Modeling Turning Points In Global Equity Market

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Modeling Turning Points In Global Equity Market

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Abstract

Turning points in financial markets are often characterized by changes in the direction and/or magnitude of market movements with short-to-long term impacts on investors’ decisions. This paper develops a Bayesian technique to turning point detection in financial equity markets. We derive the interconnectedness among stock market returns from a piece-wise network vector autoregressive model. The empirical application examines turning points in global equity market over the past two decades. We also compare the Covid-19 induced interconnectedness with that of the global financial crisis in 2008 to identify similarities and the most central market for spillover propagation.

Keywords: Bayesian inference, Dynamic Programming, Turning points, Networks, VAR
JEL: C11, C15, C51, C52, C55, C58, G01

1. Introduction

The turn of events in major financial markets since late-February 2020, following the spread of the novel coronavirus (COVID-19) from Wuhan, in China to a global pandemic, is certainly a reminder of how increased interconnectedness between markets over time play a substantial role in the contagion spreading, especially during turbulent times. As a consequence, the degree of comovements in asset markets within and across countries increases in presence of shocks, and shocks propagate to markets across countries and regions, with corresponding impacts on asset prices/returns. A clear understanding of the nature of the linkages and interconnectedness among markets is critical to understand potential contagion. Modeling financial interconnectedness have received much attention, especially after the the global financial crisis of 2007–2009, and the European sovereign debt crisis of 2010–2013 (see Ahelegbey et al., 2016b; Battiston et al., 2012; Billio et al., 2019, 2012; DasGupta and Kaligounder, 2014; Diebold and Yilmaz, 2014; Hautsch et al., 2015).

In investigating the dynamic nature of interconnectedness among institutions and markets, it has become necessary to build models that are flexible to allow also structural changes. For instance, financial institutions are often interconnected through diverse channels, ranging from inter-bank market transfers, direct deposits, relationship lending/borrowing, and exposures to common risk or market factors. These connections can change with time. Turning points in financial markets are often characterized by changes in direction and/or magnitude of market movements with short-to-long term impacts on investors’ decisions. These turning points may

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occur as a result of changes in policy regimes, fluctuations in underlying market conditions or changes in the financial health of counterparties, leading to widespread implications on the economy and in financial markets. From a modeling perspective, changes in structural relationships affect predictions, as the use of old data becomes counter-productive. However, if the conditions associated with changes in financial markets are similar to past occurrences, then such information can prove helpful by signalling the direction of market conditions. This study contributes to a stream of research useful to identify turning points in multivariate time series to better understand non-stationarity in time series observations (see Bai, 2000; Chib, 1998; Cho and Fryzlewicz, 2015; Pesaran et al., 2006; Qu and Perron, 2007; Ruggieri, 2013).

This paper contributes to the above discussion by advancing a Bayesian technique to turning point detection in financial equity markets. We formalize the derivation of the interconnectedness among stock market returns from a piece-wise vector autoregressive with residual structural equations model (VAR-RSEM). That is, we model the dynamics of the returns by a reduced-form VAR with the residuals as a system of structural equations. Financial time series data usually exhibit contemporaneous dependencies as well as temporal lag relationships over time. The VAR-RSEM specification is designed to account for the contemporaneous, serial, and cross-lagged dependencies beyond what simple stylized facts from historical data can provide. Closely related models have in recent times been applied to infer financial contagion networks (see Ahelegbey et al., 2016a,b; Barigozzi and Brownlees, 2019; Basu and Michailidis, 2015; Bianchi et al., 2019; Billio et al., 2019, 2012; Diebold and Yilmaz, 2014). In a typical moderate to large VAR model, there are often too many parameters to estimate, compared to the available observations. A natural approach to overcome over-parametrization is via variable selection to produce parsimonious and sparse VAR models. The introduction of networks operationalized as graphical VAR models presents a convenient framework to achieve parsimony while providing explainable interactions in multivariate time series (Ahelegbey et al., 2016a).

The concept adopted in this paper places our contribution within the literature on detection of changes in multivariate time series and specifically to Bayesian turning point models (Jochmann et al., 2010; Koop and Potter, 2007, 2009), turning point network models (Barnett and Ounela, 2016; Grzegorczyk et al., 2011; Lèbre et al., 2010; Xuan and Murphy, 2007), high dimensional Bayesian models (Ahelegbey et al., 2016b; Gruber and West, 2017; Koop et al., 2019), and the application of graphical models to cope with overfitting issues arising in high dimensional models (see Ahelegbey et al., 2016a,b; Corander and Villani, 2006; Gruber and West, 2017; Paci and Consonni, 2020). Although there has been substantial progress in modeling non-homogeneous structure of dependence among random variables, scalability to high dimensional models and large data problems remains an open issue. In this paper, we adopt a specification closely related to the Bayesian turning point model of Jochmann et al. (2010), the changing dependency structure of Xuan and Murphy (2007), and extension of the Bayesian graphical VAR model of Ahelegbey et al. (2016a) to allow for structural changes.

Several techniques for estimating turning point locations have dominated the literature on Bayesian solutions to multiple turning point problems. Prominent among such techniques are the Markov chain Monte Carlo (MCMC) and Gibbs sampling algorithms (Barry and Hartigan, 1993; Erdman and Emerson, 2008; Green, 1995; Western and Kleykamp, 2004) and recursive dynamic programming algorithms (Fearnhead, 2006; Fearnhead and Liu, 2007; Ruggieri, 2013). However, turning point estimation in high dimensional models characterized by complex interactions and large datasets with many data time points present a number of inferential and computational challenges. In such settings, standard MCMC-based techniques
usually suffer from slow mixing and high computational cost per MCMC iteration. We therefore adopt a modified version of the Bayesian sequential turning point algorithm of Ruggieri and Antonellis (2016), that employs a recursive dynamic programming scheme for multiple turning point detection problems. This scheme proves to be very effective in inferring the number and timing of the turning points, reducing the computational burden of the inference problem, especially when dealing with large data sets.

We apply our proposed model to study the equities market by considering the 15 major stock market indices, including G10 countries. The dataset consists of daily prices from Bloomberg, covering January 2000 to October 2020. The empirical application examines: 1) the turning points in the global equity market; and 2) compares the Covid-19 induced interconnectedness with that of the global financial crisis, to identify similarities and the most central market for spillover propagation.

The paper is organized as follows: Section 2 presents the piece-wise network VAR model and discuss the Bayesian estimation. Section 3 presents a description of the data and report the results in Section 4. Section 5 concludes the paper.

2. A Bayesian Graphical Piece-Wise Vector Autoregression

2.1. Piece-Wise VAR Model

Let \( Y_t = (Y_{1,t}, \ldots, Y_{n,t}) \) be \( n \)-variable vector of return observations at time \( t \), where \( Y_{i,t} \) is the time series of market-i at time \( t \). Suppose there exist \( k \) turning points at \( 1 = \tau_0 < \tau_1 < \tau_2 < \ldots < \tau_k < \tau_{k+1} = T \). We represent the dynamics of \( Y_t \) as piece-wise stationary vector autoregressive (VAR) model of order \( p \) given by

\[
Y_t = \sum_{s=1}^{p} B_s I(t < t^*) Y_{t-s} + U_t
\]

\[
U_t = B_0 I(t < t^*) U_t + \epsilon_t
\]

where \( p \) is the lag order, \( t^* \in [\tau_{l-1}, \tau_l] \), for \( l = 1, \ldots, k+1 \), \( B_s \) is \( n \times n \) matrix of coefficients with \( B_{ij} \) measuring the effect of \( Y_{j,t-s} \) on \( Y_{i,t} \), \( I(a) \) denotes the indicator function that takes value 1 if \( a \) is true and 0 otherwise, \( U_t \) is a vector independent and identically normal residuals with covariance matrix \( \Sigma_u \), \( B_0 \) is a zero diagonal matrix that records the contemporaneous effect of shocks, and \( \epsilon_t \) is a vector of orthogonalized disturbances with covariance matrix \( \Sigma_{\epsilon} \). From (2), the \( \Sigma_u \) can be expressed in terms of \( B_0 \) and \( \Sigma_{\epsilon} \) as

\[
\Sigma_u = (I - B_0)^{-1} \Sigma_{\epsilon} (I - B_0)^{-1}^T
\]

2.2. Network Models

Following Ahelegbey et al. (2016a), equation (1) can be operationalize a network model, where the variables in \( Y \) are defined by nodes joined by a set of links, describing the statistical relationships between a pair of variables. The introduction of networks in VAR models helps to interpret the serial, temporal and contemporaneous relationships in a multivariate time series. To analyze (1) and (2) through networks, we assign to each coefficient in \( B_s \) a corresponding latent indicator in \( G_s \in \{0, 1\}^{n \times n} \), such that for \( i, j = 1, \ldots, n \), and \( s = 0, 1, \ldots, p \):

\[
B_{ij,s} = \begin{cases} 
0 & \text{if } G_{ij,s} = 0 \implies Y_{j,t-s} \not\rightarrow Y_{i,t} \\
\beta_{ij} \in \mathbb{R} & \text{if } G_{ij,s} = 1 \implies Y_{j,t-s} \rightarrow Y_{i,t}
\end{cases}
\]
where $Y_{j,t-s} \not\rightarrow Y_{i,t}$ means that $Y_j$ does not influence $Y_i$ at lag $s$, including $s = 0$, which correspond to contemporaneous dependence.

Let $B^* = \sum_{s=0}^{p} B_k$ and $G^* = \sum_{s=0}^{p} G_k$. Following (4), we define two zero diagonal matrices $A \in \{0, 1\}^{n \times n}$ and $A^w \in \mathbb{R}^{n \times n}$, whose $ij$-th element is given by:

$$A_{ij} = \begin{cases} 0, & \text{if } G_{ij}^* = 0 \\ 1, & \text{otherwise} \end{cases}, \quad A^w_{ij} = \begin{cases} 0, & \text{if } B^*_{ij} = 0 \\ B^*_{ij}, & \text{otherwise} \end{cases}$$

where $A_{ij}$ specifies that $Y_j \rightarrow Y_j$ exist if there is a contemporaneous or lagged directed link from $Y_j$ to $Y_i$. $A^w_{ij}$ specifies the weights of such a relationship obtained as a sum of the estimated contemporaneous and lagged coefficients. The correspondence between $(G, B)$ and $(A, A^w)$ is such that the former captures the short-run dynamics in $Y_t$ while the latter can be viewed as long-term direct relationships when $Y_t = Y_{t-1} = \ldots = Y_{t-p}$. Defining a sparse structure on $(G, B)$ induces parsimony of the short-run model and sparsity on the long-run relationship matrices $(A, A^w)$.

2.3. Bayesian Estimation of A Piece-Wise Network VAR

Following standard application, we select the appropriate lag order of the VAR via a Bayesian information criterion (BIC). Thus, the parameters left to estimate in a piece-wise network VAR are $(k, V_{\tau,k}, G_{1:k}, B_{1:k}, \Sigma_{e,1:k})$, where $k$ is the number of turning points, $V_{\tau,k} = (\tau_1, \tau_2, \ldots, \tau_k)$ is the the turning point locations, $G_{1:k} = \{G^{(1)}, \ldots, G^{(k+1)}\}$ is the collection of network graphs over the segments, $B_{1:k} = \{B^{(1)}, \ldots, B^{(k+1)}\}$ is the collection coefficient matrices over the segments, and $\Sigma_{e,1:k} = \{\Sigma^{(1)}_e, \ldots, \Sigma^{(k+1)}_e\}$ is the collection of error covariance matrices over segments. Estimating these parameters jointly is a challenging problem and a computationally intensive exercise. We complete the Bayesian formulation with prior specification and posterior approximations to draw inference on the model parameters.

2.3.1. Prior Specification

We specify the prior distributions over $(k, V_{\tau,k}, G_{1:k}, B_{1:k}, \Sigma_{e,1:k})$ as follows:

\[
\begin{align*}
  k &\sim U(0, k_{\text{max}}), \\
  V_{\tau,k} &\sim U(1, T), \\
  G_{ij}^{(k)} &\sim \text{Ber}(\pi_{ij}), \\
  \Sigma_{e}^{-(k)} &\sim W(\delta, \Lambda_0)
\end{align*}
\]

where $k_{\text{max}}, \eta, \pi_{ij}, \delta, \text{and } \Lambda_0$ are hyper-parameters.

The specification for $k$ is a discrete uniform prior on the set $\{0, \ldots, k_{\text{max}}\}$, with density

$$P(k) = \frac{1}{k_{\text{max}} + 1} 1_{\{k \in \{0, k_{\text{max}}\}\}}$$

The choice of the discrete uniform prior is relatively non-controversial since it is similar to the truncated Poisson in (Grzegorczyk et al., 2011; Nobile and Fearnside, 2007).

We consider $\tau_1, \ldots, \tau_k$ as order statistics and define $d_r = \tau_r - \tau_{r-1}$ as the distance between successive turning points. We consider $V_{\tau,k}$ to be uniformly distributed on $[1, T]$ with density

$$P(V_{\tau,k}|k) = \frac{1}{N_k}, \quad N_k \approx \binom{T_d}{k}$$
The likelihood function is given by

\[ P(Z_{1:h}(\Sigma)) = (2\pi)^{-m/2} |\Sigma|^{-\frac{h}{2}} \text{etr}\left(-\frac{1}{2} \Sigma^{-1} \hat{S}\right) \]  

(8)

where \( \text{etr}(\cdot) \) is the exponential of the standard trace function, \( \hat{S} \) is the sample sum of squared matrix of dimension \( m \). It can be shown that \( \Sigma \) contains the structural parameters \( (B, \Sigma_c) \). Under the assumption that \( \Sigma \) is inverse-Wishart distributed, \( P(\Sigma) \sim IW(\nu, \Lambda) \), with prior expectation \( \frac{1}{\nu} \Lambda \) and \( \nu > m \) degrees of freedom, we follow the Bayesian framework of Geiger and Heckerman (2002) to integrate out the structural parameters analytically, thus, obtaining a marginal likelihood function given by

\[ P(Z|h) = \frac{1}{2\pi} - \frac{h}{2} (\nu) \frac{m}{2} \prod_{i=1}^{\nu} \frac{\Gamma\left(\frac{\nu h + 1 - i}{2}\right)}{\Gamma\left(\frac{\nu + h - i}{2}\right)} |\hat{\Sigma}|^{-\frac{1}{2}(\nu + h)} \]  

(9)

where \( \hat{\Sigma} = (\Lambda + \hat{S})/(\nu + h) \) is the posterior covariance matrix. Clearly, we can notice that \( h \) controls the window size which is related to the turning point locations. From the above representation, we notice that except for \( \hat{\Sigma} \) whose computation depends on the observed data, the rest can be pre-computed for different values of \( h \), \( 1 \leq h \leq T \) with \( \nu = m + 2 \).

This allows us to apply an efficient sampling algorithm to sample the model parameters in blocks. The algorithm proceeds as follows:

1. Sample \([k, V_{\tau,k}|Y, \hat{\rho}]\) following Ruggieri and Antonellis (2016) by
   (a) Sample \( k \) from the marginal distribution: \([k|Y]\)
   (b) Recursively sample \( V_{\tau,k} \) from the conditional distribution: \([V_{\tau,k}|k, Y]\)

2. Sample \([G_0, G_{1,\hat{\rho}}, B_0, B_{1,\hat{\rho}}, \Sigma_c|Y, \hat{\rho}, k, V_{\tau,k}]\) by
   (a) Sample via a Metropolis-within-Gibbs \([G_0, G_{1,\hat{\rho}}|Y, \hat{\rho}, k, V_{\tau,k}]\) by
     i. Sampling from the marginal distribution: \([G_{1,\hat{\rho}}|Y, \hat{\rho}, k, V_{\tau,k}]\)
     ii. Sampling from the conditional distribution: \([G_0|Y, \hat{\rho}, G_{1,\hat{\rho}}, k, V_{\tau,k}]\)
   (b) Sample from \([B_0, B_{1,\hat{\rho}}, \Sigma_c|Y, \hat{G}_0, \hat{G}_{1,\hat{\rho}}, \hat{\rho}, k, V_{\tau,k}]\) by iterating the following steps:
i. Sample \([B_{i,\pi_1:1:p}, Y, \hat{G}_{1:p}, \hat{G}_0, B_0, \Sigma_\varepsilon] \sim N(\hat{B}_{i,\pi_1:1:p}, D_{\pi_1})\) where

\[
\hat{B}_{i,\pi_1:1:p} = \sigma_{u,i}^{-2} D_{\pi_1} Z_{\pi_1}^t Y_{i,1}, \quad D_{\pi_1} = (\eta^{-1} I_{d_z} + \sigma_{u,i}^{-2} Z_{\pi_1}^t Z_{\pi_1})^{-1}
\]

where \(Z_{\pi_1} \in Z\) corresponds to \((\hat{G}_{y_1:z_1:1:p}, 1) = 1\), \(\eta_{u,i}\) is the \(i\)-th diagonal element of \(\hat{\Sigma}_u = (I - \hat{B}_0) - 1 \hat{\Sigma}_u (I - \hat{B}_0)^{-1}\), and \(d_z\) is the number of covariates in \(Z_{\pi_1}i\).

ii. Sample \([B_{i,\pi_1:0}, Y, \hat{G}_0, \hat{G}_{1:p}, B_{1:p}, \Sigma_\varepsilon] \sim N(\hat{B}_{i,\pi_1:0}, Q_{\pi_1})\) where

\[
\hat{B}_{i,\pi_1:0} = \sigma_{\varepsilon,i}^{-2} Q_{\pi_1} \hat{U}_{\pi_1}^t \hat{U}_{\pi_1}, \quad Q_{\pi_1} = (\eta^{-1} I_{d_u} + \sigma_{\varepsilon,i}^{-2} \hat{U}_{\pi_1}^t \hat{U}_{\pi_1})^{-1}
\]

where \(\hat{U} = Y - Z \hat{B}_{1:p}, \hat{U}_{\pi_1} \in \hat{U}_{-i}\) is the contemporaneous predictors of \(\hat{U}_i\) that corresponds to \((\hat{G}_{y_i:z_i:0}, 1) = 1\), and \(d_u\) is the number of covariates in \(U_{\pi_1}\).

iii. Sample \([\Sigma_\varepsilon^{-1}, Y, \hat{G}_{1:p}, \hat{G}_0, B_{1:p}, B_0] \sim W(\delta + N, \Lambda_N)\) where

\[
\Lambda_N = \Lambda_0 + (\hat{U} - \hat{U}_0)^t (\hat{U} - \hat{U}_0)
\]

A detailed description on how to sample the parameters is available in Appendix A.

3. Data Description

Our study makes use of daily data from Bloomberg, covering between January 2000 to October 2020, and includes 15 major stock market indices, including all G10 economies. We consider only one index per country, which typically contains the stock prices of the largest companies listed in the nation’s largest stock exchange. The countries can be grouped into three regions: the Americas (Brazil, Canada, and the United States), Asia-Pacific (Australia, China, Hong Kong, India, Japan, and South Korea), and Europe (France, Germany, Italy, Russia, Spain, and the United Kingdom). A description of the market indices chosen for the selected countries is presented in Table 1.

<table>
<thead>
<tr>
<th>Region</th>
<th>No.</th>
<th>Country</th>
<th>Code</th>
<th>Description</th>
<th>Index</th>
</tr>
</thead>
<tbody>
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<td>Brazil</td>
<td>BR</td>
<td>Brazil Bovespa</td>
<td>IBOV</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Canada</td>
<td>CA</td>
<td>Canada TSX Comp.</td>
<td>SPTSX</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>United States</td>
<td>US</td>
<td>United States S&amp;P 500</td>
<td>SPX</td>
</tr>
<tr>
<td>Asia-Pacific</td>
<td>4</td>
<td>Australia</td>
<td>AU</td>
<td>Australia ASX 200</td>
<td>AS51</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>China</td>
<td>CN</td>
<td>China SSE Comp.</td>
<td>SHCOMP</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Hong Kong</td>
<td>HK</td>
<td>Hong Kong Hang Seng</td>
<td>HSI</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>India</td>
<td>IN</td>
<td>India BSE Sensex</td>
<td>SENSEX</td>
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<tr>
<td></td>
<td>8</td>
<td>Japan</td>
<td>JP</td>
<td>Japan Nikkei 225</td>
<td>NKY</td>
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<td></td>
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<td>KR</td>
<td>South Korean KOSPI</td>
<td>KOSPI</td>
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<td>FR</td>
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<td>CAC</td>
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<td>11</td>
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<td>DE</td>
<td>Germany DAX 30</td>
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<td>15</td>
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<td>UK</td>
<td>UK FTSE 100</td>
<td>UKX</td>
</tr>
</tbody>
</table>

Table 1: Description of stock market indices of countries classified according to regions.
We report in Figure 1 the daily series of closing prices on a logarithmic scale. We scale the prices to a zero mean and unit variance and add the absolute minimum value of each series to avoid negative outcomes. This standardizes the scale of measurement for the different series. The figure shows that over the past two decades, global financial markets have experienced several catastrophic events within and across different markets. Among these events is 1) the dotcom “tech” induced crisis of 2000–2003 which was fuelled by the adoption of the internet in the late 1990s, triggering inflated stock prices that gradually went downhill and disrupted global market operations; 2) the global financial crisis of 2007–2009 which was triggered by the massive defaults of sub-prime borrowers in the US mortgage market; 3) the European sovereign debt crisis of 2010–2013 which emanated from the inability of a cluster of EU member states to repay or refinance their sovereign debt and bailout heavily leveraged financial institutions without recourse to third party assistance; and 4) the distress to world economy and global financial markets caused by the novel coronavirus pandemic in 2020.

<table>
<thead>
<tr>
<th>Country</th>
<th>Code</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
<th>Skew</th>
<th>Ex.Kurt</th>
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</thead>
<tbody>
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<td>Brazil</td>
<td>BR</td>
<td>-15.993</td>
<td>28.824</td>
<td>0.047</td>
<td>1.861</td>
<td>0.318</td>
<td>16.057</td>
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<td>CA</td>
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<td>1.124</td>
<td>-0.898</td>
<td>16.780</td>
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<td>-12.765</td>
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<td>1.446</td>
<td>-0.302</td>
<td>10.849</td>
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<td>AU</td>
<td>-10.203</td>
<td>6.766</td>
<td>0.014</td>
<td>1.014</td>
<td>-0.707</td>
<td>8.524</td>
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<td>9.401</td>
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<td>1.528</td>
<td>-0.271</td>
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<td>HK</td>
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<td>13.407</td>
<td>0.016</td>
<td>1.446</td>
<td>-0.087</td>
<td>7.600</td>
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<td>1.437</td>
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<td>6.174</td>
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<td>0.015</td>
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<td>1.530</td>
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<tr>
<td>United Kingdom</td>
<td>UK</td>
<td>-11.512</td>
<td>9.384</td>
<td>-0.001</td>
<td>1.191</td>
<td>-0.329</td>
<td>7.697</td>
</tr>
</tbody>
</table>

Table 2: Statistics of daily returns for stock market indices over the sample period.

We compute daily returns as the log differences of successive daily closing prices, that is,
\[ Y_{i,t} = 100 \left( \log C_{i,t} - \log C_{i,t-1} \right), \] with \( C_{i,t} \) the daily closing price of market \( i \) on trading day \( t \).

Table 2 reports a set of summary statistics for the index returns over the sample period. The table shows that almost all index returns have a near-zero mean and a relatively high standard deviation. The highest standard deviations, indicating individual market volatilities, are those of the emerging markets of Russia and Brazil. Almost all the return indices exhibit fairly symmetric behaviour, i.e., they are characterized mostly by small but consistent positive gains and, occasionally, large negative returns. The excess kurtosis varies between 5.32 (China) and 16.78 (Canada), which confirms the stylized facts of leptokurtic behavior of daily return series.

4. Empirical Findings

We begin by studying the turning points in the equity market interconnectedness over the sample period. We characterize the dynamics of the connectedness via a yearly rolling windows of 240 trading days. We monitor the daily changes in the interconnectedness by setting the increments between successive rolling windows to one day. Thus, we set the first window of our study from January 28, 1999, to January 3, 2000, followed by January 29, 1999, to January 4, 2000; the last window is from November 21, 2019, to October 30, 2020. In total, we consider 5318 rolling windows. We select the appropriate lag of the VAR via a Bayesian information criterion (BIC) for different lag orders (\( p \in \{1, \ldots, 7\} \)). The optimal lag order according minimum BIC score is \( p = 1 \).

We report in Figure 2 the plot of rolling-window Net-Density and the posterior probability of a turning point for each day between January 3, 2000 to October 30, 2020. The figure shows that the network density of the COVID-19 period is much greater than any period of market crisis in the last 20 years, and this include the period of the dotcom “tech” induced crisis of 2000–2003, the global financial crisis (GFC) of 2007–2009, and the European sovereign debt crisis of 2010–2013. The spike in the network density after February 2020 is typified by the fear and panic that greeted the global financial market, and thus, intensifying sell-off of assets. The increased interconnectedness during the pandemic beyond the level recorded during the GFC suggest stronger evidence of contagion in the coronavirus pandemic than during the financial and eurozone crisis.
4.1. Turning Points in Global Equity Markets

For clarity, we extract the turning point dates, as they emerge from Figure 2. Table 3 lists the turning point dates with their posterior probabilities and possible financial market events that characterize the identified dates.

<table>
<thead>
<tr>
<th>Date</th>
<th>Probability</th>
<th>Financial Market Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/09/2001</td>
<td>0.967</td>
<td>September 11 Effect</td>
</tr>
<tr>
<td>01/10/2001</td>
<td>0.997</td>
<td>Turn-around in Financial markets</td>
</tr>
<tr>
<td>08/07/2003</td>
<td>0.706</td>
<td>Turn-around after SARS induced crisis</td>
</tr>
<tr>
<td>23/07/2007</td>
<td>0.528</td>
<td>Panic in the asset-backed commercial paper market</td>
</tr>
<tr>
<td>15/09/2008</td>
<td>0.896</td>
<td>Bankruptcy of Lehman Brothers</td>
</tr>
<tr>
<td>06/11/2008</td>
<td>0.976</td>
<td>IMF prediction of deep recession</td>
</tr>
<tr>
<td>07/07/2009</td>
<td>0.945</td>
<td>End of the great recession</td>
</tr>
<tr>
<td>12/07/2016</td>
<td>0.584</td>
<td>Rising oil prices and Aftermath of Brexit</td>
</tr>
<tr>
<td>21/02/2020</td>
<td>0.999</td>
<td>Beginning of Covid-19 induced global stock market crash</td>
</tr>
<tr>
<td>08/04/2020</td>
<td>0.921</td>
<td>End of Covid-19 induced global stock market crash</td>
</tr>
</tbody>
</table>

Table 3: Turning point dates with their posterior probabilities and possible financial market events.

The first turning point (event #1) in financial market over the past two decades is the September 11, 2001 attack that led to one of the largest single-day point decline in major markets. This was followed by a turn-around in at the beginning of October 2001 (event #2).

The timeline of the Severe Acute Respiratory Syndrome (SARS) outbreak shows that the illness first appeared in Guangdong Province, China in November 2002 and spread to 37 countries. This affected the Asian financial market and began to affect stock market integration around early 2003. The World Health Organisation (WHO) database shows that the SARS infection period was from 16/11/2002 – 05/07/2003 (see World Health Organization, 2003). Thus, the third turning point (event #3) captures the turn-around in financial market as a result of WHO’s declaration that SARS outbreaks have been contained worldwide.

The fourth turning point (event #4) marks the contraction in the asset-backed commercial paper (ABCP) market that began in late July 2007, which triggered fears and panic across the financial market. As documented by Covitz et al. (2013) and in Financial Crisis Inquiry Commission (2011), the collapse of the ABCP market played a central role in transforming concerns about the credit quality of mortgage-related assets into a global financial crisis. Early July 2007 also experienced the collapse of two Bear Stearns hedge funds that had speculated heavily in mortgage-backed securities.

The fifth turning point (event #5) marks September 15, 2008 when stock markets experienced the worst sell-off in the last 20 years. It was a Monday that followed a weekend turmoil of triple trouble. That is, Lehman Brothers (the fourth-largest U.S. investment bank at the time) filed for Chapter 11 bankruptcy protection; Merrill Lynch was acquired by Bank of America; and the American International Group (AIG - the world’s largest insurance company) presented an unprecedented request for short-term financing from the Federal Reserve. According to the Financial Crisis Inquiry Commission (2011), the risk exposures of AIG are concentrated among the largest international banks (both US and European) across a wide array of product types (bank lines, derivatives, securities lending, etc.). Thus, AIG’s failure could trigger significant counterparty losses to these firms. The fears and panic across financial markets led to increased interconnectedness which amplified the shocks, affecting a broader aspect of the US financial system and many other correlated markets and economies.
By the end of October 2008, many advanced economies like the US, Europe and Japan were already facing their deepest recession since the 1930s. With the global financial market in turmoil, producers and consumers were losing confidence in the financial system. As documented in the World Economic Outlook report published on November 6th 2008 (see IMF, 2008), the IMF predicted a worldwide “deep recession” in 2009 following the deteriorated global growth of world GDP over the past month. This forecast coupled with financial conditions continued to present serious downside risks, pushing the world over the edge with reaction across major stock markets. Thus, the sixth turning point (event #6) marks the date of the IMF prediction with its effect on the global equity market.

The seventh turning point (event #7) identifies July 7th 2009 as the day that marks the beginning of global economy recovery from the great recession. In its World Economic Outlook report published on July 8th 2009 (see IMF, 2009), the IMF projected receding contractionary forces with a positive but weak recovery between 2009–2010.

Rising crude oil prices in July 2016 affected oil producing countries as well oil dependent nations, sending market participants worried about the possible impact of rising crude oil prices on global stock market activities. The period also coincides with the aftermath of the Brexit which saw Britain voting to exit the EU in June 2016. The uncertainty surrounding the Brexit had significant effect on many investors thereby altering financial market activities across the globe. The failure of the Bank of England to ease the shocks that followed the Brexit vote also contribute to a mild turning point (i.e. event #8).

Discussions of the first Coronavirus case dates back to mid-November 2019. A global pandemic was triggered when the severity and scale of the impact of the novel coronavirus led to what can be best described as “hibernation” of world activities, i.e., a temporary sleep or “artificial coma”. February 21st 2020 (event #9) marked the day Covid-19 outbreak began to affect Europe and the U.S., plunging many stock markets into turmoil. Despite its impact, the GFC is incomparable to the Covid-19 outbreak in terms of the scale and magnitude of its effects. The uncertainty at the onset of latter was accompanied by the existential threat from which many markets may not recover in the sense that it could cripple if not wipe out nations and economies completely. This assertion is evidence by the fact that it recorded the fastest fall in global stock markets (see Figure 1). Unlike any of the past crises, within just a month of reported coronavirus cases in Europe and the US, major national indices began to record their worst ever historical prices in history.

The last turning point (event #10) mark the beginning of financial market recovery from the Covid-19 outbreak. Although, the global stock market is in a recovery phase, major world economies are currently in a recession or depression.

4.2. Dynamic Interconnectedness In Global Equity Markets

We analyze the dynamic nature of the interconnectedness among the major stock markets to assess which markets over the past two decades. Using the turning point dates, we divide the sample into four sub-periods of tranquil and turbulent times: (03/01/2000 – 12/09/2008), (15/09/2008 – 06/07/2009), (07/07/2009 – 20/02/2020) and (21/02/2020 – 30/10/2020). We report in Figure 3 the network topology over the sub-periods. Each network is represented with color-coded links and nodes. Red-links indicate negative weights and green-links denote positive weights. Red-color nodes represent American markets, blue-nodes for European markets, and green-nodes for markets in Asia-Pacific countries. The size of the nodes is proportional to their hub scores.
Figure 3: Sub-period Interconnectedness. Red nodes represent America markets, blue for European, and green for Asia-Pacific. The size of the nodes are weighted out-degree. Red links denote negative effects and green for positive interactions. The node position is based on eigendecomposition of the networks.

The figure provides strong evidence of clustering among the markets. More importantly, the Asian-Pacific markets (green-nodes) seem to move together, likewise, the European markets (blue-nodes) due to similarities in underlying market conditions. The US, however, appears separated from the others most of the time, as the rest of the American markets (red-nodes) are usually closer to the European ones. We notice that the US is usually the biggest sized node and strongly positively connected to the rest of the markets. Based on the node sizes, the US appears to be the most influential in almost all the sub-periods except 21/02/2020 – 30/10/2020, where Japan seems to dominate.

We compare the sub-period networks in terms of average degree, density, clustering coefficient, and average path length (see Appendix B for a description of these network statistical metrics). Table 4 shows that network statistics extracted for the four sub-period connectivity structures. We notice that two sub-periods record tranquil (non-crisis) conditions (i.e., 03/01/2000 – 12/09/2008 and 07/07/2009 – 20/02/2020), while the other two (15/09/2008 – 06/07/2009, and 21/02/2020 – 30/10/2020) experienced stressful conditions. The tranquil periods are characterized by relatively low average degree of interconnectedness, lowe density and clustering index, and a relatively high average path length. This suggests a lower degree
of equity market integration before and after the global financial crisis. It also shows that in the event a shock to a major market or a group of major markets, these shocks will take a much longer time to propagate to other markets to cause a systemic breakdown.

The turbulent periods, on the other hand, are characterized by relatively high average degree of interconnectedness, high density and clustering index, and a relatively low average path length. The lower average path length shows that it takes a relatively shorter time for a shock to a major market or a group of major markets to propagate to other markets, leading to a systemic breakdown. The network statistics of 15/09/2008 – 06/07/2009 and 21/02/2020 – 30/10/2020 periods shows that stock market integration during the GFC is strikingly similar to the behavior exhibited most recently at the height of the Coronavirus pandemic in 2020. More importantly, the spikes in the network density at onset of both crisis periods (see Figure 2) indicate elevated levels of unusualness in stock markets with a rise in volatility and market co-movements.

4.3. Global Financial Crisis vs Covid-19 Outbreak

We compare the interconnectedness of markets during the GFC and the Covid-19 outbreak. For this comparison, we extract the intersection and differences between the networks. Figure 4 presents the similarity and difference between the structure of interconnectedness during the GFC and Covid-19 sub-periods. Figure 4(a) depicts the network links during the GFC but not in the Covid-19 period. Figure 4(b) display the network links common to both periods, and Figure 4(c) shows only links in the Covid-19 period but not present during the GFC. Overall, we found 77 common connections between both networks. The GFC recorded 44 extra links that not present in the Covid-19 network, and the latter also report 54 new connections that was not in existence during the financial crisis. Surprisingly, majority of the new stock market connections center around Japan, Germany and India, whereas the GFC period centered around the US.

We now turn our attention to assess and compare the most critical (or central) market to during what appears to be the two most severe equity market crisis over the last two decades. Table 5 reports the summary of the centrality ranking of the most influential markets over the two crisis sub-periods. The table shows the top three transmitters of spillover propagation during the GFC are the US, UK and France, while the top three receivers of shocks during the period were Canada, Germany and France. During the Covid-19 outbreak, this most central markets for transmitting shocks are Japan, India and the US, while Japan, Australia, and Russia ranks high at the receiving end of shocks. Thus, not only has the structure of the nature of interconnectedness changed over the two crisis, but the most central markets for spillover propagation has also changed in recent times.
Figure 4: Comparing the Global Financial Crisis (GFC) and Covid-19 outbreak Network. Red nodes represent America markets, blue for European, and green for Asia-Pacific. The size of the nodes are weighted out-degree. Red links denote negative effects and green for positive interactions. The number in parenthesis signifies the total links in each network. Note: $A^c$ - Complement of $A$.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hub</td>
<td>Auth</td>
</tr>
<tr>
<td>1</td>
<td>US (0.798)</td>
<td>CA (0.383)</td>
</tr>
<tr>
<td>2</td>
<td>UK (0.265)</td>
<td>DE (0.359)</td>
</tr>
<tr>
<td>3</td>
<td>FR (0.256)</td>
<td>FR (0.328)</td>
</tr>
<tr>
<td>4</td>
<td>BR (0.249)</td>
<td>ES (0.319)</td>
</tr>
<tr>
<td>5</td>
<td>CA (0.228)</td>
<td>IT (0.292)</td>
</tr>
<tr>
<td>6</td>
<td>IT (0.182)</td>
<td>JP (0.281)</td>
</tr>
<tr>
<td>7</td>
<td>KR (0.161)</td>
<td>UK (0.252)</td>
</tr>
<tr>
<td>8</td>
<td>HK (0.101)</td>
<td>HK (0.238)</td>
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<tr>
<td>9</td>
<td>JP (0.098)</td>
<td>KR (0.235)</td>
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<tr>
<td>10</td>
<td>DE (0.097)</td>
<td>BR (0.221)</td>
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<tr>
<td>11</td>
<td>AU (0.092)</td>
<td>IN (0.208)</td>
</tr>
<tr>
<td>12</td>
<td>IN (0.074)</td>
<td>AU (0.190)</td>
</tr>
<tr>
<td>13</td>
<td>RU (0.068)</td>
<td>RU (0.157)</td>
</tr>
<tr>
<td>14</td>
<td>ES (0.063)</td>
<td>CN (0.133)</td>
</tr>
<tr>
<td>15</td>
<td>CN (0.049)</td>
<td>US (0.068)</td>
</tr>
</tbody>
</table>

Table 5: Centrality ranking during Global Financial Crisis and Covid-19 Pandemic.

5. Conclusion

This paper studies the nature of turning point in financial equity markets. We propose a Bayesian technique for turning point detection in a piece-wise network vector autoregressive model that approximates the interconnectedness among stock market returns. The empirical application examines turning points in global equity market over the past two decades. We also compare the Covid-19 induced interconnectedness with that of the global financial crisis to identify similarities and the most central markets for spillover propagation.

Our proposed approach proves to be effective in identifying financial market turning points in relevant periods, like the September 11 attack of 2001, the turn-around after SARS induced crisis in 2003, the panic in the asset-backed commercial paper market in 2007, the Bankruptcy of Lehman Brothers in 2008, the beginning and the end of the 2008–2009 great recession,
the rising oil prices and aftermath of Brexit in 2016, and the beginning and end of Covid-19 induced global stock market crash in 2020. We document a significant change in the structure of stock market integration during the global financial crisis and the Covid-19 outbreak. The result shows that the Covid-19 induced market interconnections record the highest density, suggesting stronger evidence of spillovers and contagion in the Covid-19 outbreak than during the global financial crisis.

References


Appendix A. Details of Sampling Approach of the Parameters

This section provides a detailed description of the sampling approach of the parameters.

Appendix A.1. Sampling Number of Turning Points

Using Bayes rule, the posterior distribution on the number of turning points is given as:

\[
P(K = k | Z) = \frac{P(K = k)P(V_{\tau,k}|K = k)P(Z|K = k, V_{\tau,k})}{P(Z)}
\]

\[
= \frac{1}{k_{\text{max}} + 1} \frac{1}{N_k} P(Z|K = k, V_{\tau,k}) \frac{P(V_{\tau,k}|K = k)P(K = k)}{P(Z)}
\]

where

\[
P(Z) = \sum_{K} \sum_{V_{\tau,k}} P(Z|K = k, V_{\tau,k})P(K = k, V_{\tau,k})
\]

(A.1)
For $t = 1, \ldots, T$, we denote with $\Psi_k(1, h) = P(Z_{1:h}| K = k, V_t, k)$, the density of $Z_{1:h}$ with $k > 0$ turning points defined by

$$
\Psi_k(1, h) = \sum_{l<h} \Psi_{k-1}(1, t)P(Z_{t+1:h})
$$

(A.3)

for $h = (d_r + 1), \ldots, T$, where the above density for $k = 0$ is initialized by $\Psi_0(1, h) = P(Z_{1:h})$, and $d_r = \tau_l - \tau_{l-1}$ is the distance between two successive turning points.

Appendix A.2. Sample Turning Point Locations
Following the process of filtering recursion (see Fearnhead, 2006; Ruggieri, 2013), the posterior distribution of the first turning point is given by

$$
P(\tau_1 = s|Z) = \frac{\Psi_0(1, s) P(Z_{s+1:T})}{\sum_{s<T} \Psi_0(1, s) P(Z_{s+1:T})}
$$

(A.4)

and the posterior distribution of the first turning point is given by

$$
P(\tau_k = t|\tau_{k+1}, Z) = \frac{\Psi_{k-1}(Z_{1:t}) P(Z_{t+1:|\tau_{k+1}})}{\sum_{t \in [k-1, \tau_{k+1}]} \Psi_{k-1}(Z_{1:t}) P(Z_{t+1:|\tau_{k+1}})}
$$

(A.5)

Appendix A.3. Sampling The Network
Let $V_y = (y_1, \ldots, y_n)$ be the vector of indices of response variables, and $V_z = (z_1, \ldots, z_{np})$ the indices of the lagged observations. The network relationship from $z_v \in V_z$ to $y_i \in V_y$ can be represented by $(G_{y_i, z_v} = 1)$. Following Geiger and Heckerman (2002), the closed-form expression of the local marginal likelihood is given by

$$
P(Y|G_{y_i, z_v}) = \frac{\pi^{-\frac{1}{2}}}{\nu_0^{\frac{1}{2}} \nu_n^{\frac{1}{2}}} \frac{\Gamma\left(\frac{\nu_0+n_x}{2}\right)}{\Gamma\left(\frac{\nu_0-n_x}{2}\right)} \frac{|Z'_\psi Z'_{\psi} + \nu_0 I_{nx}|^{\frac{1}{2}}}{|X_i X_i + \nu_0 I_{nx}|^{\frac{1}{2}}}
$$

(A.6)

where $\Gamma(\cdot)$ is the gamma function, $X_i = (Y_i, Z'_{\psi})$, $I_d$ is a $d$-dimensional identity matrix, $n_\psi$ is the number of covariates in $Z_{\psi}$, $n_x = n_\psi + 1$, $\nu_0 > n_x$ is a degree of freedom hyper-parameter of the prior precision matrix of $(Y, Z)$, and $\nu_n = \nu_0 + N$. Equation (A.6) indicates that only the ratio of the posterior sum of squares depend on the data. Thus, we reduce computational time by pre-computing the part of (A.6) that is independent of the data, for different values of $n_x \in [1, m]$ and for fixed $\nu_0 = m + 2$ and $N$. We also pre-compute the posterior of the full sum of squares matrix and extract the sub-matrices that relates to $\{Z_{\psi}\}$ and $\{(Y_i, Z_{\psi})\}$. For computational details of the score function (see Ahelegbey et al., 2016a). See Algorithms 1 and 2 for a pseudo code of the lagged and contemporaneous network sampling steps.

For our empirical application, we set the hyper-parameters as follows: $\pi_{ij} = 0.5$ (which leads to a uniform prior on the graph space), $\eta = 100$, $\delta = n + 2$ and $\Lambda_0 = \delta I_n$. We set the number of MCMC iterations to sample 50,000 graphs and we ensured that the convergence and mixing of the MCMC chains are tested via the potential scale reduction factor (PSRF) of Gelman and Rubin (1992).
Algorithm 1 Sampling $[G_{1;\tilde{p}}|Y,\tilde{p}]$

1: **Require:** Set of responses $V_y = (y_1, \ldots, y_n)$ and lagged attributes $V_z = (z_1, \ldots, z_{n\tilde{p}})$
2: Initialize $G_{1;\tilde{p}} = \emptyset$
3: for $y_i \in V_y$ do
   4: for $z_j \in V_z$ do
      5: Compute $\phi_a = P(Y|G_{y_i,\emptyset}^{(1)};1;\tilde{p})$ and $\phi_b = P(Y|G_{y_i,z_j}^{(1)};1;\tilde{p})$
      6: if $\phi_b > \phi_a$ then $G_{y_i,z_j}^{(1)} = 1$ else $G_{y_i,z_j}^{(1)} = 0$
   7: for $h = 2$ to Total iterations do
      8: for $y_i \in V_y$, set $G_{y_i;\tilde{p}}^{(h)} = G_{y_i;\tilde{p}}^{(h-1)}$
      9: Randomly draw $z_k \sim V_z$
     10: Add/remove link from $z_k$ to $y_i$: $G_{y_i,z_k}^{(h)} = 1 - G_{y_i,z_k}^{(h-1)}$
     11: Compute $\phi = \exp \left[ \log P(Y|G_{y_i}^{(h)};1;\tilde{p}) - \log P(Y|G_{y_i}^{(h-1)};1;\tilde{p}) \right]$. Draw $u \sim U(0, 1)$.
     12: if $u < \min\{1, \phi\}$ then $G_{y_i}^{(h)} = G_{y_i}^{(h-1)}$ else $G_{y_i}^{(h)} = G_{y_i}^{(h-1)}$

Algorithm 2 Sampling $[G_0|Y,\hat{G}_{1;\tilde{p}},\hat{p}]$

1: **Require:** Set of attributes $V_y = (y_1, \ldots, y_n)$ and estimated lag network $\hat{G}_{1;\tilde{p}}$
2: Initialize $G_0 = \emptyset$ and $G_{0;\tilde{p}} = [G_0, \hat{G}_{1;\tilde{p}}]$
3: for $y_i \in V_y$ do
   4: Set $V_{y_i} = V_y \setminus \{y_i\}$ and $\{z_\pi : \hat{G}_{y_i,z_\pi} = 1\}$
   5: for $y_j \in V_{y_i}$ do
      6: Set $\pi_i = (y_j \cup z_\pi)$. Compute $\phi_a = P(Y|G_{y_i,z_\pi}^{(1)};0;\tilde{p})$ and $\phi_b = P(Y|G_{y_i,\pi_i}^{(1)};0;\tilde{p})$
      7: if $\phi_b > \phi_a$ then $G_{y_i,\pi_i}^{(1)} = 1$ else $G_{y_i,\pi_i}^{(1)} = 1$
   8: for $h = 2$ to Total iterations do
      9: for $y_i \in V_y$, set $G_{y_i;\tilde{p}}^{(h)} = G_{y_i;\tilde{p}}^{(h-1)}$
     10: Randomly draw $y_k \sim V_{y_i}$
     11: Add/remove link from $y_k$ to $y_i$: $G_{y_i,y_k}^{(h)} = 1 - G_{y_i,y_k}^{(h-1)}$
     12: Compute $\phi = \exp \left[ \log P(Y|G_{y_i}^{(h)};0;\tilde{p}) - \log P(Y|G_{y_i}^{(h-1)};0;\tilde{p}) \right]$. Draw $u \sim U(0, 1)$.
     13: if $u < \min\{1, \phi\}$ then $G_{y_i}^{(h)} = G_{y_i}^{(h-1)}$ else $G_{y_i}^{(h)} = G_{y_i}^{(h-1)}$

Appendix B. Network Statistics

**Average Degree**

Average degree is simply the average number of edges per node in the graph. It can be computed numerically as: Average Degree = Total Edges/Total Nodes.

**Network Density**

Let $A$ be an $n$-node graph without self-loop. We characterize (through numerical summaries) the time-varying nature of interconnections by monitoring the network density: Network Density = Total Edges/n(n−1).
**Clustering Coefficient**

Network clustering index is a measure of the tendency for nodes in a network form clusters or triangles. We apply the global clustering index of Barrat and Weigt (2000) which corresponds to the social network concept of transitivity and can be captured numerically as:

\[
CC = \frac{3 \times (\text{number of triangles})}{(\text{number of open triads})} \tag{B.1}
\]

where open triads are defined as a connected sub-graph consisting of three nodes and two edges. The index takes values between 0 and 1. It can be viewed as the probability of two neighbors of a node link to each other.

**Average Path Length**

The average path length is the average number of steps along the shortest paths for all possible pairs of network nodes. The average path length for a network with \(n\)-nodes is

\[
APL = \frac{1}{n(n-1)} \sum_{i \neq j} d_{i,j} \tag{B.2}
\]

where \(d_{i,j}\) is the shortest path between the nodes \(i\) and \(j\).

**Node Centrality**

Node centrality in networks addresses the questions of how important a node/variable is in the network. Commonly discussed centrality measures include in-degree (number of in-bounds links), out-degree (number of out-bound links), authority, and hub scores. Let \(A^w\) be an \(n\)-node weighted graph without self-loop.

1. The authority score of node-\(i\) is a weighted sum of the power/hub score of the vertices with directed links towards node-\(i\). They can be obtained via absolute value of the eigenvectors associated with the largest eigenvalue of \((A^w A^w')\). An authority node has a large in-degree.
2. The hub score of node-\(j\) is the weighted sum of the power/authority score of vertices with a directed link from node-\(j\). They can be obtained via absolute value of the eigenvectors associated with the largest eigenvalue of \((A^w' A^w)\). A hub node usually has a large out-degree.

From a financial viewpoint, nodes with high authority scores/in-degree are highly influenced by others, while high hub scores/out-degree nodes are the influencers.