$</td>
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<td>0.1994</td>
</tr>
<tr>
<td>$CE_s$</td>
<td>-0.4635</td>
<td>-0.0047</td>
</tr>
<tr>
<td>$CE_{RANK}$</td>
<td>-14.1443</td>
<td>0.8198</td>
</tr>
</tbody>
</table>

The intuition behind this result is the following. Under the benchmark model, public debt decreases in real terms when the stimulus is money financed through the benchmark MFFS. This is due to the inflationary effect of the policy that reduces the real value of debt. As a consequence, savers which are the owners

---

\textsuperscript{11} The values of $D$ equal to 0.1, 0.5 and 1 correspond respectively to SS borrowing to borrower’s labor income ratios, $\frac{D}{\pi N_b}$, equal to 0.1, 0.5 and 1.
of public assets are worse off. Also, the real value of private debt reduces as inflation increases, so that borrowers are better off. This brings about a positive wealth effect on borrowers and a negative one on savers, which reflects into a larger increase in borrowers consumption relatively to that of savers. Despite this, savers consumption increases under a MFFS while it decreases under a DFFS. The positive reaction of savers consumption is due to the decrease of the real interest rate brought about by monetary injection and the increased aggregate demand. In the RANK model all agents are savers and thus the crowding-in effect on consumption is still present though lower than in a TANK model, where aggregate consumption mostly increase because of the presence of borrowers. Further, the RANK model is also characterized by a negative effect on welfare triggered by the labor supply. Savers labor supply decreases in the DFFS by a larger amount than under a MFFS. The overall effect implies a higher increase in welfare under a DFFS than under a MFFS and the consumption equivalent welfare is then negative.

In the TANK model the positive effects on borrowers welfare dominate the negative ones on savers and the benchmark MFFS is preferable in terms of aggregate welfare than the DFFS. If instead the monetary authority fully stabilizes the public debt savers are better off under a MFFS than under a DFFS, both in the RANK and in the TANK model. In this case, in fact, the negative wealth effect of the debt are absent.

To sum up, we can state the following.

**Result 4:** The benchmark MFFS is preferable in terms of aggregate welfare only in the benchmark TANK model. When the MFFS is accompanied by a monetary authority which fully stabilizes the debt a MFFS implies an higher welfare than a DFFS both in the RANK and in the TANK model.

5 Model Dynamics in a ZLB scenario

In this section, as in Galì (2017), we analyze the effectiveness of the fiscal stimulus in stabilizing the economy in the face of an adverse demand shock, under both a money-financed and a debt-financed regime. The shock is large enough to push the nominal interest rate on the ZLB. The ZLB constraint can be incorporated formally in the set of the log-linearized equilibrium conditions above by replacing savers’ money demand with

\[
\left(\hat{i}_t - \log (\beta)\right) \left(\hat{m}_{s,t} - \hat{C}_{s,t} + \hat{\eta}_t\right) = 0
\]

(48)

for all \(t\), where

\[
\hat{i}_t \geq \log (\beta)
\]

(49)

is the nominal interest rate bounded to zero and

\[
\hat{m}_{s,t} \geq \hat{C}_{s,t} + \hat{\eta}_t
\]
represents savers’ demand for real balances.

In addition, in the case of debt financing, condition (36) must be replaced with:

\[
\left( \hat{i}_t - \log (\beta^*) \right) \left( \hat{i}_t - \phi_\pi \left( \log \left( \frac{\pi_t}{\pi} \right) \right) \right) = 0
\]

for all \( t \), which guarantees that the Taylor rule is met as long as the ZLB constraint is not binding.

Next we analyze several scenarios, each defined by a specific combination of monetary and fiscal policy responses to the demand shock. In particular, as in Galì (2017), we assume that the demand shock \( \hat{\varepsilon}_t = \varepsilon_0 \) for \( t = 0, 1, 2, \ldots T \) and \( \hat{\varepsilon}_t = 0 \) for \( t = T + 1, T + 2, \ldots \). This can be interpreted as a temporary adverse demand shock that causes the drop of the natural interest rate up to period \( T \), vanishing thereafter. In particular, we assume \( \gamma = 0.06 \) and \( T = 5 \). We assume that the shock is unanticipated, however once it is realized, the trajectory of the shock and corresponding policy responses are known with certainty.

We start by considering the benchmark case of no fiscal response to the shock (i.e. \( \hat{\gamma}_t = 0 \), for \( t = 0, 1, 2, \ldots \)) with the central bank implementing the Taylor rule subject to the ZLB constraint. As discussed in Galì (2015, chapter 5) and Galì (2017), that policy lowers the nominal rate to zero for the duration of the shock (i.e. up to \( T \)), and then it reverts back to a simple stabilizing rule\(^{12}\).

Formally,

\[
\hat{i}_t = \max \left[ \log (\beta^*), \hat{\rho}_t + \phi_\pi \left( \log \left( \frac{\pi_t}{\pi} \right) \right) \right]
\]

where \( \phi_\pi > 1 \).

- Figures 6 about here -

Figure 6 shows the responses of the economy to the adverse demand shock. Under all regimes, public expenditure increases by 1 percent and the fiscal stimulus lasts for the duration of the adverse demand shock.

Figure 6 shows that the benchmark MFFS is the most effective only in normal times. In a ZLB scenario instead the benchmark MFFS redistributes from borrowers to savers and implies a strong recession, though less persistent than under the DFFS. On the contrary, the alternative MFFS becomes the most effective financing regime in a ZLB scenario. It redistributes from savers to borrowers and, by sustaining higher level of inflation, is able to avoid the economy to enter into a recession. The intuition for this last result is rather simple. An adverse demand shock pushes the economy into a ZLB scenario. As a consequence of the negative demand shock, as shown in Figure 6, the real GDP falls down bringing about an increase of the real value of public

\(^{12}\)See, e.g. Eggertsson and Woodford (2003). Following Galì (2017), our goal is to characterize the effect of different fiscal interventions. The case of discretionary monetary policy in the absence of fiscal response is just a useful benchmark with respect to which we measure the effectiveness of different fiscal interventions.
debt, not only under a DFFS, but also under the benchmark MFFS. Thus, savers who are the owners of the public debt are better off, while borrowers are worse off. Though the real interest rate falls down under the benchmark MFFS, it is not sufficient to bring about a consumption crowding-in. Both consumption of borrowers and savers decreases on impact and the policy results less effective than in normal times. On the contrary, the alternative MFFS is much more effective in ZLB than in normal times. In fact, under this alternative regime the seigniorage is adjusted to keep unchanged the real value of public debt, by generating higher inflation than in the benchmark MFFS. The stronger inflationary effect of the alternative MFFS pushes the real interest rate down more than under the benchmark one and the policy redistributes from savers to borrowers, so that borrowers consumption increases and so does aggregate consumption (as shown in Figure 7). As a result, the policy pushes up the aggregate demand and results so effective to avoid the economy to enter into a recession.

Finally, Figure 8 shows the responses of output and consumption under the three financing schemes compared to those of a RANK model. It is rather important to notice that the strong effectiveness of the alternative MFFS does not hold in a RANK model, where the redistributive channel is absent. Again, this suggests that the redistributive channel is very important and that it cannot be neglected by policy makers.

To sum up, we can state the following.

**Result 5:** The alternative MFFS is the most effective regime in a ZLB scenario. In a TANK model this regime is so effective to avoid the recessionary effects implied by the ZLB. Remarkably, this result does not hold in a RANK model, where the redistributive channel is absent.

6 Conclusion

By considering a Two-Agents New-Keynesian model in a Borrower-Saver framework, this paper argues that redistribution is a key channel through which the fiscal financing regimes affect the macroeconomic aggregates. In particular, it analyzes the redistributive channel of a MFFS compared to that of a DFFS. The idea that a future inflation tax can reduce the ability of money to stimulate the aggregate demand has been central in the economic debate that followed the recent crisis. However, in a borrowers-savers model, the inflation tax can be used to redistribute from one household to the other one further amplifying or reducing the effect of the stimulus. For this reason, in this paper we consider two types of MFFS: if the seigniorage is adjusted in each period to keep unchanged the real value of public debt and to avoid a prolonged inflationary effect of the policy (Galì, 2017) or not.

We show that the effectiveness of a MFFS is state dependent - due to the redistribution channel. In normal times, the most effective regime is a MFFS with no additional intervention by the Central Bank to stabilize real public debt.
using inflation, whereas in a ZLB scenario, a MFFS accompanied by real debt stabilization - through the adjustment of seigniorage - is the most effective one.

The contribution of the paper can be summarized in five main results.

First, in normal times, all the regimes imply a redistributive effect from savers to borrowers. However, thanks to the accommodative monetary policy, borrowers gains are much larger under our a MFFS without public debt stabilization. The consumption ratio between borrowers and savers is indeed five times higher under the MFFS without real public debt stabilization than under a DFFS, so that the first one is followed by a stronger expansionary effect on output than the latter one. As a consequence, the redistribution channel strongly amplifies the impact of the fiscal intervention with respect to a RANK model, increasing the effectiveness of the MFFS. Public debt stabilization instead only mildly amplifies the effect of a MFFS with respect to a DFFS. In this case, the transmission mechanism is indeed less effective since the redistributive channel of public bonds and that of inflation are muted to stabilize the real value of the debt.

Second, we compute consumption and output fiscal multipliers associated to the three different financing regimes and we find that, under our benchmark MFFS fiscal multipliers are largely higher than one. Interestingly, these multipliers increase exponentially as the share of borrowers increases. Also, we find that fiscal multipliers are an increasing function of the borrowing limit.

Third, our benchmark MFFS is preferable in a TANK model since the expansionary effects of government spending are larger, thanks to very high fiscal multipliers. Fourth, in a RANK model it is instead welfare detrimental with respect to a DFFS.

Fifth, while in normal times, the most effective regime is the benchmark MFFS, in a ZLB scenario the most effective one is a MFFS accompanied by the stabilization of real public debt - through the adjustment of seigniorage. In a TANK model this regime is so effective to avoid the recessionary effects implied by the ZLB. Remarkably, this result does not hold in a RANK model, where the redistributive channel is absent.

Finally, we show that contrary to the common wisdom a MFFS is followed by a moderate increase of inflation which is only temporarily higher than the target. This suggests that the redistribution channel cannot be neglected by policy makers.

References


A The money demand for borrowers

We now show that borrowers’ optimal money demand is always zero. Suppose that $M_{b,t} > 0$, in this case borrowers will consume less today, implying a loss in utility of consumption and a gain in terms of utility of money balances today. Money saved today will be consumed tomorrow. In other words, borrowers will choose $M_{b,t} > 0$ if and only if:

$$-U^{C_{t}} + U_{m_{t}} + \beta_b U^{C_{t+1}} = 0$$  \hspace{1cm} (51)

which can be rewritten as

$$U_{m_{t}} \big|_{m_{t} > 0} = U^{C_{t}} - \beta_b U^{C_{t+1}}$$  \hspace{1cm} (52)

This means that borrowers will optimally choose $M_{t}^b = 0$, every time

$$U_{m_{t}} \big|_{m_{t} = 0} < U^{C_{t}} - \beta_b U^{C_{t+1}}$$  \hspace{1cm} (53)

Further, since the borrower is constrained in terms of bond, we know from the Euler equation that

$$U^{C_{t}} = (1 + i_t) \beta_b U^{C_{t+1}} + \phi_t,$$  \hspace{1cm} (54)

and thus that

$$\beta_b U^{C_{t+1}} = \frac{U^{C_{t}}}{1 + i_t} - \frac{\phi_t}{1 + i_t}$$  \hspace{1cm} (55)

which can be substituted into (53) implying that

$$U_{m_{t}} \big|_{m_{t} = 0} < \frac{i_t}{1 + i_t} + \frac{\phi_t}{1 + i_t}$$  \hspace{1cm} (56)

Now, remember that if $M_{t}^b = 0$, the constraint on money demand is binding and then

$$\frac{U_{m_{t}}}{U^{C_{t}}} = \left( \frac{i_t}{1 + i_t} \right) - \phi_t M_{t}^b$$  \hspace{1cm} or

$$U_{m_{t}} = \left( \frac{i_t}{1 + i_t} \right) U^{C_{t}} - \phi_t M_{t}^b U^{C_{t}}$$  \hspace{1cm} (58)

equation (58) can be substituted into (56) so that

$$U_{m_{t}} \big|_{m_{t} = 0} = \left( \frac{i_t}{1 + i_t} \right) U^{C_{t}} - \phi_t M_{t}^b U^{C_{t}} < \frac{i_t}{1 + i_t} + \frac{\phi_t}{1 + i_t}$$  \hspace{1cm} (59)

simplifying

$$-\phi_t M_{t}^b U^{C_{t}} < \frac{\phi_t}{1 + i_t}$$  \hspace{1cm} (60)
Notice that since the borrowing constraint is always binding \( \phi_t > 0 \) for each \( t \), since with \( M_b^t = 0 \), \( \phi_t^{M'} > 0 \) this equation is always satisfied, implying that borrowers do not save either in terms of bonds nor in terms of money.

Similarly we now show that savers will always choose a demand for money with \( M_s^t > 0 \) and thus implying that \( \phi_t^{M'} = 0 \). Since \( \phi_t = 0 \) for savers, equation (59) for savers will be:

\[
U_{m_s^t}|_{m_s^t > 0} = \left( \frac{i_t}{1 + i_t} \right) U_{C_s^t} = U_{C_s^t} \frac{i_t}{1 + i_t}
\]

which is always satisfied.

B Robustness

B.1 Alternative Labor Market Structures

B.1.1 Households

There is a continuum of households indexed by \( i \in [0, 1] \). Households in the interval \([0, \lambda]\) consume their available labor income and borrow in each period. Households in the interval \((\lambda, 1]\) hold assets and smooth consumption. The period utility function is common across households and it has the following separable form:

\[
U_t = \ln (C_{i,t}) - \chi \frac{\left( \frac{m_{i,t}}{\bar{m}_{i,t}} \right)^{1+\sigma}}{1 + \sigma} \frac{N_{i,t}^{1+\phi}}{1 + \phi}.
\]

We assume a continuum of differentiated labor inputs indexed by \( j \in [0, 1] \). As in Schmitt-Grohe and Uribe (2006), agent \( i \) supplies each possible type of labor input. Wage-setting decisions are made by labor type specific unions indexed by \( j \in [0, 1] \). Given the wage \( W_j^t \) fixed by union \( j \), agents stand ready to supply as many hours to the labor market \( j \), \( N_j^t \), as required by firms, that is: \( N_j^t = \left( \frac{W_j^t}{W_t} \right)^{-\theta_e} N_i^t \), where \( \theta_e \) is the elasticity of substitution between labor inputs. Here \( N_i^t \) is aggregate labor demand and \( W_t \) is an index of the wages prevailing in the economy at time \( t \). Formal definitions of labor demand and of the wage index can be found in the section devoted to firms. Agents are distributed uniformly across unions; hence aggregate demand for labor type \( j \) is spread uniformly across the households.\(^{13}\) It follows that the individual quantity of hours worked, \( N_t \), is common across households, and we denote it as \( N_t \).

This must satisfy the time resource constraint \( N_t = \int_0^1 N_j^t dj \). Combining the latter with labor demand we obtain \( N_t = \int_0^1 \left( \frac{W_j^t}{W_t} \right)^{-\theta_e} \). The labor market structure rules out differences in labor income between households without the

\(^{13}\)Thus a share \( \lambda \) of the members of each union are borrowers, while the remaining portion is composed of Ricardian agents.
need to resort to contingent markets for hours. The common labor income is
given by \( L^d \int_0^t W_t^j \left( \frac{W_t}{W_t} \right)^{-\theta_w} dj \). Notice that each union pools the labor income
of agents, leading borrowers and savers to work for the same amount of time.

**Savers** Savers face the following flow budget constraint in nominal terms:

\[
P_t C_{s,t} + B_{s,t}^H + P_t A_{s,t} + M_{s,t} + \Omega_{s,t} P_t V_t \leq (1 + \delta_{t-1}) B_{s,t-1}^H + (1 + \delta_{t-1}) P_t A_{s,t} + M_{s,t-1} + \Omega_{s,t-1} P_t (V_{t-1} + \Gamma_t) + N_t^d \int_0^1 N_t^j \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} - T_{s,t}.
\]

A saver has labor income \( N_t^d \int_0^1 W_t^j \left( \frac{W_t}{W_t^j} \right)^{-\theta_w} dj \). \( A_{s,t-1} \) is the real value
at beginning of period \( t \) of total private assets held in period \( t \), a portfolio of one-period bonds issued in \( t - 1 \) on which the household receives the nominal interest \( \delta_{t-1} \). \( V_{t-1} \) is the real market value at time \( t \) of shares in intermediate
good firms, \( \Gamma_t \) are real dividend payoffs of these shares, \( \Omega_{s,t} \) are share holdings,
\( T_{s,t} \) is the lump-sum tax, \( B_{s,t}^H \) are the savers’ holdings of nominally riskless one-
period government bonds (paying an interest \( \delta_t \)). The nominal debt \( B_{s,t}^H \) pays
one unit in nominal terms in period \( t + 1 \).

After defining the aggregate price level as \( P_t = \left( \int_0^1 P_t (z)^{1-\tau} dz \right)^{\frac{1}{1-\tau}} \), as
well as real debt as, \( b_t^H = B_t/P_t \), optimality is characterized by the following
first-order conditions for savers:

\[
\beta_s E_t \left\{ \frac{C_{s,t} (1 + \delta_t)}{C_{s,t+1}^s \pi_{t+1}} \right\} = 1, \quad (64)
\]
\[
\beta_s E_t \left\{ \frac{C_{s,t} V_{t+1} + \Gamma_{t+1}}{C_{s,t+1}^s} \right\} = 1, \quad (65)
\]
\[
\chi \left( \pi - \frac{m_{s,t}}{C_{s,t}} \right)^\sigma = \left( \frac{\delta_t}{1 + \delta_t} \right). \quad (66)
\]

**Borrowers** Borrowers (indicated with the subscript \( b \)) face a borrowing con-
straint at all times. They face the following budget constraint:

\[
P_t C_{b,t} + P_t A_{b,t} + M_{b,t} = (1 + \delta_{t-1}) P_t A_{b,t-1} + M_{b,t-1} + N_t^d \int_0^1 W_t^j \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} dj - T_{b,t}, \quad (67)
\]

and, the borrowing constraint (on borrowing in real terms) at all times \( t \):

\[-A_{b,t} \leq \bar{D}\]
the usual \( C_{b,t} \geq 0 \) and

\[ M_{b,t} \geq 0. \]

Agents belonging to this group consume disposable income and borrow in each period and delegate wage decisions to unions. For these reasons there is no first order condition with respect to labor supply but optimality is characterized by the first-order conditions:

\[ C_{b,t}^{-1} = \beta_b E_t \left( \frac{1 + \delta_t}{\pi_{t+1}} C_{b,t+1}^{-1} \right) + \phi_t \]

and,

\[ M_{b,t} = 0. \]

B.1.2 Wage Setting

As in Galì and Lopez-Salido (2007) the nominal wage newly reset at \( t \), \( W_t \), is chosen to maximize a weighted average of agents’ lifetime utilities. The weights attached to the utilities of savers and borrowers are \((1 - \lambda)\) and \( \lambda \), respectively. The union problem is

\[
\max_{W_t} \left\{ [\lambda \ln(C_{s,t}) + \lambda \ln(C_{b,t})] - \lambda M^t_{s,t} \right\} 
\]

subject to \( N_t = \int_0^1 N_t^j dj \), (63) and (67). The FOC with respect to \( W_t \) is

\[
\left[ \frac{\lambda}{MRS_{b,t}} + (1 - \lambda) \frac{1}{MRS_{s,t}} \right] W_t \cdot \frac{P_t}{P_t} - \mu^w \cdot 0
\]

or

\[
\frac{W_t}{P_t} = \mu^w \left[ \frac{\lambda}{C^t_{b,L}^{\prime}} + (1 - \lambda) \frac{1}{C^t_{s,L}^{\prime}} \right]^{-1}
\]

substituting for \( MRS_{b,t} = C^t_{b,L} N^t_{l} \) and \( MRS_{s,t} = C^t_{s,L} N^t_{l} \).

B.1.3 Aggregation and Market Clearing

The equilibrium allocation \( Y_t = C_t + G_t + \frac{\delta_t}{2} (\pi_t - 1)^2 \) is based on additional markets clearing conditions,

\[ C_t = \lambda C_{b,t} + (1 - \lambda) C_{s,t}; \]

\[ M_t = \lambda M_{b,t} + (1 - \lambda) M_{s,t}; \]

respectively, aggregate consumption and money market clearing condition.
The clearing of labor markets requires:

\[ N^d_t = \left( \frac{W^t_j}{W^t} \right)^{-\theta_w} N^d_t \quad \forall j \quad N_t = \int_0^1 N^j_t \ dj \]  

(71)

where \( Y^d_t = C_t \) represents aggregate demand, \( N^d_t = \int_0^1 N^j_t \ dz \) is total aggregate demand of labor input \( j \) and \( N_t = \int_0^1 N_t \ (z) \ dz \) denotes firms’ aggregate demand of the composite labor input \( N_t \).

Figures 9-13 compare the impulse response functions (IRFs) implied by our TANK model in presence of wage-setting by unions under a MFFS with those obtained under a DFFS accompanied by a monetary policy described by the simple interest rate Taylor rule. As shown, there are no qualitative differences with respect to the benchmark model neither at aggregate level nor at disaggregate level. However, the presence of unions amplifies the redistribution channel from savers to borrowers, leading to a larger expansion under both regimes. This is due to the fact that the income effect on borrowers cannot be absorbed by decreasing their labor supply, but only by their consumption boom. And, it explains also why, under both regimes, the expansionary effect of a fiscal stimulus in a TANK model is larger than in a RANK one, as Figures 11 shows. Finally Figures 12 ad 13 show the implied fiscal multipliers. Again, the results of the baseline model are reinforced in a unionized labor market economy.

**Figures**

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![Figure 1: The effects of a Fiscal Stimulus](image-url)
Figure 2: The redistributive effects of a Fiscal Stimulus

Figure 3: RANK vs TANK in Normal times
Figure 4. Fiscal Multipliers of Output and Consumption.

Figure 5. Cumulative Fiscal Multipliers: changing the borrowing limit.
Figure 6: Dynamic Effects of an increase of government expenditure in a liquidity trap

Figure 7: Redistributive Effects of an increase of government expenditure in a liquidity trap
Figure 8: RANK vs TANK in a ZLB scenario

Figure 9: The effects of a Fiscal Stimulus (in presence of unions)
Figure 10: The redistributive effects of a Fiscal Stimulus (in presence of unions)

Figure 11: RANK vs TANK (in presence of unions)
Figure 12: Fiscal Multipliers of Output and Consumption (in presence of unions)

Figure 13: Cumulative Fiscal Multipliers: changing the borrowing limit (in presence of unions)