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Abstract

The paper examines the relationships among market assets during stressful times, using two recently proposed econometric modeling techniques for tail risk measurement: the extreme downside hedge (EDH) and the extreme downside correlation (EDC). We extend both measures taking into account the sensitivity of asset's return to innovations not only from the overall market index, but also from its components, by means of network modelling. Applying our proposal to the cryptocurrencies market, we find that crypto-assets can be clustered in two groups: speculative assets, such as Bitcoin, which are mainly "givers" of tail contagion; and technical assets, such as Ethereum, which are mainly "receivers" of contagion.

Keywords: Crypto-assets, Extreme downside hedge, Extreme downside correlation, Network Models, Systematic risk, Systemic risk.

JEL: C31, C58, G01, G12

1. Introduction

Estimating risks is important to achieve the best investment decisions. Typically, individuals consider a trade-off between expected return and risk in investment decisions (Bera and Kannan, 1986; Puspitaningtyas, 2018; Scott and O'Brien, 2003). Within the context of market investments, the risk of a portfolio is usually measured by estimating their return sensitivity to risk factors, through the market beta coefficient. However, there is a wide consensus that the relationship between asset returns and market risk varies, and depends on market conditions. For example, a stronger correlation could be exhibited by asset returns during volatile periods, and especially in the case of extreme market downturns (see Ang and Chen, 2002; King and Wadhwani, 1990; Longin and Solnik, 2001).

In light of this observation, Kadan et al. (2016) generalized the concept of systematic risk to a broader class of risk measures. They proposed an equilibrium framework that generalizes the Capital Asset Pricing Model, and an axiomatic approach which leads to a systematic risk measure as the unique solution to a risk allocation problem. Both approaches extend the traditional market beta to capture the multiple dimensions of risk. Systematic market factors are not the only cause of return volatilities. Especially after the recent financial crisis, researchers have understood the importance of systemic risk - inherent vulnerability of the financial system that propagates initial shocks to leading to the failure of many institutions, whose cascading effects may endanger the whole system (see Acemoglu et al., 2015; Battiston

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et al., 2012; Billio et al., 2012; Diebold and Yilmaz, 2014; Elliott et al., 2014; Härdle et al., 2016; Ladley, 2013). Systemic risk can be thought of as a widespread failure of financial institutions or as a freezing up of capital markets, which can substantially reduce the supply of such critical intermediation. Failures of financial institutions or capital markets can have an important externality on the rest of the economy, and the recent financial crisis provides ample evidence of the importance of containing systemic risks.

The available definitions of systemic risk focus on different aspects of the phenomenon, such as imbalances, collapse of confidence, correlated exposures of financial institutions, negative impact on the real economy, information asymmetry, asset bubbles, contagion, and negative externalities. For a comprehensive review, see Ahelegbey (2016); Bisias et al. (2012); Brunnermeier and Oehmke (2013); De Bandt and Hartmann (2000); Eijffinger (2011); Oosterloo and de Haan (2003). From a regulatory viewpoint, widespread financial regulations, such as Basel I and Basel II, are designed to limit financial risk (market, credit and operational risk) seen in isolation; they are not sufficiently focused on systemic risk. This even though the systemic risk is often the rationale provided for such regulation Acharya et al. (2017). Basel III attempts to include systemic risk, but it does so to a limited extent. This may be due to a lack of consensus in the systemic risk literature.

The limited consensus on the definition of systemic risk is reflected in a large number of measurement methods available. Among them are Banking System's (Portfolio) Multivariate Density (BSPMD; Segoviano and Goodhart, 2009), conditional value-at-risk (CoVaR; Adrian and Brunnermeier, 2016), absorption ratio (AR; Kritzman et al., 2011), marginal expected shortfall (MES; Acharya et al., 2017), distressed insurance premium (DIP; Huang et al., 2012), dynamic causality index with principal component analysis systemic risk measures (DCI, PCAS; Billio et al., 2012), network connectedness measures (NCMs; Billio et al., 2012; Diebold and Yilmaz, 2014). Other recent contributions include Wang et al. (2019), who proposed a new measure of systemic risk named CSRISK, which identifies a financial institution's capital shortfall under the worst scenario, conditional on a substantial market decline; Brunnermeier and Cheridito (2019) who developed a framework for measuring systemic risk, SystRisk, that captures the a priori cost to society for providing tail-risk insurance to the financial system; Bianchi et al. (2019) who developed a scheme in which latent states are identified based on a novel weighted eigenvector centrality measure; Brownlees and Engle (2017) who introduced SRISK, which measures the capital shortfall of a firm, conditional on a severe market decline and is a function of size, leverage, and risk of the firm itself.

Another cause of return volatilities, additional to systematic and systemic risk, is tail risk. The importance of tail risk in financial markets has been highlighted because of the turbulence of financial markets over the last years and, in particular, in crypto-asset markets. Many studies have documented the considerable impact of this risk on expected returns, see for instance Barro (2006); Gabaix (2012); Gillman et al. (2015); Rietz (1988); Wachter (2013). From an econometric viewpoint, Harris et al. (2019) proposed two complementary measures of systematic tail risk and showed that the first measure, named extreme downside correlation (EDC), is based on the tendency of asset returns to crash at the same time as the market, while the second measure, named extreme downside hedge (EDH), measures the sensitivity of asset returns to market tail risk.

Related to this contribution, several studies have examined the relationship between tail risk and asset returns. For example, Chabi-Yo et al. (2018) proposed a systematic tail risk measure, Lower Tail Dependence (LTD), based on the estimated crash sensitivity of an individual asset to a market crash; Van Oordt and Zhou (2016) proposed a systematic tail risk

measure that captures the sensitivity of asset returns to market returns, conditional on market tail events and showed that this measure is associated with future asset returns; Almeida et al. (2017) introduced a tail risk measure that is based on the risk-neutral excess expected shortfall of a cross-section of asset returns.

In this paper we extend the proposed systematic risk measures, EDH and EDC, taking into account not only market (systematic) tail risk, as the previous contributions, but also a systemic tail risk. In this way, we consider as potential explanations of returns volatilities: systematic risk, systemic risk, and tail risk. We thus contribute to the market risk literature with a model that combines tail risk not only with systematic risk but also with systemic risk. To exemplify our proposal, we consider the leading assets from the crypto-asset market, thereby also extending the recent work of Borri (2019), who used CoVaR to show that crypto assets are highly exposed to tail risk from their market, but not from traditional assets. Our work is related to a recent research line that aims to explain bubbles in crypto prices in terms of interconnectedness between them or between exchange markets: (see for example Agosto and Cafferata, 2020; Bouri et al., 2019; Corbet et al., 2018; Giudici et al., 2019). Our work presents a more general methodology aimed at combining systemic interconnectedness with tail risk (which is indeed related to bubbles).

The paper is organized as follow.: Section 2 presents our proposed methodology. For our empirical application, we present a description of the data and report the results in Section 3 and a sensitivity analysis in Section 4. Section 5 concludes the paper with a brief discussion and suggestions for future research.

2. Methodology

In this section, we describe our extension of the extreme downside correlation (EDC) and of the extreme downside hedge (EDH) measures, aimed at estimating systemic tail risk dependence among return series of assets. We also present how to summarise systemic tail dependence by the means of network centrality measures.

2.1. Extreme Downside Correlation (EDC)

The EDC is a correlation-based technique that measures the marginal relationship between a pair of continuous variables, focusing on the tail of their joint return distributions. It is a non-parametric measure of tail risk co-movement of financial assets. Let $X_{i,t}$ be the returns of assets i (or X_i) at time t and denote with μ_i the historical mean of asset i. The EDC measures the tail correlation between assets i and j given by

$$EDC_{\tau,ij} = \frac{\sum_{t=1}^{T} \left[(X_{i,t} - \mu_i)_\tau (X_{jt} - \mu_j)_\tau \right]}{\left[\sum_{t=1}^{T} (X_{i,t} - \mu_i)_\tau^2 \right]^{1/2} \left[\sum_{t=1}^{T} (X_{jt} - \mu_j)_\tau^2 \right]^{1/2}} \tag{1}$$

$$\text{where } (X_{i,t} - \mu_i)_\tau = \begin{cases} (X_{i,t} - \mu_i), & \text{if } X_{i,t} < X_{\tau,i} = F_X^{-1}(\tau) \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

where $X_{\tau,i}$ is the left-side τ-quantile of the standardized distribution on X_i, $\tau \in (0,1)$, and $F_X(\tau) = Pr(X_i \leq \tau)$ is the cumulative distribution function (cdf) of X_i. The value of τ defines the percentage confidence level, $100(1-\tau)\%$. If $j = m$ is a market index, then $EDC_{\tau,im}$ captures the systematic relationship between asset-i and the market.

The tail of the return distribution technically corresponds to either extremely low gains (left tail) or very high returns (right tail). Following standard applications, we set our focus on the left tail to study the co-movement in returns of assets during stressful times which are usually characterized by losses. Following standard practice, we use the $\tau = 5\%$ quantile level which corresponds to a 95% confidence level in our empirical application. We also conduct robustness checks with other τ-quantile levels to validate the sensitivity of the findings.

For multivariate time series observations, the co-movements in the tail distributions of the joint return series can be operationalized as a network graph where nodes represent the assets and the edges denote the undirected association between the nodes. We therefore define an $n \times n$ zero diagonal weighted (G_C^W) and unweighted adjacency matrix (G_C^U) such that the ij-th element of both matrices are given by

$$G_{C,ij}^W = \left\{ \begin{array}{ll} EDC_{\tau,ij}, & \text{if } EDC_{\tau,ij} \neq 0 \\ 0, & \text{otherwise} \end{array} \right. , \qquad G_{C,ij}^U = \left\{ \begin{array}{ll} 1, & \text{if } G_{C,ij}^W \neq 0 \\ 0, & \text{otherwise} \end{array} \right. \tag{3}$$

2.2. Extreme Downside Hedge (EDH)

The extreme downside hedge (EDH) measures the sensitivity of returns to innovations in the tail risk of the market and/or of other counterparties. The variables of interest for the EDH model are the return series of the assets and a measure of innovation in the tail risk of the conditioning set of variables. Recent measures for assessing the riskiness of assets is the expected shortfall (also referred to as conditional value at risk - CoVaR or CVaR) (see Adrian and Brunnermeier, 2016; Alexander, 2009; Bali et al., 2009).

Let $X_t = (X_{1,t}, \ldots, X_{n,t})$ be n-variable vector of return observations at time t, where $X_{i,t}$ is the time series of asset-i at time t. Let $X_{\tau,i}$ denote the left-side τ-quantile of the distribution on X_i, for $\tau \in (0,1)$. Following Gaivoronski and Pflug (2005), we compute the $CVaR_\tau(X_i)$ as a proxy for the tail risk by

$$CVaR_\tau(X_i) = E(X_i) - C_\tau(X_i) \tag{4}$$

$$C_\tau(X_i) = \left(\frac{1}{\tau} F_X(\tau)\right) E\left(X_i | X_i \leq X_{\tau,i}\right) + \left(1 - \frac{1}{\tau} F_X(\tau)\right) X_{\tau,i} \tag{5}$$

where $F_X(\tau) = Pr(X_i \leq X_{\tau,i})$ is the cdf of X_i. We denote with $CVaR_{i,t}$ - the $CVaR_\tau(X_i)$ at time t. We employ $\Delta CVaR$ as a proxy for the innovation in the tail risk.

Following Harris et al. (2019), we start the EDH model with the systematic tail risk of an asset as the sensitivity of returns of asset-i with respect to $\Delta CVaR$ of the market index as

$$X_{i,t} = \alpha_i + \beta_{i|m} \, \Delta CVaR_{m,t} + \epsilon_{i,t} \tag{6}$$

where $\Delta CVaR_{m,t} = CVaR_{m,t} - CVaR_{m,t-1}$, α_i is the intercept, $\epsilon_{i,t}$ is the error term, and $\beta_{i|m}$ is the response of the asset returns to changes in market tail risk.

The EDH for systematic risk expresses the "contagion" effect of the market tail risk on asset returns. It does not, however, capture other channels such as exposure to the tail risk of other assets. This application extend the EDH to consider a "systemic" version that estimate the sensitivity of the returns of a single index to the innovation in the CVaR of other indices.

More formally, we can define the single index model of the EDH systemic risk by

$$X_{i,t} = \alpha_i + \sum_{i \neq j = 1}^{n-1} \beta_{i|j} \, \Delta CVaR_{j,t} + \epsilon_{i,t} \tag{7}$$

where $\Delta CVaR_{j,t} = CVaR_{j,t} - CVaR_{j,t-1}$, $\beta_{i|j}$ is the response of the stock return of asset-i to changes in the tail risk of other assets.

A further approximation is a mixed EDH models that combines the right-hand side of (6) and (7) in the single index model. Thus, the mixed covariates model is given by

$$X_{i,t} = \alpha_i + \sum_{i \neq j = 1}^{n-1} \beta_{i|j} \, \Delta CVaR_{j,t} + \beta_{i|m} \, \Delta CVaR_{m,t} + \epsilon_{i,t} \tag{8}$$

The parameters of the EDH models can be estimated via maximum likelihood. Following the financial network literature, estimates of $\beta = \{\beta_{i|j}\}$ can be used to construct network adjacency matrices that describe conditional independence relationships. More precisely, we denote with G_β^U and G_β^W the unweighted and weighted adjacency matrices such that the ij-th element are given by

$$G_{\beta,ij}^W = \begin{cases} \beta_{i|j}, & \text{if } \beta_{i|j} \neq 0 \\ 0, & \text{otherwise} \end{cases}, \qquad G_{\beta,ij}^U = \begin{cases} 1, & \text{if } G_{\beta,ij}^W \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

2.3. Network Analysis

We now present a brief description of the network metrics applied in our empirical analysis to summarize the information from the estimated networks. For purposes of interpretable diagram representation, we condense the information contained in matrices into an $n \times n$ zero diagonal weighted (A^W) and unweighted adjacency matrix (A^U) such that the ij-th element of both matrices are given by

$$A_{ij}^W = \begin{cases} 0, & \implies X_j \nrightarrow X_i \\ \mathbb{R}, & \implies X_j \rightarrow X_i \end{cases}, \qquad A_{ij}^U = \begin{cases} 0, & \implies X_j \nrightarrow X_i \\ 1, & \implies X_j \rightarrow X_i \end{cases} \tag{10}$$

The unweighted in-degree of node-i, $\overleftarrow{D}_i^U$, and the unweighted out-degree of node-j, $\overrightarrow{D}_j^U$, can be defined by

$$\overleftarrow{D}_i^U = \sum_j A_{ij}^U, \qquad \overrightarrow{D}_j^U = \sum_i A_{ij}^U \tag{11}$$

where $\overleftarrow{D}_i^U$ counts the number of links directed towards node-i, while $\overrightarrow{D}_j^U$ is the number of links going out of node-j. The weighted in-degree of node-i, $\overleftarrow{D}_i^W$, and the weighted out-degree of node-j, $\overrightarrow{D}_j^W$, can be defined by

$$\overleftarrow{D}_i^W = \sum_j A_{ij}^W, \qquad \overrightarrow{D}_j^W = \sum_i A_{ij}^W \tag{12}$$

where $\overleftarrow{D}_i^W$ and $\overrightarrow{D^W}_i$ are the row and column sums of A^W respectively.

We can also calculate centrality measures that take into account the importance of neighborhood of a node in a network graph. For example, the hub and authority centrality assign a score to nodes in the network in a way that is proportional to the importance of its neighbours. For a given unweighted network graph, this involves solving the following problem

$$\left(A^{U'}A^U\right) h_u = \lambda_h^u h_u, \qquad\qquad \left(A^U A^{U'}\right) a_u = \lambda_a^u a_u, \qquad (13)$$

where h_u and a_u are the hub score and authority score eigenvectors, corresponding to λ_h^u and λ_a^u, the largest eigenvalues of $A^{U'}A^U$ and $A^U A^{U'}$ respectively. The weighted network graphs, hub and authority scores involve solving the following

$$\left(A^{W'}A^W\right) h_w = \lambda_h^w h_w, \qquad\qquad \left(A^W A^{W'}\right) a_w = \lambda_a^w a_w, \qquad (14)$$

where h_w and a_w are the hub score and authority score eigenvectors, corresponding to λ_h^w and λ_a^w, the largest eigenvalues of $A^{W'}A^W$ and $A^W A^{W'}$ respectively.

3. Empirical Findings

We apply our proposed methodology to the return time series of the first 10 crypto assets, in terms of market capitalization, over the period September 13, 2017 – October 23, 2019 (771 daily observations). More precisely, we analyze the return series of Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Tether (USD), Bitcoin Cash (BCH), Litecoin (LTC), Binance Coin (BNB), Eos (EOS), Stellar (XLM), Tron (TRX) (in order of market capitalization). For market index, we consider the CRIX Crypto Index (CRI), (see Hardle and Trimborn, 2018).

To understand the time dynamics, we plot the normalized crypto-asset log price series in Figure 1. Due to differences in the values, plotting the original log prices would make it difficult to visualize some of them. To overcome this limitation, we standardize each series to a zero mean and unit variance and add the absolute minimum value of each series. This keeps the values positive and standardizes the scale of measurement for the different series.
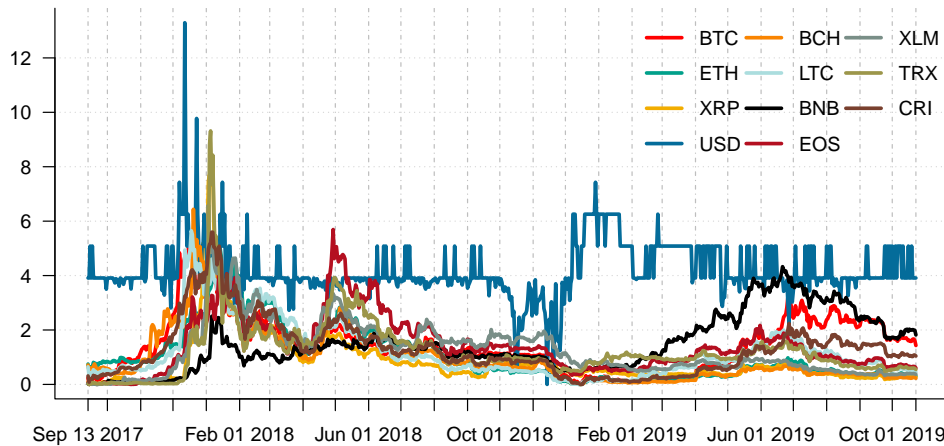

Figure 1: Normalized crypto asset price series.

Figure 1 confirms well-known features of cryptocurrencies, such as their overall high volatility, except for the stable coin (USD); and their strong co-movement, as pointed out by Corbet et al. (2018) and Bouri et al. (2019).

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To better understand the available data, we now calculate summary statistics. We would like, however, to calculate them on the returns, and on the prices. For this purpose, let $P_{i,t}$ be the daily close price of crypto asset i on trading day t. We compute the daily returns as the differences between the logarithms of successive daily closing prices, that is:

$$X_{i,t} = 100 \left(\log P_{i,t} - \log P_{i,t-1}\right)$$

Name	Code	Mean	Sdev	Min	Max	Skew.	Kurt.
Bitcoin	BTC	0.0858	4.3995	-20.7530	22.5119	-0.0693	3.3541
Ethereum	ETH	-0.0694	5.2720	-25.8859	23.4731	-0.3328	2.8990
Ripple	XRP	0.0404	6.6688	-35.3279	60.6885	1.8943	17.0350
Tether	USD	-0.0000	0.6545	-4.7402	5.7158	0.4571	12.1055
Bitcoin Cash	BCH	-0.1134	7.5222	-40.9658	43.1582	0.4924	6.4695
Litecoin	LTC	-0.0283	6.0010	-39.5151	38.9338	0.6604	8.0194
Binance Coin	BNB	0.3741	6.8343	-34.2318	48.2413	0.7909	7.7071
Eos	EOS	0.1710	7.4623	-30.1615	34.7309	0.6020	3.9429
Stellar	XLM	0.1772	7.6414	-32.8337	66.6779	1.4543	11.2702
Tron	TRX	0.2564	9.9517	-38.2167	78.6667	2.0892	15.3649
Crypto Index	CRI	0.0671	4.4851	-25.3340	19.8541	-0.6589	4.3882

Table 1: Descriptive Statistics for Cryptocurrency returns (September 14, 2017 – October 17, 2019).

Table 1 shows that the average of the daily return series is all close to zero, in line with the economic theory regarding asset returns. However, the ten crypto assets exhibit different variability in returns. In particular: USD shows the lowest relative variability; this is in line with the fact that this a stable coin, and its price should not deviate too much. On the other hand, TRX shows the highest standard deviation; indeed, this particular crypto asset witnesses a period of high fluctuations during the considered sample period. The skewness of the returns varies between -0.33 and - 2.1, with the majority of cryptocurrencies exhibiting positive skewness. The kurtosis varies between 2.89 to 17, indicating a Leptokurtic behavior of the daily return series. Most crypto assets exhibit values which reflects the non-Gaussian and heavy tailed behaviour of their associated distribution. This is particularly true for XLM and XRP, as already noted by Agosto and Cafferata (2020) .

The above considerations suggest that tail risk is to be taken into account, in the calculation of both systematic and systemic risk measures for crypto assets, in line with our proposed methodology, and consistently with what remarked by Bouri et al. (2019) and Agosto and Cafferata (2020) in the context of price bubble determination. In the next subsection, we show our main empirical findings.

3.1. EDC for Cryptocurrencies

To compute the τ-quantile of the returns for the EDC analysis, we standardize the return series to a zero mean and unit variance. We can then calculate the EDH Systemic for the Crypto assets. Figure 2 shows the result of the extreme downside correlation (EDC) matrix at 5%-quantile level, with correlations coefficients with lower than the 5% significance level set to zeros. Figure 3 displays the corresponding graphical representation of the EDC as a weighted network graph. The sign of the links are depicted with color codes such that green represents positive associations and negative relationships are colored in red. The size of the vertices corresponds to the degree of the nodes.

	BTC	ETH	XRP	USD	BCH	LTC	BNB	EOS	XLM	TRX	CRI
BTC	1.00	0.34	0.30	0.16	0.08	0.58	0.23	0.61	0.56	0.39	0.66
ETH	0.34	1.00	0.62	0.05	0.16	0.46	0.56	0.28	0.56	0.63	0.48
XRP	0.30	0.62	1.00	0.07	0.02	0.74	0.91	0.40	0.81	0.92	0.71
USD	0.16	0.05	0.07	1.00	-0.05	0.15	0.06	0.14	0.14	0.11	0.15
BCH	0.08	0.16	0.02	-0.05	1.00	0.09	-0.00	0.42	0.05	0.01	0.10
LTC	0.58	0.46	0.74	0.15	0.09	1.00	0.63	0.63	0.83	0.80	0.90
BNB	0.23	0.56	0.91	0.06	-0.00	0.63	1.00	0.33	0.72	0.84	0.61
EOS	0.61	0.28	0.40	0.14	0.42	0.63	0.33	1.00	0.57	0.47	0.70
XLM	0.56	0.56	0.81	0.14	0.05	0.83	0.72	0.57	1.00	0.85	0.87
TRX	0.39	0.63	0.92	0.11	0.01	0.80	0.84	0.47	0.85	1.00	0.77
CRI	0.66	0.48	0.71	0.15	0.10	0.90	0.61	0.70	0.87	0.77	1.00

Figure 2: Extreme downside correlation (EDC) matrix at 5%-quantile level. The light (dark) green color indicates weak (strong) positive correlations.
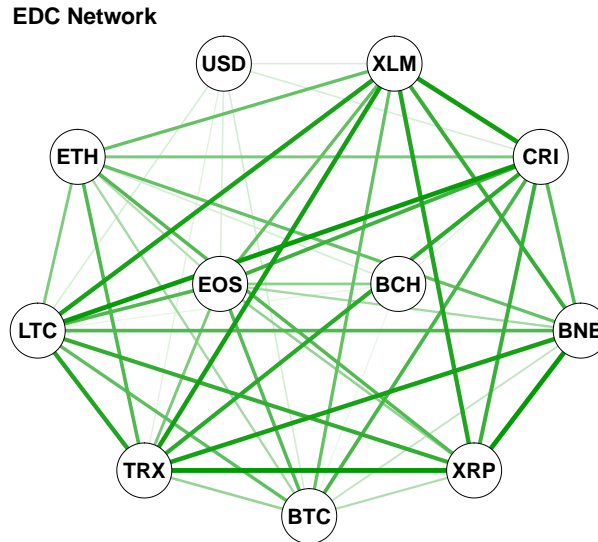
Figure 3: Graphical representation of the extreme downside correlation (EDC) matrix at 5%-quantile level. The size of the vertices corresponds to the node-degrees.

The figures depict positive relationships between crypto-assets which is not surprising since most people view cryptocurrencies as the same and cannot differentiate between them. According to Table 2 and from a systematic perspective, we observe that left tail series of the market index, CRI, is highly and strongly related to LTC and XLM (with correlation coefficients around 0.9), followed by TRX, XRP, EOS, and BTC. The USD and BTC are the only assets uncorrelated with the market index.

From the systemic perspective, we observed some form of clusters with centroids ETH on one hand and BTC on the other. The top correlated assets with ETH are TRX, XRP, BNB, and XLM, while that of BTC are EOS, LTC and XLM. Thus, of the two clusters, XLM appears to be the only crypto asset with the strongest and same level of association with ETH and BTC. Thus, XLM can be viewed as a bridge between the two clusters. The evidence of two clusters with highly correlated features around ETH and BTC corroborates

with the results by Bouri et al. (2019) and Agosto and Cafferata (2020). From an economic viewpoint, this confirms the different nature of the two groups of crypto assets with (ETH, TRX, XRP) constituting the "professional/technical" assets, while (BTC, EOS, LTC) forms the "speculative" cryptocurrencies. The result confirms Bouri et al. (2019) emphasizing that Bitcoin is not so strongly related to Ethereum, in terms of tail behavior. We notice that the stable coin Tether (USD), as expected, is uncorrelated with the others. Bitcoin Cash (BCH) is also uncorrelated with the majority of the assets due to its change over time.

To better understand the centrality of the crypto assets, we summarize the EDC network using standard network measures as shown in Table 2. Since the EDC is an undirected network, the in-degree and out-degree of nodes are the same for weighted and unweighted networks. The same is true for hub and authority centrality measures. The table shows that if centrality is expressed by the number of connected counterparties (degrees), then apart from the market index (CRI), the most important crypto assets are Bitcoin (BTC), alongside Litecoin (LTC) and EOS, followed by Ethereum (ETH), Stellar (XLM) and Tron (TRX). The least connected assets are Tether (USD) and Bitcoin cash (BCH). If centrality, however, depends on the importance of an individual's neighbors (eigenvector, i.e, either hub or authority), then the order of ranking of assets coincides with that of the degrees.

	InDeg.U = OutDeg.U	Hub.U = Auth.U	InDeg.W = OutDeg.W	Hub.W = Auth.W
BTC	**10**	**0.3345**	3.8868	0.2468
ETH	9	0.3117	4.0879	0.2677
XRP	8	0.2925	5.3992	0.3582
USD	6	0.2238	0.8600	0.0559
BCH	5	0.1875	0.8480	0.0483
LTC	**10**	**0.3345**	5.8231	0.3667
BNB	8	0.2925	4.8347	0.3269
EOS	**10**	**0.3345**	4.5465	0.2715
XLM	9	0.3154	**5.9161**	**0.3771**
TRX	9	0.3154	5.7836	0.3732
CRI	**10**	**0.3345**	**5.9389**	0.3703

Table 2: Centrality Measures for EDC network according to unweighted in-degree (InDeg.U), unweighted out-degree (OutDeg.U), unweighted hub centrality (Hub.U), unweighted authority centrality (Auth.U), weighted in-degree (InDeg.W), weighted out-degree (OutDeg.W), weighted hub centrality (Hub.W), and weighted authority centrality (Auth.W). Boldface values indicate the best choice for each metric.

In terms of measuring centrality via weighted networks, the table shows Stellar (XLM) as the top-ranked and the most important crypto asset based on weighted degree and eigenvector. This is not surprising since the position of XLM makes it serve as the borderline between the two clusters identified in Figure 2. By this line of reasoning, it is safe to conclude that Stellar plays an important role in the cryptocurrency market as an intermediating asset between the "professional/technical" group and the "speculative" ones.

3.2. Extreme Downside Hedging for Cryptocurrencies

To compute the τ-quantile of the returns for the EDH analysis, we standardize the return series to a zero mean and unit variance. We then estimate the EDH measure considering three different case scenarios. The first case is that of the EDH systematic model with n systems of equations, where each equation models the return of each asset expressed as a function of the $\Delta CVaR$ of the market index. The second scenario is the EDH systemic model with
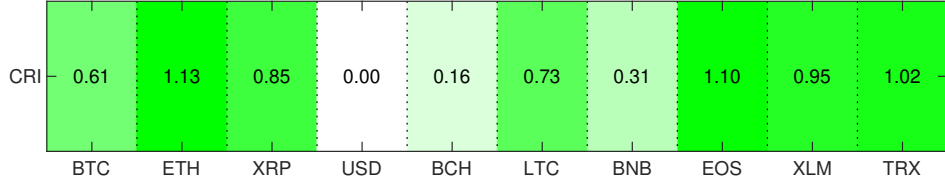
n systems of equations, where the i-th equation expresses the return of the i-th asset as a function of the $\Delta CVaR$ of the remaining $n-1$ assets. The third case combines the above two scenarios in a single model: we refer to this case as the EDH Mix model. The results from the calculation of the EDH systematic are presented in Figures 4 and 5.

Panel A of Figures 4 and 5 shows that the systematic EDH is positive for all cryptocurrencies, except for the stable coin USD, which is not affected by CRI. This result is consistent with the fact that CRI is a weighted average, essentially, of the most capitalized cryptocurrencies. On the other hand, Tether is pegged to the dollar and, thus, it is a diversifier for the other cryptos, exactly as the dollar would be. The results of the systematic EDH and the EDC reveal a positive and statistically significant relationship between the tail risk of the crypto assets and market index, which confirms the findings of Harris et al. (2019).

Panel B of Figure 4 shows the result of the systemic extension of the EDH. From the figure, we quickly notice two blocks of assets based on the color-coding of the cells viewed by columns. On one block are the red columns of the negative effect of downside risk on the returns of the majority of assets. This group is centered around BTC, XRP, and BCH. On the other hand, is the block of green columns which indicate a positive effect of tail risk on the returns of many crypto assets. This group consists of ETH, LTC, EOS, and XLM. Although USD and BNB seem to exhibit the characteristics of the former, their effects are quite weak compared to those identified above. A similar result is observed when we include the market index as depicted in Panel C of Figure 4. Focusing on BTC and ETH, we see from their respective rows of Panels B that the returns of BTC is positively sensitive to the tail risk of ETH, LTC, EOS, and XLM, while that of ETH is only positively exposed to LTC, EOS, and XLM. Thus, both BTC and ETH are sensitive to the same set of assets except that ETH affects BTC but not the reverse. The effect of ETH on BTC, however, vanishes when the model is conditioned on the market index.

To place our findings in existing studies, the evidence of a negative systemic effect of the tail risk of Bitcoin on other cryptocurrencies confirms the results in Agosto and Cafferata (2020); Bouri et al. (2019) and Borri (2019), which suggests that Bitcoin acts as a safe haven for "diversification" purposes in the crypto market. The positive systemic impact of ETH makes it serve a "complementary" role. Further observations from the results suggest that, although the tail returns of EOS and LTC appear strongly correlated with BTC as a "speculative" class of assets according to EDC, the EDH reveals them as acting more similar to the ETH. Thus, the Eos and Litecoin can be classified as "speculative" crypto assets that play a more "complementary" role. Besides, XRP and TRX - although classified as "professional" assets due to strong relationship with ETH according to EDC, these two cryptocurrencies serve more as "diversification" assets. This leads us to classify the assets into four groups based on the results of the EDC and EDH. The identified groups are: 1) "speculative" and "diversification", e.g. Bitcoin; 2) "professional" and "complementary", e.g. Ethereum; 3) "speculative" and "complementary", e.g. Eos and Litecoin; and 4) "professional" and "diversification", e.g. Ripple and Tron. The "diversification" role of Ripple may be due to the fact that it is controlled by a large consortium of banks.
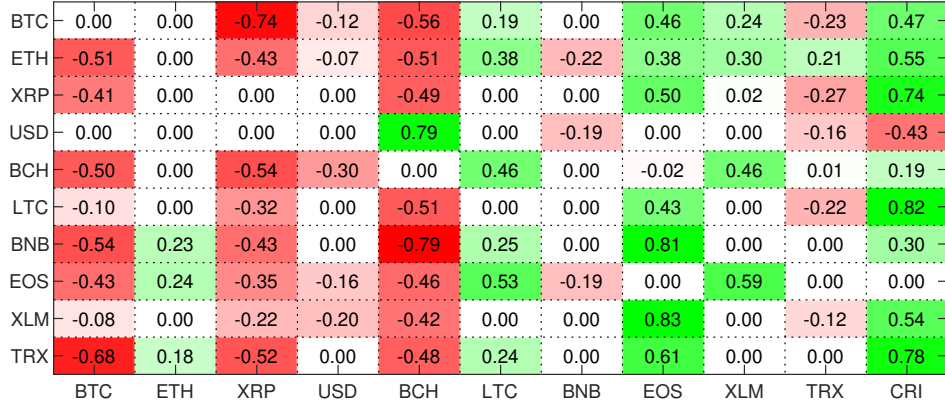
To better understand the centrality of the crypto assets, Table 3 contains the calculated summary measures. The result shows that, differently from what occurs to the EDC, IN and OUT centrality measures are different. In terms of IN-measures (those who receive shocks) Ethereum, followed by Bitcoin, are the most central: they can indeed be thought of representative of the two groups of cryptos: speculative rather than professional. In terms of OUT measures, instead, Bitcoin Cash and Bitcoin are the most central. The asymmetric nature of
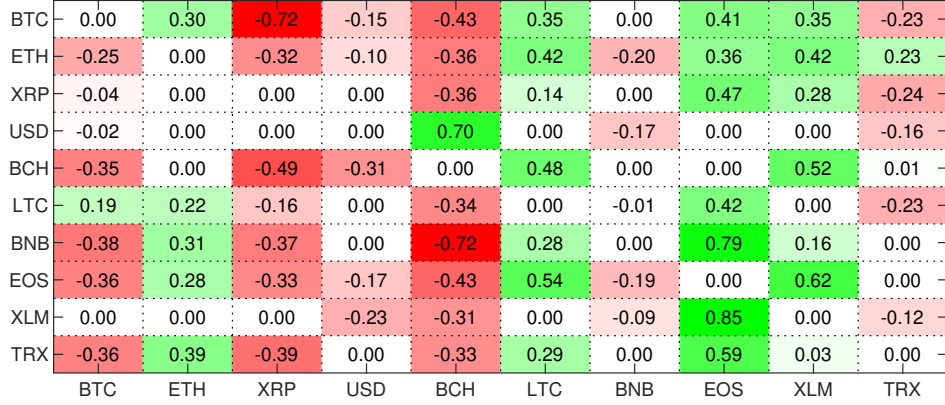
Figure 4: Extreme downside hedging (EDH) estimates for Panel A: Systematic, Panel B: Systemic, and Panel C: Combined Systematic and Systemic. The light (dark) green color indicates weak (strong) positive effects, and light (dark) red color indicates weak (strong) negative sensitivity to tail risk. Column labels of Panel B and C are $\Delta CVaR$ (Explanatory Variables) and row labels are $X_{i,t}$ (Dependent Variables).

the centrality measures confirms our economic classification of crypto-asset: speculative ones on one hand, which mainly distribute contagion; professional ones on the other hand which mainly receive contagion. Within them, BTC and ETH are their champions. Comparing the systematic EDC and EDH note also that, when we consider the lower 95 percent of our observations (EDC), in comparison with the 5 percent above (EDH), the results are different. All correlations are positive in the former case and negative in the second. It means that, in normal times, cryptocurrencies have positive correlations with each other, but in crash time many of them are affected in a negative direction. This is consistent with what found by Bouri et al. (2019) and Borri (2019).

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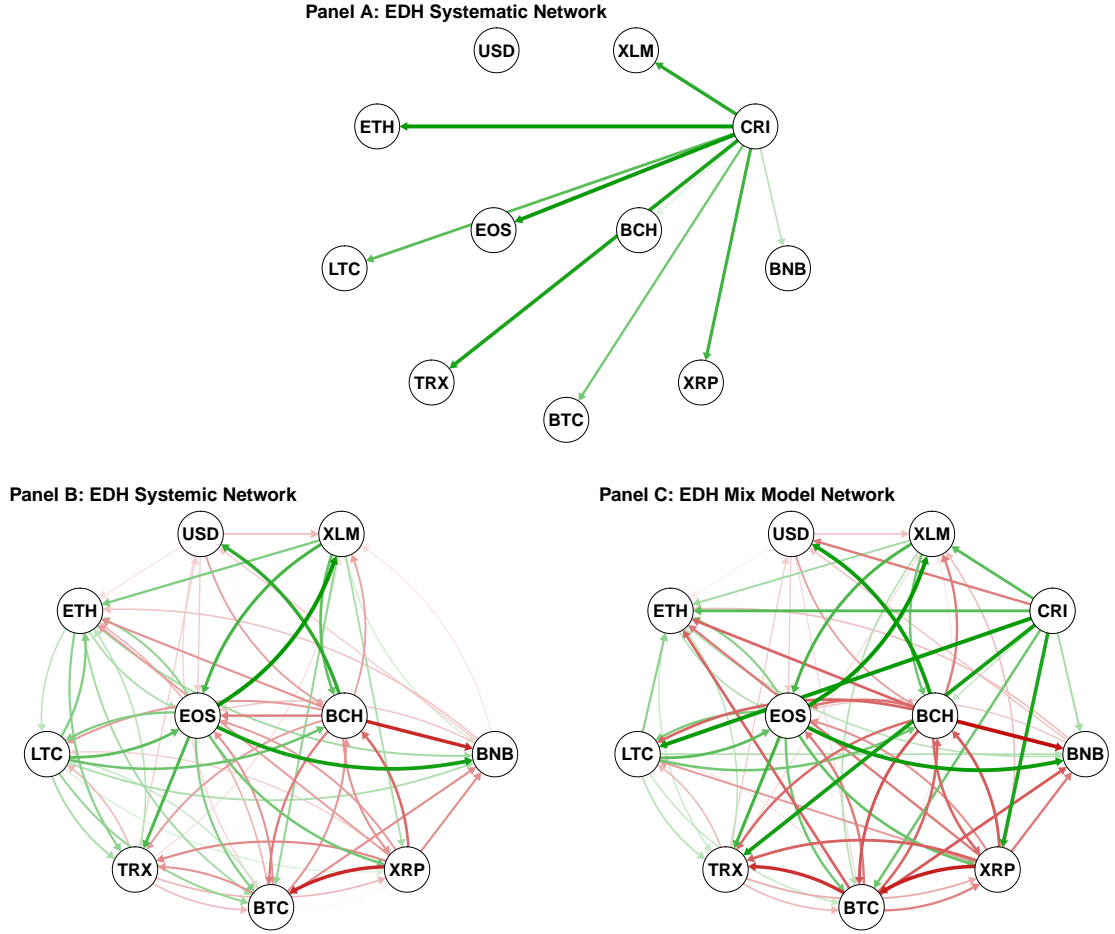
Figure 5: Graphical representation of the Extreme downside hedging (EDH) for Panel A: EDH Systematic, Panel B: EDH Systemic, and Panel C: EDH Mix.

	InDeg.U	OutDeg.U	Hub.U	Auth.U	InDeg.W	OutDeg.W	Hub.W	Auth.W
BTC	8	8	0.3613	0.3637	-0.1210	-1.5492	0.2573	0.4063
ETH	**9**	5	0.2461	**0.4106**	0.1927	1.5037	0.2173	0.3431
XRP	6	7	0.3446	0.2974	0.2467	-2.7971	0.3905	0.2301
USD	4	5	0.2322	0.1827	**0.3526**	-0.9634	0.1142	0.1483
BCH	6	**9**	**0.4043**	0.2752	-0.1419	-2.5791	0.4870	0.2533
LTC	7	7	0.3421	0.3144	0.0891	2.4994	0.3390	0.2212
BNB	7	5	0.2118	0.3399	0.0761	-0.6694	0.0738	**0.4707**
EOS	8	7	0.3269	0.3567	-0.0415	**3.8829**	**0.5007**	0.3532
XLM	5	7	0.3421	0.2110	0.1055	2.3852	0.3094	0.2445
TRX	7	7	0.2950	0.3399	0.2273	-0.7273	0.1308	0.3509

Table 3: Centrality Measures for EDH Systemic according to unweighted in-degree (InDeg.U), unweighted out-degree (OutDeg.U), unweighted hub centrality (Hub.U), unweighted authority centrality (Auth.U), weighted in-degree (InDeg.W), weighted out-degree (OutDeg.W), weighted hub centrality (Hub.W), and weighted authority centrality (Auth.W). Boldface values indicate the best choice for each metric.

4. Sensitivity Analysis

To validate the sensitivity of our empirical results, and our conclusions, we have conducted several robustness checks, as described by the following tables. The stability of the obtained results confirms the validity of our results.
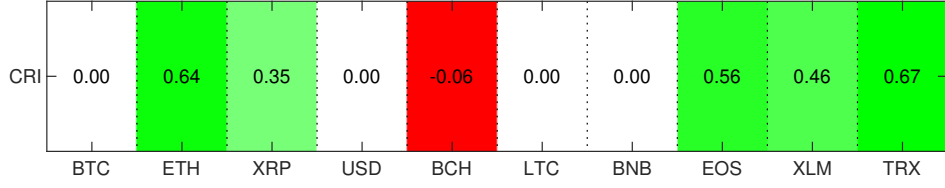

(a) **Panel A: EDC** ($\tau = 1\%$)

(b) **Panel B: EDC** ($\tau = 10\%$)

Figure 6: Extreme downside correlation (EDC) matrix for Panel A: EDC ($\tau = 1\%$) and Panel B: EDC ($\tau = 10\%$). The light (dark) green color indicates weak (strong) positive correlations.

5. Conclusions

In the paper, we proposed a methodology to measure systemic tail risk, extending the tail systematic measures introduced by Harris et al. (2019) into a multivariate network modeling framework. Doing so we also extend the EDC and EDH to study bubble inter-contentedness to crypto-asset market, very well known for the presence of extreme risks.

The results of the systematic EDH and the EDC reveal a positive and statistically significant relationship between the tail risk of the crypto assets and the weighted average market index. This corroborates the results of a significantly positive tail risk premium by Harris et al. (2019). The extension of EDC to systemic tail risk analysis reveals two main clusters of crypto assets. On one hand is the group of assets with "speculative" behavior, like Bitcoin, EOS and Litecoin, and one the other hand is the group with "professional" outlook, like
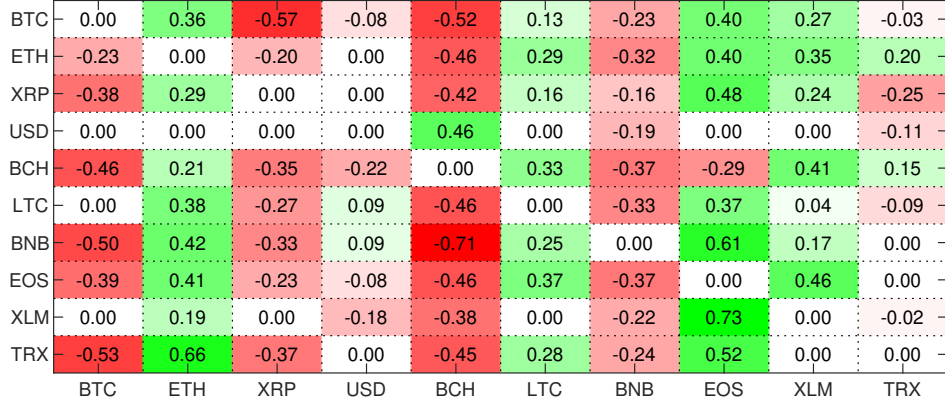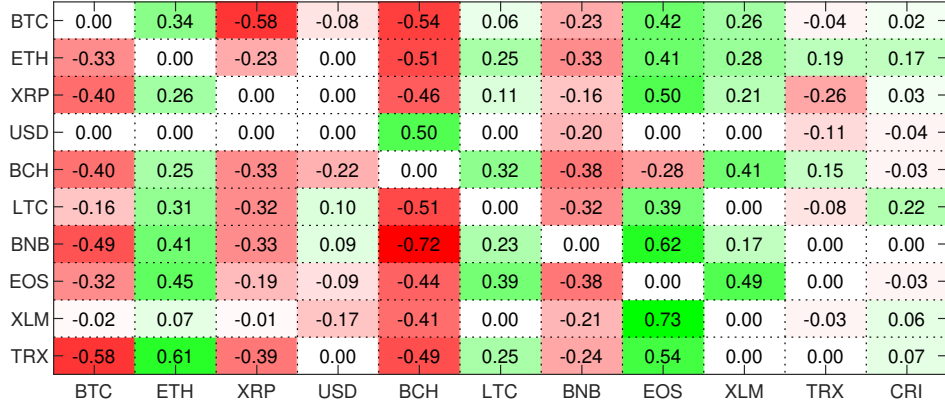
Figure 7: Sensitivity ($\tau = 1\%$) EDH estimates for Panel A: Systematic, Panel B: Systemic, and Panel C: Combined Systematic and Systemic. The light (dark) green color indicates weak (strong) positive effects, and light (dark) red color indicates weak (strong) negative sensitivity to tail risk. Column labels of Panel B and C are $\Delta CVaR$ (Explanatory Variables) and row labels are $X_{i,t}$ (Dependent Variables).

Ethereum, Tron, and Ripple. We find evidence of Stellar acting as a bridge between the two clusters. Within these two groups, Bitcoin and Ethereum can be thought of as the "champions". Stable coins, as expected, are a world on its own. The results of the EDH, however, shows that the two group cluster of the EDC can be decomposed further into four, consisting of 1) "speculative" and " diversification", e.g. Bitcoin; 2) "professional" and "complementary", e.g. Ethereum; 3) "speculative" and "complementary", e.g. Eos and Litecoin; and 4) "professional" and " diversification", e.g. Ripple and Tron. The centrality of the EDC and EDH networks shows the asymmetric nature of the two groups: while Bitcoin and Bitcoin cash are mainly agents of tail contagion, Ethereum and Ripple are the most vulnerable.

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Figure 8: Sensitivity ($\tau = 10\%$) EDH estimates for Panel A: Systematic, Panel B: Systemic, and Panel C: Combined Systematic and Systemic. The light (dark) green color indicates weak (strong) positive effects, and light (dark) red color indicates weak (strong) negative sensitivity to tail risk. Column labels of Panel B and C are $\Delta CVaR$ (Explanatory Variables) and row labels are $X_{i,t}$ (Dependent Variables).

A possible limitation of the study may stem from the fact that although estimating the EDH model with maximum likelihood may seem convenient, the downside to its implementation for the network construction is the inability to quantify the uncertainty that characterizes network link prediction. This may be propagated through multiple testing, thereby affecting the estimated network and its implication. Further study to authenticate the findings will be to adopt a Bayesian approach, which handles the link uncertainty problem by incorporating prior information where necessary and applying model averaging (see Ahelegbey et al., 2016).

Future research may consider the application of the methodology to other financial markets and, possibly, the study of its implications for portfolio allocation models.

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Acknowledgments

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