Monopolistic Competition, As You Like It

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Abstract

We study monopolistic and other forms of imperfect competition with asymmetric preferences over a variety of goods provided by heterogeneous firms. We show how to compute equilibria through the Morishima elasticities of substitution. Simple pricing rules and closed-form solutions emerge under monopolistic competition when demands depend on common aggregators. This is the case for Generalized Additively Separable preferences (encompassing additive preferences and their Gorman-Pollak extensions), implicitly additive preferences and others. For applications to trade, with markups variable across goods of different quality and countries of different income, and to macroeconomics, with markups depending on aggregate variables, we propose specifications of indirectly additive, self-dual addilog and implicit CES.

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Which prices should emerge in markets where firms sell differentiated goods? Such a basic question is at the core of modern economic theories that depart from the perfectly competitive paradigm by adopting models of monopolistic and othe forms of imperfect competition inspired by the works of Chamberlin (1933) and Robinson (1933). Unfortunately, most of these theories rely on a simplified model of competition with constant elasticity of substitution (CES) preferences based on Dixit and Stiglitz (1977, Section I), which delivers constant markups, either across countries and among firms in trade models (Krugman, 1980; Melitz, 2003) or over time in macroeconomic models (Blanchard and Kyotaki, 1987; Woodford, 2003). Only a few applications use more general but still symmetric preferences (Dixit and Stiglitz, 1977, Section II; Bertoletti and Etro, 2016), even when considering variable productivity across firms (as in Melitz and Ottaviano, 2008, Bertoletti and Etro, 2017, Parenti *et al.*, 2017, Arkolakis *et al.*, 2018, Dhingra and Morrow, 2018) and over time (as in Kimball, 1995, or Bilbiie *et al.*, 2012). In an attempt to capture the features of monopolistic competition in the spirit of Chamberlin,[2] in this work we consider heterogeneous firms supplying genuinely differentiated commodities. This suggests a richer way of thinking about markup variability across firms and markets as well as over time, which can be useful for applications to trade and macroeconomics. At the same time, this variability matters theoretically because it introduces the possibility of sources of inefficiency both at the firm level (in the choice of the production mix) than in aggregate across sectors which are additional with respect to the case of symmetric preferences.

We consider demand systems derived from preferences over a fixed number of different commodities that can be represented by any well-behaved utility functions. Each commodity is produced with a constant, idiosyncratic marginal cost. Our question is simply which choices should be made by firms in such a market. The starting point is the analysis of Cournot and Bertrand equilibria in which firms choose either their quantities or their prices taking as given the strategies of the competitors and the demand systems. We naturally generalize the familiar monopoly pricing condition by expressing the equilibrium of a firm in terms of its market share and of the substitutability of its own product with respect to those sold by competitors. Substitutability is measured by the average of the Morishima Elasticities of Substitution, as rediscovered and formalized by Blackorby and Russell (1981).[3] On this basis, we discuss how to solve for Cournot and Bertrand equilibria by computing the Morishima measures.

Then we move to competition among a large number of firms where, in the spirit of Dixit and Stiglitz (1977, 1993), market shares are negligible and

[2] Chamberlin (1933) defined monopolistic competition with reference to factors affecting the shape of the demand curve, and certainly did not intend to limit his analysis to the case of symmetric goods. And he saw no discontinuity between its own market theory and "the theory of monopoly as familiarly conceived" (Chamberlin, 1937, p. 562), claiming *inter alia* that "monopolistic competition embraces the whole theory of monopoly. But it also looks beyond, and considers the interrelations, wherever they exist, between monopolists who are in some appreciable degree of competition with each other." (p. 571-2).

[3] The Morishima Elasticity of Substitution was originally proposed by Morishima (1967) in a book review written in Japanese.

firms "perceive" demand elasticity as given by the average Morishima elasticity. This approach allows us to define monopolistic competition even when demands depend in asymmetric ways on the strategies of the competitors. We exploit it to study imperfect competition for examples of homothetic preferences, namely when demands are derived from translog or generalized linear preferences.

Next we consider monopolistic competition for a wide type of preferences, the Generalized Additively Separable (GAS) preferences introduced by Pollak (1972). These include the large classes of directly and indirectly additive preferences (whose symmetric versions have been used respectively by Dixit and Stiglitz, 1977 and Bertoletti and Etro, 2017), and an additional class of preferences that we call Gorman-Pollak preferences after the contributions of Gorman (1970a, 1987) and Pollak (1972). With GAS preferences, the demand functions depend on a common aggregator of firm strategies. Intuition suggests that to take this aggregator as given while computing the elasticity of demand should be approximately correct (i.e., profit maximizing) when market shares are negligible. We show that this is indeed the case with GAS preferences, in the sense that the perceived demand elasticities are approximately equal to the Morishima measures when shares are negligible. In addition, the equilibrium strategies do not depend on whether prices or quantities are chosen by the firms, implying that imperfectly competitive choices do actually "converge" to those of monopolistic competition. This provides a simple approach to solve for asymmetric equilibria, and it allows the computation of prices in closed-form solution for a variety of examples. Actually, in the case of indirectly additive preferences, equilibrium pricing is independent across firms and the price of each firm depends on its marginal cost and product substitutability, and on the consumers' willingness to pay for its quality as well as on their income. The special case of constant markups that differ across goods emerges in case of "power" additive subutilities and with the more general and unexplored family of "self-dual addilog" preferences (see Houthakker, 1965). These examples, in which firms sell goods of different qualities at different markups in different markets, is naturally useful for trade applications.

The same approach to monopolistic competition can be extended to demand functions that depend on two common aggregators, as exemplified by the case of a restricted Almost Ideal Demand System (Deaton and Muellbauer, 1980). We focus on two types of preferences. The first, that exploits the possible separability of marginal utilities, includes a generalization of the preferences employed by Melitz and Ottaviano (2008) in international economics. The second is provided by the implicitly additive preferences of Hanoch (1975), for which we can again prove that to take the aggregators (one coincides with the same utility level) as given is approximately correct when shares are negligible. This case encopasses the Kimball (1995) homothetic aggregator widely used in macroeconomics, as well as the "implicit CES" preferences (Gorman, 1970a,b, and Blackorby and Russell, 1981) which deliver markups common across goods that vary with the utility level and therefore consumers' expenditure. Since it provides a channel of propagation of aggregate shocks through markup variability (countercyclical markups can magnify positive temporary shocks by reducing the relative price of the final goods and increasing real wages), the latter, unexplored specification should be useful for the macroeconomic analysis of business cycle. In conclusion, we remark that our approach to monopolistic competition in principle can be further extended to other preferences delivering demand functions that depend on several aggregators.

Our work is related to different literatures. The analysis of Bertrand and Cournot competition with differentiated products is well known under quasilinear preferences (Vives, 1999). We generalize and reframe its setting in terms of the Morishima measures, whose role was introduced by Bertoletti and Etro (2016) in a symmetric environment.[4] Few papers have analyzed monopolistic competition with asymmetric preferences. The original work of Dixit and Stiglitz (1977, Section III) touched on this topic only peripherally. The earliest treatement we are aware of is in the interesting work of Pascoa (1997), who focused on an example with Stone-Geary preferences and a continuum of goods. More recently, the trade literature with heterogeneous firms, started by Melitz (2003) and Melitz and Ottaviano (2008), has usually considered monopolistic competition with symmetric preferences for a continuum of goods. Only a few works have considered asymmetries to model quality differentials among goods (for instance Baldwin and Harrigan, 2012 and Crozet *et al.*, 2012), but retaining the CES structure of the Melitz model.[5] We follow the spirit of this literature generalizing it to genuinely asymmetric preferences that deliver different markups.[6]

The work is organized as follows. Section 1 presents alternative equilibria of imperfect competition for the same demand microfoundation. Section 2 studies monopolistic competition under generalized additive separability of preferences. Section 3 extends such an approach to the case of other separable preferences. Section 4 concludes.

1 A Model of Imperfect Competition

We consider identical consumers with preferences over an exogenous (finite) number n of commodities represented by the following direct and indirect utility functions:

$$U = U(\mathbf{x}) \quad \text{and} \quad V = V(\mathbf{s}), \tag{1}$$

where $\mathbf{x}$ is the n-dimensional vector of quantities and $\mathbf{s} = \mathbf{p}/E$ is the corresponding vector of prices normalized by income/expenditure E. We assume that preferences are well-behaved, and in particular that the utility maximizing choices are unique, interior ($\mathbf{x}, \mathbf{p} > \mathbf{0}$) and characterized by the first-order conditions. Therefore, the inverse and direct (Marshallian) demand systems are

[4]See Etro (2016, 2017) for applications to macroeconomics and trade in a symmetric environment.

[5]Recent exceptions include Feenstra and Romalis (2014), who use an "implicit" CES structure, and our earlier work based on indirect additivity.

[6]See d'Aspremont and Dos Santos Ferreira (2017) for an alternative equilibrium concept, and Hottman *et al.* (2016) for an empirical approach based on a nested CES utility system.

delivered by Hotelling-Wold's and Roy's identities:

$$s_i(\mathbf{x}) = \frac{U_i(\mathbf{x})}{\widetilde{\mu}(\mathbf{x})}, \qquad x_i(\mathbf{s}) = \frac{V_i(\mathbf{s})}{\mu(\mathbf{s})}, \tag{2}$$

where

$$\widetilde{\mu}(\mathbf{x}) = \sum_{j=1}^{n} U_j(\mathbf{x})\, x_j, \qquad \mu(\mathbf{s}) = \sum_{j=1}^{n} V_j(\mathbf{s})\, s_j \tag{3}$$

and U_i and V_i denote marginal utilities, $i = 1, .., n$. Here $\widetilde{\mu}$ is the marginal utility of income *times* the expenditure level, and $|\mu(\mathbf{s})| = \widetilde{\mu}(\mathbf{x}(\mathbf{s}))$, as can be verified by "adding up" the market shares $b_j = s_j x_j$.

Firm i produces good i at the marginal cost c_i and obtains per-consumer variable profits given by:[7]

$$\pi_i = (p_i - c_i)x_i. \tag{4}$$

We begin from the case in which each firm correctly perceives its demand function and chooses its profit-maximizing strategy. In the tradition of industrial organization we have to consider two separate equilibria, where each firm simultaneously chooses either its production level (Cournot competition) or its price (Bertrand competition). We then move to monopolistic competition.

1.1 Cournot competition

Let us start considering firms that choose their quantities on the basis of the inverse demand functions $s_i(\mathbf{x})$ in (2). Correctly anticipating the quantities $\mathbf{x}'_{-i} = [x_1, .., x_{i-1}, x_{i+1}, .., x_n]$ produced by the competitors, each firm i chooses x_i to equate its marginal revenue to its marginal cost c_i. The relevant "individual marginal revenue" of firm i is $MR_i = \partial(p_i x_i)/\partial x_i$, where $p_i(\mathbf{x}) = s_i(\mathbf{x})E$. It can be written as:

$$\begin{aligned} MR_i &= \frac{[U_i(\mathbf{x}) + U_{ii}(\mathbf{x})x_i]\,\widetilde{\mu}(\mathbf{x}) - U_i(\mathbf{x})x_i \left[U_i(\mathbf{x}) + \sum_{j=1}^{n} U_{ji}(\mathbf{x})x_j\right]}{\widetilde{\mu}(\mathbf{x})^2} E \\ &= p_i(\mathbf{x}) \left[1 - s_i(\mathbf{x})x_i - \sum_{j=1}^{n} \epsilon_{ij}(\mathbf{x}) s_j(\mathbf{x}) x_j\right], \end{aligned}$$

where the (gross) Morishima Elasticity of Complementarity (MEC) between varieties i and j is defined as:[8]

$$\epsilon_{ij}(\mathbf{x}) = -\frac{\partial \ln\{s_i(\mathbf{x})/s_j(\mathbf{x})\}}{\partial \ln x_i} = \frac{U_{ji}(\mathbf{x})x_i}{U_j(\mathbf{x})} - \frac{U_{ii}(\mathbf{x})x_i}{U_i(\mathbf{x})}. \tag{5}$$

[7] Since expenditure is assumed exogenous and independent from profits, no explicit role is left here to play for the size of the market or for possible fixed costs (but see the extension to entry sketched in Section 1.5).

[8] See Blackorby and Russell (1981) on the corresponding *net* measure which applies to compensated demands. The larger is ϵ_{ij} the smaller is the possibility of good j to substitute for good i. Notice that $\epsilon_{ii} = 0$ and that in general $\epsilon_{ij} \neq \epsilon_{ji}$ for $i \neq j$.

These inverse measures of substitutability depend on preferences and not on the specific utility functions which are chosen to represent them. Since substitutability can differ among goods, let us compute the weighted average of the MECs for good i with respect to all the other goods j, with weights based on the expenditure shares $b_j(\mathbf{x}) = s_j(\mathbf{x})x_j$, namely:

$$\overline{\epsilon}_i(\mathbf{x}) = \sum_{j \neq i}^{n} \epsilon_{ij}(\mathbf{x}) \frac{b_j(\mathbf{x})}{1 - b_i(\mathbf{x})}. \tag{6}$$

It is immediate to verify that the marginal revenue can be rewritten as $MR_i = p_i(\mathbf{x})[1 - b_i(\mathbf{x})]\,[1 - \overline{\epsilon}_i(\mathbf{x})]$, and that the equilibrium quantities satisfy the system:

$$p_i(\mathbf{x}) = \frac{c_i}{1 - \epsilon_i^C(\mathbf{x})} \quad \text{for } i = 1, 2, ..., n, \tag{7}$$

where the left hand side comes from the inverse demand given in (2) and the right hand side depends on:

$$\epsilon_i^C(\mathbf{x}) = b_i(\mathbf{x}) + [1 - b_i(\mathbf{x})]\overline{\epsilon}_i(\mathbf{x}). \tag{8}$$

Here ϵ_i^C is an increasing function of the market share of firm i and of its average Morishima elasticity $\overline{\epsilon}_i$, and must be smaller than unity.[9] Intuitively, a firm's markup is higher when it supplies a good that is on average less substitutable with the other goods (high $\overline{\epsilon}_i$), and its market share is larger (high b_i). The Cournot equilibrium system (7) is operational, in the sense that for given preferences and cost distribution one can directly compute the Cournot quantities and then obtain the equilibrium prices.

Let us consider the Cournot equilibria in the well-known example of CES preferences with utility $U = \sum_j x_j^{1-\epsilon}$, where $\epsilon \in [0, 1)$ is a parameter corresponding to a common and constant MEC. Closed form solutions emerge in a few cases. For instance, in a Cournot duopoly we can solve explicitly for:

$$x_i^C = \frac{(1-\epsilon)c_j^{1-\epsilon}E}{\left(c_i^{1-\epsilon} + c_j^{1-\epsilon}\right)^2 c_i^{\epsilon}} \quad \text{and} \quad p_i^C = \frac{c_i}{1-\epsilon}\left[1 + \left(\frac{c_j}{c_i}\right)^{1-\epsilon}\right]. \tag{9}$$

The price of each firm is increasing in its cost (though less than proportionally) and in the relative cost of the other firm, while it is independent from income (due to homotheticity). When goods are perfect substitutes ($\epsilon = 0$) the equilibrium price is the sum of the marginal costs.[10]

[9] Throughout this work we assume that the first-order condition for profit maximization characterizes firm behaviour. Of course, existence and unicity of the solution require that the demand system satisfies a number of regularity conditions (for a related discussion see Vives, 1999, Ch. 6).

[10] In this case we can also solve for the oligopoly equilibrium obtaining $p_i^C = n\bar{c}/(n-1)$, where $\bar{c}$ is the arithmetic average of marginal costs.

1.2 Bertrand competition

Consider now firms that choose their prices on the basis of the direct demand $x_i(\mathbf{s})$ in (2), while correctly anticipating the prices of the competitors $\mathbf{s}'_{-i} = [s_1, .., s_{i-1}, s_{i+1}, .., s_n]$. The elasticity of the Marshallian direct demand of firm i can be computed as:

$$\left|\frac{\partial \ln x_i}{\partial \ln p_i}\right| = -\frac{s_i}{x_i(\mathbf{s})} \frac{V_{ii}(\mathbf{s})\mu(\mathbf{s}) - V_i(\mathbf{s})\left[V_i(\mathbf{s}) + \sum_{j=1}^n V_{ji}(\mathbf{s})s_j\right]}{\mu(\mathbf{s})^2}.$$

Consider the (gross) Morishima Elasticity of Substitution (MES) between goods i and j:[11]

$$\varepsilon_{ij}(\mathbf{s}) = -\frac{\partial \ln\{x_i(\mathbf{s})/x_j(\mathbf{s})\}}{\partial \ln s_i} = \frac{s_i V_{ji}(\mathbf{s})}{V_j(\mathbf{s})} - \frac{s_i V_{ii}(\mathbf{s})}{V_i(\mathbf{s})}, \tag{10}$$

which again depends on preferences and not on their specific representations, and compute the weighted average:

$$\bar{\varepsilon}_i(\mathbf{s}) \equiv \sum_{j\neq i}^n \varepsilon_{ij}(\mathbf{s}) \frac{b_j(\mathbf{s})}{(1 - b_i(\mathbf{s}))}, \tag{11}$$

where, with a little abuse of notation, $b_j(\mathbf{s}) = s_j x_j(\mathbf{s})$ is now the expenditure share of firm i as a function of the normalized prices.

We can now rewrite the demand elasticity $|\partial \ln x_i/\partial \ln p_i|$ as:

$$\varepsilon_i^B(\mathbf{s}) = b_i(\mathbf{s}) + [1 - b_i(\mathbf{s})]\bar{\varepsilon}_i(\mathbf{s}), \tag{12}$$

which needs to be larger than 1 to define the Bertrand equilibrium through the following system:

$$p_i = \frac{\varepsilon_i^B(\mathbf{s})\, c_i}{\varepsilon_i^B(\mathbf{s}) - 1} \quad \text{for } i = 1, 2, .., n. \tag{13}$$

Firms set higher markups if their goods are on average less substitutable than those of competitors (low $\bar{\varepsilon}_i$) and their market shares larger (high b_i). The system (13) allows one to directly compute all Bertrand equilibrium prices for given costs and preferences.

It is well known that the Bertrand prices do not coincide with those obtained under competition in quantities. As an example, consider again CES preferences, whose indirect utility can be written as $V = \sum_j s_j^{1-\varepsilon}$. The parameter $\varepsilon > 1$ corresponds to the constant and symmetric MES and is the reciprocal of the MEC parameter $\epsilon = 1/\varepsilon$ in the previous representation of the same preferences. In this case we can derive an implicit expression for the best response functions:

$$p_i = c_i \left[\frac{\varepsilon}{\varepsilon - 1} + \frac{p_i^{1-\varepsilon}}{(\varepsilon - 1)\sum_{j\neq i} p_j^{1-\varepsilon}}\right],$$

[11] See Blackorby and Russell (1981) and Blackorby *et al.* (2007). The higher is ε_{ij} the greater is the possibility of good j to substitute for good i. Notice that $\varepsilon_{ii} = 0$ and that in general $\varepsilon_{ij} \neq \varepsilon_{ji}$ for $i \neq j$.

which shows the strategic complementarity among price choices. In a Bertrand duopoly this gives:

$$p_i = \frac{c_i \left[\varepsilon + \left(\frac{p_j}{p_i} \right)^{\varepsilon - 1} \right]}{\varepsilon - 1} , \tag{14}$$

which implies markups different from the Cournot ones.

1.3 Monopolistic competition

The remainder of this paper is dedicated to analyze monopolistic competition. There are different ways to make sense of this concept but, in the spirit of Dixit and Stiglitz's (1993) reply to Yang and Heidra (1993), we interpret monopolistic competition as the result of having firms that correctly perceive market shares as negligible. In fact, what Dixit and Stiglitz (1977) did in their symmetric setting amounts to neglect any term of order $1/n$ in the demand elasticities, where n is a number of firms assumed sufficiently large to make the omitted terms small. Similarly, in our setting, when there are many goods we should expect consumers to spread their expenditure if preferences are well-behaved and not too asymmetric, so that the market shares should be small for all goods.[12] On this basis, our previous results suggest to approximate the relevant demand elasticities with the corresponding averages of the Morishima measures.

Accordingly, we will consider as monopolistically competitive an environment where market shares are negligible, that is $b_i \approx 0$ for any $i = 1, .., n$, and where firms, correctly anticipating the value of actual demands, "perceive" the relevant elasticities as given by the average Morishima measures. This approach actually leads to two approximations according to whether we refer either to quantity or to price competition. In the first case we approximate (7) by using the expression:

$$p_i(\mathbf{x}) = \frac{c_i}{1 - \bar{\epsilon}_i(\mathbf{x})} \quad \text{for } i = 1, 2, .., n. \tag{15}$$

In the second case we approximate (13) by:

$$p_i = \frac{\bar{\varepsilon}_i\left(\mathbf{s}\right) c_i}{\bar{\varepsilon}_i\left(\mathbf{s}\right) - 1} \quad \text{for } i = 1, 2, .., n. \tag{16}$$

These simplified systems need to be solved to derive the prices and quantities which arise in the monopolistic competition equilibrium (that ought to imply negligible market shares). Once we depart from symmetry this may still be a formidable task, but in the next sections we will consider several types of asymmetric preferences for which explicit solutions can be actually derived.

We can learn something more about this approach to monopolistic competition by considering the cross demand elasticities. They can be computed

[12] Sufficient conditions on preferences to deliver this result are studied in Vives (1987).

as:

$$\begin{aligned}\frac{\partial \ln p_j(\mathbf{x})}{\partial \ln x_i} &= \frac{U_{ji}(\mathbf{x})\,x_i}{U_j(\mathbf{x})} - \sum_{h=1}^{n} \frac{U_{hi}(\mathbf{x})x_i}{U_h(\mathbf{x})} b_h(\mathbf{x}) \\ &= \epsilon_{ij}(\mathbf{x}) - \bar{\epsilon}_i(\mathbf{x}) + b_i(\mathbf{x})\bar{\epsilon}_i(\mathbf{x}),\end{aligned}$$

$$\begin{aligned}\frac{\partial \ln x_j(\mathbf{s})}{\partial \ln s_i} &= \varepsilon_{ij}(\mathbf{s}) - \left|\frac{\partial \ln x_i(\mathbf{s})}{\partial \ln p_i}\right| \\ &= \varepsilon_{ij}(\mathbf{s}) - \bar{\varepsilon}_i(\mathbf{s}) - b_i(\mathbf{s})\left(1 - \bar{\varepsilon}_i(\mathbf{s})\right).\end{aligned}$$

When shares are indeed negligible the cross effects should be perceived as negligible too whenever the differences $\epsilon_{ij} - \bar{\epsilon}_i$ and $\varepsilon_{ij} - \bar{\varepsilon}_i$ are small *and* the perceived own elasticities are not very large. Apparently, this is the case that Dixit and Stiglitz (1993) had in mind, and we expect it to apply to the typical monopolistic competition equilibrium with positive markups. Notice that the former condition is satisfied in any equilibrium of a symmetric environment. However, both conditions might be violated in our asymmetric setting: in similar cases the perceived cross demand elasticities can be large, and associated to a large own demand elasticity and therefore to small equilibrium markups. In other words, it can happen that goods are perceived as highly substitutable and that monopolistic competition pricing approximates marginal cost pricing,[13] as we will see in a variety of examples, namely in the case of translog preferences in next section and later on with quadratic preferences à la Melitz and Ottaviano (2008) and with restricted AIDS preferences.

1.4 The case of homothetic preferences

Benassy (1996) studied examples of monopolistic competition with symmetric homothetic preferences.[14] Here we are concerned with the more general case of asymmetric homothetic preferences, because they provide a good starting point to analyze our proposed equilibria. Let us normalize the indirect utility function to be:

$$V = \frac{E}{P(\mathbf{p})} = \frac{1}{P(\mathbf{s})}, \tag{17}$$

where P is homogeneous of degree 1 and, as is well known, a fully-fledged price index. Roy's identity delivers direct demands and market shares:

$$x_i = \frac{P_i(\mathbf{s})}{P(\mathbf{s})} \quad \text{and} \quad b_i = \frac{s_i P_i(\mathbf{s})}{P(\mathbf{s})}, \tag{18}$$

[13] Notice that, in general, the value of these cross demand elasticities need not be negligible in a strategic setting. In fact, if they were null there would be no reason for strategic interaction and we could think of those producers as "isolated monopolists".

[14] Also see Feenstra (2003, 2014), who considers the case of heterogeneous firms with a symmetric version of the so-called "quadratic mean of order r" (QMOR) preferences, which include the well-known translog specification.

which are homogeneous respectively of degree -1 and 0. This allows us to compute the MES as:

$$\varepsilon_{ij}(\mathbf{s}) = \frac{s_i P_{ji}(\mathbf{s})}{P_j(\mathbf{s})} - \frac{s_i P_{ii}(\mathbf{s})}{P_i(\mathbf{s})},$$

which is homogeneous of degree 0, being the difference of two functions that are both homogeneous of that degree. The fact that both the market share b_i and the average MES $\bar{\varepsilon}_i$ are homogeneous of degree zero implies immediately that pricing is independent from income.[15] Similar results can be derived starting from the direct utility (which can be written as a consumption index) and using the inverse demand system and the average MEC to study quantity competition.

Translog preferences To illustrate, let us consider the homothetic translog preferences (Christensen *et al.*, 1975) represented by the following price index:

$$P(\mathbf{s}) = \exp\left[\ln \alpha_0 + \sum_i \alpha_i \ln s_i + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln s_i \ln s_j\right], \tag{19}$$

where we assume without loss of generality $\beta_{ij} = \beta_{ji}$, and we need $\sum_i \alpha_i = 1$ and $\sum_j \beta_{ij} = 0$ to satisfy the linear homogeneity of P (Feenstra, 2003 uses a symmetric version of these preferences). The direct demand for good i is:

$$x_i(\mathbf{s}) = \frac{P_i(\mathbf{s})}{P(\mathbf{s})} = \frac{\alpha_i + \sum_j \beta_{ij} \ln s_j}{s_i},$$

which delivers the market share $b_i = \alpha_i + \sum_j \beta_{ij} \ln s_j$. Profits of firm i are then given by:

$$\pi_i = \frac{(p_i - c_i)\left[\alpha_i + \sum_j \beta_{ij} \ln(p_j/E)\right] E}{p_i},$$

whose direct maximization provides the Bertrand equilibrium conditions:

$$p_i = c_i \left(1 + \frac{b_i}{\beta_i}\right), \tag{20}$$

where the positiveness of $\beta_i \equiv -\beta_{ii}$ is necessary to ensure $\varepsilon_i^B = 1 + \beta_i/b_i > 1$.[16]

[15] Moreover, when preferences are homothetic and symmetric, this implies that Morishima elasticities and markups in a symmetric equilibrium can be at most a function of the number of goods. While this result has been used elsewhere (for instance in Bilbiie *et al.*, 2012), we are not aware of a formal proof. We are thankful to Mordecai Kurz for pointing this out.

[16] We can rewrite (20) as:

$$p_i = c_i \left(1 + \frac{\alpha_i + \sum_{j \neq i} \beta_{ij} \ln p_j}{\beta_i} - \ln p_i\right).$$

Taking logs and approximating for small markups we get:

We can obtain the same result, as well as the monopolistic competition equilibrium, by deriving the Morishima elasticity between goods i and j as:

$$\varepsilon_{ij} = 1 + \frac{\beta_i}{b_i} + \frac{\beta_{ji}}{b_j},$$

so that the average MES is:

$$\bar{\varepsilon}_i = \sum_{j\neq i}^{n} \varepsilon_{ij} \frac{b_j}{1-b_i} = 1 + \frac{\beta_i}{(1-b_i)\, b_i}.$$

This allows one to get (20) from (16), and to obtain the monopolistic competition prices:

$$p_i = c_i \left[1 + \frac{(1-b_i)\, b_i}{\beta_i}\right]. \tag{21}$$

Notice that the latter are below the Bertrand prices (20),[17] and that, when market shares are indeed negligible ($b_i \approx 0$), the average MES is large, goods are highly substitutable and prices must be close to the marginal costs ($\widehat{p}_i \approx c_i$).

Generalized linear preferences Let us now discuss an example of asymmetric homothetic preferences due to Diewert (1971). Suppose that preferences can be represented by the following direct utility/consumption index:

$$U = \sqrt{\mathbf{x}}' \mathbf{A} \sqrt{\mathbf{x}} = \sum_i \sum_j \sqrt{x_i} a_{ij} \sqrt{x_j}, \tag{22}$$

where, without loss of generality, we can take the matrix $\mathbf{A}$ to be symmetric. To satisfy the standard regularity conditions we assume that $a_{ij} \geq 0$ for any i, j (notice that parameters a_{ii}, $i = 1, .., n$ have no impact on the Hessian D^2U). Here we obtain $U_i = \sum_j a_{ij}\sqrt{x_j}/\sqrt{x_i}$ and $\widetilde{\mu} = U$, with market shares $b_i = (\sqrt{x_i}\sum_j a_{ij}\sqrt{x_j})/U(\mathbf{x})$. Since the MECs can be computed as:

$$\epsilon_{ij} = \frac{1}{2}\left[1 + \frac{a_{ij}\sqrt{x_i}}{\sum_h a_{jh}\sqrt{x_h}} - \frac{a_{ii}\sqrt{x_i}}{\sum_h a_{ih}\sqrt{x_h}}\right],$$

we obtain the average MEC:

$$\bar{\epsilon}_i = \frac{1}{2}\left\{1 - \frac{a_{ii}\sqrt{x_i}}{\sum_h a_{ih}\sqrt{x_h}} + \frac{b_i - a_{ii}x_i/U(\mathbf{x})}{1-b_i}\right\},$$

$$\begin{bmatrix} \ln p_1^B \\ \ln p_2^B \\ .. \\ \ln p_n^B \end{bmatrix} \approx \begin{bmatrix} 2 & -\frac{\beta_{12}}{\beta_1} & .. & -\frac{\beta_{1n}}{\beta_1} \\ -\frac{\beta_{21}}{\beta_2} & 2 & .. & -\frac{\beta_{2n}}{\beta_2} \\ .. & .. & .. & .. \\ -\frac{\beta_{n1}}{\beta n} & -\frac{\beta_{n2}}{\beta n} & .. & 2 \end{bmatrix}^{-1} \begin{bmatrix} \ln c_1 + \frac{\alpha_1}{\beta_1} \\ \ln c_2 + \frac{\alpha_2}{\beta_2} \\ .. \\ \ln c_n + \frac{\alpha_n}{\beta_n} \end{bmatrix}.$$

[17] Under full symmetry, these equilibrium prices correspond to those reported in Bertoletti and Etro (2016), whereas demand parameters are endogenous as in Feenstra (2003).

which allows us to determine the equilibrium conditions.[18] $\bar{\epsilon}_i$ is strictly positive for every good, implying positive markups, unless $a_{ij} = 0$ for any $i \neq j$ (in which case commodities would be perfect substitutes).

A simple case emerges when $a_{ii} = 0$ for any i, which implies $\bar{\epsilon}_i = 1/\left[2\left(1 - b_i\right)\right]$. This allows us to express Cournot prices as:

$$p_i = \frac{2c_i}{1 - 2b_i}, \tag{23}$$

and the (smaller) monopolistic competition prices as:

$$p_i = \frac{2\left(1 - b_i\right) c_i}{1 - 2b_i}. \tag{24}$$

With these preferences the markups do not vanish when the market shares become negligible, but rather converge to twice the marginal cost: indeed we get $\widehat{p}_i \approx 2c_i$ when $b_i \approx 0$.

1.5 Outside goods and endogenous entry

Concluding this section we mention briefly two important extensions of our setting that are conceptually straightforward but can be useful for general equilibrium applications. First, one can add a good representing the outside economy. Pricing within the monopolistically competitive sector carries on unchanged after imposing independent pricing for the outside good (for instance marginal cost pricing if perfect competition holds in that sector) and taking this into account in the computation of the average Morishima elasticity. Instead, solving for Cournot or Bertrand competition is more complex in the presence of an outside good (see d'Aspremont and Dos Santos Ferreira, 2017).

Second, one can consider free entry in the same spirit as Melitz (2003). Given an *ex ante* probability distribution over parameters indexing the goods to be produced, firms would enter the market until the expected profits cover the entry cost. This would leave unchanged the competition stage whenever costs and market size attract a number of firms large enough to justify the assumption of small market shares. Interestingly, one can limit *ex ante* heterogeneity to differences in productivity and analyze *ex post* choices of product differentiation that depend on them. An example (with a continuum of goods) is provided by Bertoletti and Etro (2017, Section 2.3), who studies endogenous quality choices within a setting with heterogeneous firms .

2 Monopolistic competition with GAS preferences

Until now we have implicitly analyzed demand systems that depend on prices in a general way. However, by depending on indices (as in the canonical CES case)

[18] Notice that in the special, fully symmetric case with $a_{ij} = a > 0$ and $x_i = x$ for $i, j = 1, .., n$, one gets $\bar{\epsilon}_i = 1/2$.

demand systems are sometimes simpler, which allows us to study an alternative approach to monopolistic competition. In this section we explore preferences that generate direct demand functions that depend on the own price and *one* common aggregator of all prices or, equivalently, inverse demand functions that depend on the own quantity and *one* common aggregator of all quantities. Pollak (1972) termed these as *generalized additively separable* (GAS) preferences and showed that they encompass the following main classes: the directly and indirectly additive preferences (Houthakker, 1960), whose symmetric versions have been used to model monopolistic competition respectively by Dixit and Stiglitz (1977) and Bertoletti and Etro (2017),[19] what we will call the "Gorman-Pollak preferences", that have been discussed by Gorman (1970a, 1987) and Pollak (1972) but, as far as we know, never applied before,[20] and the implicit CES preferences which we will present in next section. In this environment we will show that an equilibrium of monopolistic competition can be identically defined starting from either price or quantity competition and having firms to perceive as given the value of the common aggregator of individual behaviors. This approach is entirely consistent with that adopted by Dixit and Stiglitz (1977) who suggested, in a setting with symmetric and directly additive preferences, to neglect the impact of an individual firm on the marginal utility of income (the relevant aggregator in their setting), provided that this is sufficiently small to make this behaviour approximately correct.

Pollak (1972) defined *Generalized Additively Separable* (GAS) preferences as those exhibiting demand functions that can be written as:

$$s_i = s_i(x_i, \xi(\mathbf{x})) \quad \text{and} \qquad x_i = x_i(s_i, \rho(\mathbf{s})), \tag{25}$$

where $\partial s_i/\partial x_i$, $\partial x_i/\partial p_i < 0$ and $\xi(\mathbf{x})$ and $\rho(\mathbf{s})$ are common functions ("aggregators") of respectively quantities and prices. Notice that $s_i = x_i^{-1}(x_i; \xi(\mathbf{x}))$ is the partial inverse of $x_i(\cdot)$ with respect to its first argument, and that $\xi(\mathbf{x}) = \rho(\mathbf{s}(\mathbf{x}))$.

It turns out that GAS preferences provide an ideal setting to study monopolistic competition, since we can naturally define it as the environment in which each firm correctly anticipates the value of the aggregators ρ and ξ, but takes ("perceives") them as given while choosing its strategy to maximize profits:

$$\pi_i = (s_i E - c_i) x_i (s_i, \rho) = (s_i(x_i, \xi) E - c_i) x_i. \tag{26}$$

It is important to stress that in this case the price and quantity equilibria of monopolistic competition do coincide. Since the "perceived" inverse demand of a commodity is just the inverse of the "perceived" direct demand, the corresponding elasticities ϵ_i and ε_i are simply related by the condition $\varepsilon_i = 1/\epsilon_i$ (as in a monopoly). Finally, in Appendix A we prove that, provided that the market shares are negligble, to take the aggregator as given is approximately

[19] Also see Mrázová and Neary (2017).

[20] Gorman (1987) writes: "I have not seen this system tried, which is a pity, since it is easily understood, is related to a leading theoretical model, and would be very useful should it fit."

correct for firms, since the perceived demand elasticity is approximately equal to the average Morishima measure. Formally:

PROPOSITION 1. *When preferences are of the GAS type and the market shares become negligible, the perceived demand elasticity does approximate the average Morishima measure.*

Accordingly, the monopolistic competition equilibrium where firms take aggregators as given approximates the i$\vec{m}$perfect competition equilibria of Section 1, which in this sense do "converge", when market shares become negligible.

The first-order conditions[21] for profit maximization of (26) taking as given either ρ or ξ define a system of pricing or production rules, say:

$$p_i = \underline{p}_i(c_i, \rho) \quad \text{and} \quad x_i = \underline{x}_i(c_i, \xi). \tag{27}$$

These rules, together with the budget constraint $\sum_j p_j x_j = E$ and the assumption that firms correctly anticipate the actual demands, can be used to derive the equilibrium value of the aggregators as a function of the cost vector $\mathbf{c}$ and of income E, and therefore the equilibrium prices $\widehat{p}_i(\mathbf{c}, E)$ and quantities $\widehat{x}_i(\mathbf{c}, E)$. To illustrate our solution concept we will now use it in a number of examples of preferences of the GAS type. In contrast to what happens in the case of the equilibria described before, it turns out that in several cases we can obtain closed-form solutions.

2.1 Directly additive preferences

Preferences are directly additive when they can be represented by the direct utility:

$$U = \sum_{j=1}^{n} u_j(x_j), \tag{28}$$

where the sub-utility functions u_j are increasing and concave. The inverse demand system is given by

$$s_i(x_i, \xi(\mathbf{x})) = \frac{u_i'(x_i)}{\xi(\mathbf{x})},$$

where $\xi = \widetilde{\mu} = \sum_j x_j u_j'$ and $x_i(s_i, \rho) = u_i'^{-1}(s_i \xi)$. These preferences clearly belong to the GAS type, and were originally used by Dixit and Stiglitz (1977) in the symmetric version with $u_j(x) = u(x)$ for all j.[22] We can express the profits of each firm i as:

$$\pi_i = \left[\frac{u_i'(x_i) E}{\xi} - c_i \right] x_i. \tag{29}$$

[21] Of course it is also necessary that the second-order conditions are satisfied, i.e., basically that the "perceived" marginal revenues are decreasing.

[22] For a further analysis of symmetric, additive preferences see Zhelobodko *et al.* (2012) and Bertoletti and Epifani (2014).

The profit-maximizing condition[23] with respect to x_i, taking ξ as given, is:

$$u_i''(x_i)x_iE + u_i'(x_i)E = \xi c_i,$$

and it can be rearranged in the pricing conditions:

$$p_i\left(x_i\right) = \frac{c_i}{1-\epsilon_i(x_i)}, \quad i = 1, 2, ..., n, \tag{30}$$

where $p_i\left(x_i\right) = u_i'(x_i)E/\xi$. Let us define the elasticity of the marginal subutility $\epsilon_i(x) \equiv -xu_i''(x)/u_i'(x)$, which corresponds to the elasticity of the inverse demand $s_i(x,\xi)$ for given ξ. Notice that ϵ_i is also the MEC ϵ_{ij} between good i and any other good $j \neq i$, therefore it coincides also with the average MEC $\bar{\epsilon}_i$ discussed in Section 1. In general, the markups can either increase or decrease in the consumption of each good depending on whether $\epsilon_i(x)$ is increasing or decreasing.

Asymmetries of preferences and costs complicate the derivation of the equilibrium because the quantity of each good depends on the quantities of all the other goods through the demand system.[24] Nevertheless, combined with the demand system, the pricing conditions can be used to solve for the production rules $\underline{x}_i(c_i,\xi)$ and the budget shares $\underline{b}_i(c_i,\xi) = s_i(\underline{x}_i(c_i,\xi),\xi)\underline{x}_i(c_i,\xi)$ in function of the common aggregator. Using the adding up constraint $\sum_j \underline{b}_j(c_j,\xi) = 1$ eventually one can solve for the equilibrium values of $\widehat{\xi}\left(\mathbf{c},E\right)$, as well as for all equilibrium quantities $\widehat{x}_i = \underline{x}_i(c_i,\widehat{\xi}\left(\mathbf{c},E\right))$ and prices $\widehat{p}_i = s_i(\widehat{x}_i,\widehat{\xi}\left(\mathbf{c},E\right))E$. As we will show next, in a few examples where the subutilities have a common functional form the equilibrium values can be also derived in closed-form solutions.

Power sub-utility The simplest asymmetric case of direct additivity is based on the sub-utility power function:

$$u_i(x_i) = \frac{\widetilde{q}_i x_i^{1-\epsilon_i}}{1-\epsilon_i}, \tag{31}$$

where the MEC parameters $\epsilon_i \in [0,1)$ and the demand shift parameters $\widetilde{q}_i > 0$, which could be interpreted as quality indexes, can differ among goods. These preferences are a special instance of the "direct addilog" preferences presented by Houthakker (1960), and also of the so-called "Constant Ratios of Elasticities of Substitution" (CRES) model of Hanoch (1975). As a straightforward non-homothetic generalization of the CES case they have been often used in applications with perfect competition.[25] Since in this special case the MECs

[23] The second-order condition requires $xu_i'(x)$ to be concave in x. These are satisfied in the next three examples.

[24] In the symmetric setting of Dixit and Stiglitz (1977) the budget constraint allows the direct computation of equilibrium price and quantity from $p = E/xn = c/(1-\epsilon(x))$.

[25] Mukerji (1963) and Dhrymes and Kurz (1964) are early examples of these functional forms as production technologies. More recently, Fieler (2011) has used them as utility functions in a trade model. They are also used within two-tier settings to models differences between nests/sectors: see e.g. Hottman *et al.* (2016).

are constant, the monopolistic competition equilibrium price is:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon_i}, \tag{32}$$

which shows a full pass-through of changes in the marginal cost and independence from the pricing behavior of competitors and income. Instead the equilibrium quantities $\widehat{x}_i$ depend on the equilibrium value $\widehat{\xi}$, but explicit solutions are not generally available. Possibly due to this lack of full tractability, recent applications with quality differences (Baldwin and Harrigan, 2011, Crozet, Head and Mayer, 2012) have retained the CES structure (essentially constraining $\epsilon_i = \epsilon$ for any good i), which allows for differences in the demand shift parameters, but precludes any differences in markups across goods.

Stone-Geary sub-utility Consider the following simple version of the well-known Stone-Geary preferences (see Geary, 1950-51 and Stone, 1954):

$$u_i(x_i) = \log(x_i + \bar{x}_i), \tag{33}$$

with every $\bar{x}_i$ positive but small enough to insure a positive demand.[26] Solving for the elasticity of the perceived inverse demand we get $\epsilon_i(x) = x/(x+\bar{x}_i)$, and then the pricing condition:

$$p_i\left(x_i\right) = c_i\left(1+\frac{x_i}{\bar{x}_i}\right).$$

The right-hand side is decreasing in $\bar{x}_i$ because a higher value of it increases demand elasticity. However, the equilibrium price of each firm cannot be derived independently from the behavior of competitors: the interdependence between firms created by demand conditions requires the following, fully-fledged equilibrium analysis.

By Hotelling-Wold identity we have:

$$s_i(x_i,\xi) = \frac{1}{(x_i+\bar{x}_i)\xi},$$

where $\xi = \sum_j x_j/\left(x_j+\bar{x}_j\right)$. Combining this with the pricing condition we can compute the quantity $x_i = \sqrt{\bar{x}_i E/(c_i\xi)} - \bar{x}_i$ and the (normalized) price rules $s_i = \sqrt{c_i/(\bar{x}_i E\xi)}$ for firm i. Define $\Psi = \sum_{j=1}^n \sqrt{\bar{x}_j c_j}$. Using the adding up constraint we obtain the condition $\frac{n}{\xi} - \left(\sqrt{E\xi}\right)^{-1}\Psi = 1$, which can be solved for the equilibrium value:

$$\widehat{\xi} = \frac{\left[\sqrt{\Psi^2+4nE}-\Psi\right]^2}{4E}.$$

[26] Simonovska (2015) has recently used a symmetric version of these preferences to study monopolistic competition among heterogeneous firms.

Replacing $\widehat{\xi}$ in the price rule we get the final closed-form solution for the monopolistic competition price of firm i:

$$\widehat{p}_i = \frac{2E\sqrt{\frac{c_i}{\bar{x}_i}}}{\sqrt{\Psi^2 + 4nE} - \Psi}. \tag{34}$$

In this example the price of each firm i is increasing less than proportionally in its marginal cost c_i (incomplete pass-through) and decreasing in the preference parameter $\bar{x}_i$. Moreover, an increase in income increases the price of each good less than proportionally (pricing to market). Note that each price is increasing in Ψ, therefore an increase in the marginal cost c_j of a competitor or in the preference parameter $\bar{x}_j$ (which reduces the associated marginal utility) induce, albeit indirectly, an increase in the price of firm i. Finally, entry of new firms tends to reduce all prices.

Quadratic sub-utility Consider quadratic sub-utilities as in:

$$u_i(x_i) = \alpha_i x_i - \frac{\gamma_i}{2} x_i^2 \tag{35}$$

with $\alpha_i, \gamma_i > 0$. This kind of preferences are a special instance of the quasi-homothetic preferences studied by Pollak (1971), and have a long tradition in economic analysis which dates back to the work of Hermann H. Gossen. Assuming that $s_i > 0$ for all i,[27] the perceived inverse demand elasticity is given by $\epsilon_i(x) = \gamma_i x/(\alpha_i - \gamma_i x)$, so that the monopolistic competition pricing condition can be computed as follows:

$$p_i\left(x_i\right) = c_i \frac{\alpha_i - \gamma_i x_i}{\alpha_i - 2\gamma_i x_i}.$$

Since $s_i(x_i, \xi) = \left(\alpha_i - \gamma_i x_i\right)/\xi$ we obtain the production and pricing rules $x_i = \left(\alpha_i - c_i \xi/E\right)/2\gamma_i$ and $s_i = \left(\alpha_i + c_i \xi/E\right)/2\xi$. Using the adding up constraint allows us to solve for the equilibrium value of the quantity aggregator as:

$$\widehat{\xi} = 2\frac{\sqrt{\frac{\Gamma\Phi}{4E^2} + 1} - 1}{\Gamma},$$

where $\Gamma = \sum_{j=1}^n \frac{c_j^2}{\gamma_j}$ and $\Phi = \sum_{j=1}^n \frac{\alpha_j^2}{\gamma_j}$, and eventually to obtain the equilibrium price:

$$\widehat{p}_i = \frac{c_i}{2} + \frac{\alpha_i \Gamma}{4\left(\sqrt{E^2 + \frac{\Gamma\Phi}{4}} - E\right)}. \tag{36}$$

The price of each firm i is increasing less than proportionally in its marginal cost c_i as well as in the intensity of preferences for the good α_i and in income E, but it is also increasing in the marginal costs of competitors and decreasing in the intensity of preferences for the goods of the latter.

[27] This requires $\alpha_i > 2\gamma_i x_i$.

2.2 Indirectly additive preferences

Preferences are indirectly additive when they can be represented by the indirect utility function:

$$V = \sum_{j=1}^{n} v_j(s_j), \tag{37}$$

with sub-utilities v_j decreasing and convex (Houthakker, 1960). Elsewhere we have used the symmetric version of these preferences ($v_j(s) = v(s)$ for any j) for the analysis of monopolistic competition. The general direct demand system is given by:

$$x_i(s_i, \rho(\mathbf{s})) = \frac{v_i'(s_i)}{\rho(\mathbf{s})},$$

with $\rho = \mu = \sum_{j=1}^{n} s_j v_j'$ and $s_i(x_i, \xi) = v_i'^{-1}(x_i\rho)$, which confirms that they belong to the GAS type. Here we can express the profits of firm i as:

$$\pi_i = \frac{(s_i E - c_i)\, v_i'(s_i)}{\rho}. \tag{38}$$

For a given value of the price aggregator ρ the elasticity of perceived demand $x_i(s_i, \rho)$ is given by $\varepsilon_i(s) = -s v_i''(s)/v_i'(s)$, which is also the MES ε_{ij} between goods i and j ($i \neq j$) and thus coincides with the average MES $\overline{\varepsilon}_i$ (see Section 1). The monopolistic competition price for each firm is then given by the solution to the price condition:[28]

$$p_i = \frac{\varepsilon_i(p_i/E)c_i}{\varepsilon_i(p_i/E) - 1}, \quad i = 1, 2, ..., n. \tag{39}$$

Remarkably, each condition (39) is now sufficient to determine the monopolistic competition price of each firm in function of its own marginal cost and income. Under weak conditions one can also insure existence and uniqueness of the equilibrium. This means that for the entire class of indirectly additive preferences each firm i can choose its price $\widehat{p}_i(c_i, E)$ under monopolistic competition independently from the behavior and the number of the competitors, as well as from their cost conditions or from parameters concerning their goods (e.g., from their "qualities").[29] All the equilibrium quantities (and the other firm-level variables, such as sales and profits) as well as welfare can then be recovered from the direct demand functions. For this reason, this class of preferences could be naturally employed in multicountry trade models, whereas the effects of differential trade costs, qualities and demand elasticities could be empirically assessed. A natural outcome of these models is that goods of higher quality or lower substitutability will generate higher revenues in a given market and therefore will be more

[28] The second-order condition requires $v_i'''(s)$ to be positive or not too negative. These are satisfied in the next examples.

[29] Moreover, entry of new firms, due for instance to a widening of the market size, does not affect the prices of the existing firms, but just creates welfare gains from variety for the consumers.

likely to be exported to more distant countries. Such Alchian-Allen effects (of "shipping the good apples out") have been explored in recent works by Baldwin and Harrigan (2011), Crozet, Head and Mayer (2012), Feenstra and Romalis (2014) and others, but always retaining the CES structure that generates identical markups across goods. The indirectly additive specification allows one to move easily beyond the case of common markups, as we will see in few examples below (for an extension to endogenous quality and free entry see also Bertoletti and Etro, 2017, Section 2.3).

Power sub-utility Consider a power sub-utility as:

$$v_i(s_i) = \frac{q_i s_i^{1-\varepsilon_i}}{\varepsilon_i - 1}, \tag{40}$$

where heterogeneity derives from the shift parameter $q_i > 0$ and the constant MES parameter $\varepsilon_i > 1$, implying that preferences are neither CES nor homothetic (unless $\varepsilon_j = \varepsilon$ for any j). This generalization of the CES case is a special instance of the "indirect addilog" preferences of Houthakker (1960), and also of the so-called "Constant Differences of Elasticities of Substitution" (CDE) model (Hanoch, 1975). The pricing of firm i under monopolistic competition is immediately derived as:

$$\widehat{p}_i = \frac{\varepsilon_i c_i}{\varepsilon_i - 1}, \tag{41}$$

which implies again full pass-through of changes of the marginal cost. It is straightforward to derive the equilibrium quantity:

$$\widehat{x}_i = \frac{q_i \left[\frac{(\varepsilon_i - 1)E}{c_i \varepsilon_i}\right]^{\varepsilon_i}}{\sum_{j=1}^n q_j \left[\frac{(\varepsilon_j - 1)E}{\varepsilon_j c_j}\right]^{\varepsilon_j - 1}},$$

and consequently sales and profits. Clearly, q_i is a shift parameter capturing the quality of good i, that leaves unchanged the price but increases profit by increasing sales. The relative production, sales and profits of firms depend on the relative quality of their goods, on their cost efficiency and demand elasticity, and on the level of income in simple ways that can be exploited in empirical work. We can also solve for equilibrium welfare as:

$$\widehat{V} = \sum_{j=1}^n \frac{q_j \left(\varepsilon_j c_j\right)^{1-\varepsilon_j} E^{\varepsilon_j - 1}}{\left(\varepsilon_j - 1\right)^{2-\varepsilon_j}},$$

which allows one to analyze the welfare impact of any parameter change by computing equivalent variations of income.

Translated power sub-utility Consider the following sub-utility:

$$v_i(s) = \frac{(a_i - s)^{1+\gamma_i}}{1 + \gamma_i}, \tag{42}$$

with $a_i, \gamma_i > 0$ (and $v_i(s) = 0$ if $s > a_i$). It delivers simple perceived demand functions, including the case of a linear demand (for $\gamma_i = 1$) and the limit cases of a perfectly rigid demand ($\gamma_i \approx 0$) and a perfectly elastic demand ($\gamma_i \to \infty$). The symmetric version has been recently applied in Bertoletti *et al.* (2018) to study the welfare impact of trade liberalization in a multicountry trade model with heterogeneous firms. Since $\varepsilon_i(s) = \gamma_i s/(a_i - s)$, the price of firm i is then:

$$\widehat{p}_i = \frac{a_i E + \gamma_i c_i}{1 + \gamma_i}, \tag{43}$$

which shows incomplete pass-through of marginal cost changes (parametrized by the firm-specific parameter γ_i) and markups increasing in the intensity of preference for each good (as captured by parameter a_i) and in income.

Summarizing, these examples provide closed-form solutions for equilibrium prices, quantities and welfare. What is important to remark, is that indirect additivity separates price decisions between firms under monopolistic competition, therefore we could obtain the equilibrium values even if the subutilities had different functional forms across goods. However, pricing does depend on the characteristics of the market, being in general affected by the willingness to pay for the product (which depends on its substitutability and on the expenditure level) and by the cost to serve it. This flexibility, together with the unique property of generating demand functions that can be described empirically by a standard multinomial logit model (see Thisse and Ushchev, 2016), make this class of preferences particularly useful for developing and estimating models with heterogeneous goods.

2.3 Gorman-Pollak preferences

Building on Gorman (1970a) and Pollak (1972),[30] Gorman (1987) has characterized the other main class of GAS preferences by the following extension of additivity. Suppose that preferences can be represented by the utility functions:

$$U(\mathbf{x}) = \sum_j u_j\left(\xi(\mathbf{x})\, x_j\right) - \phi\left(\xi(\mathbf{x})\right) \quad \text{and} \quad V(\mathbf{s}) = \sum_j v_j(\rho(\mathbf{s})\, s_j) - \theta\left(\rho(\mathbf{s})\right) \tag{44}$$

where ξ can be seen as generating the benefit of increasing the *effective* quantity of good i to ξx_i at the utility cost $\phi(\xi)$, which is equivalent (in the dual representation of preferences) to the possibility of reducing the inconvenience of consumption $\theta(\rho)$ at the cost of increasing the *effective* price of good i to ρs_i. ξ and ρ are implicitly defined by the conditions:

$$\phi'(\xi) \equiv \sum_{j=1}^n u_j'(\xi x_j) x_j \quad \text{and} \quad \theta'(\rho) \equiv \sum_{j=1}^n v_j'(\rho s_j) s_j. \tag{45}$$

[30]See Terence Gorman's collected works published in Blackorby and Shorrocks (1995).

Their role is to cancel out any cross effect on utility, as in the case of additive preferences. The demand system can then be easily computed as:

$$s_i(\mathbf{x}) = \frac{u_i'(\xi(\mathbf{x})\, x_i)}{\phi'(\xi(\mathbf{x}))} \quad \text{and} \quad x_i(\mathbf{s}) = \frac{v_i'(\rho(\mathbf{s})\, s_i)}{\theta'(\rho(\mathbf{s}))}, \tag{46}$$

which shows that preferences (44) are of the GAS type. It can be shown that $\rho = \phi'(\xi)$ and $\xi = -\theta'(\rho)$, so that the utility-maximizing choices satisfy $\rho(\mathbf{s})\, s_i = u_i'(\xi(\mathbf{x})\, x_i)$ and $\xi(\mathbf{x})\, x_i = -v_i'(\rho(\mathbf{s})\, s_i)$, and $\widetilde{\mu} = \rho\xi$. We label these as "Gorman-Pollak" preferences: as far as we know, they have never been employed in applications.

It follows from (46) that, for given value of the aggregators, the properties of the demand functions depend on the properties of the u_i and v_i functions. Preferences are indeed CES when the subutilities u_i and v_i have a common power expression, and are more generally homothetic when functions ϕ and θ are logarithmic,[31] a case which covers the GAS demand systems investigated in Matsuyama and Ushchev (2017). Obviously, the functional forms (44) have to satisfy the usual regularity conditions (explored by Fally, 2018). Let us define the elasticities:

$$\epsilon_i(z) \equiv -\frac{u_i''(z)z}{u_i'(z)} \quad \text{and} \quad \varepsilon_i(z) \equiv -\frac{v_i''(z)z}{v_i'(z)}.$$

When firms maximize their profits:[32]

$$\pi_i = \left[\frac{u_i'(\xi x_i)\, E}{\phi'(\xi)} - c_i\right] x_i = \frac{(s_i E - c_i)\, v_i'(\rho s_i)}{\theta'(\rho)}$$

taking as given the aggregators it is immediate to verify that the perceived demand elasticities of monopolistic competition are given by $\epsilon_i(\xi x_i)$ and $\varepsilon_i(\rho s_i)$, which imply the equilibrium pricing condition:

$$p_i = \frac{c_i}{1 - \epsilon_i(\xi(\mathbf{x})\, x_i)} = \frac{\varepsilon_i(\rho(\mathbf{s})\, s_i)\, c_i}{\varepsilon_i(\rho(\mathbf{s})\, s_i) - 1}. \tag{47}$$

In contrast to the case of additive preferences, within the Gorman-Pollak class the relevant demand elasticities *do* depend in general on the values of the aggregators, and *do not* directly correspond to the Morishima measures. Nevertheless our results for GAS preferences apply (see Appendix A) and the equilibrium (47) approximates the imperfect competition equilibria in which firms correctly perceive market shares when these are negligible. Moreover, the Gorman-Pollak preferences retain a tractability that is well illustrated by next example.[33]

[31] In this case the conditions (45) define aggregators that are homogeneous of degree -1, therefore the demand ratios are homogeneous of degree 0 and preferences are homothetic.

[32] The second order conditions for profit maximization can be easily derived. They are satisfied in our example below.

[33] An advantage of the Gorman-Pollak preferences is that they weaken the strict relationship that holds between price and income elasticities under additivity (Deaton, 1974).

Self-dual addilog preferences The family of "self-dual addilog"[34] preferences introduced by Houthakker (1965) and investigated by Pollak (1972) belongs to the Gorman-Pollak class. For this family of preferences the direct demand system is given by:

$$x_i(\mathbf{s}) = q_i \frac{s_i^{-\varepsilon_i}}{\rho\left(\mathbf{s}\right)^{\varepsilon_i + \frac{\delta-1}{\delta}}}, \tag{48}$$

where $q_i > 0$ is a shift parameter reflecting the quality of good i, $\varepsilon_i > 1$ governs the perceived elasticity of demand and $\rho\left(\mathbf{s}\right)$ is implicitly defined by the condition $\sum_{i=1}^n q_i s_i^{1-\varepsilon_i} \rho^{\frac{1-\delta}{\delta}-\varepsilon_i} = 1$. We assume $\delta \in (0,1)$, and $\varepsilon_i \neq \varepsilon_j$ for some i and j (otherwise preferences are CES). Moreover, the inverse demand system is given by:

$$s_i(\mathbf{x}) = \widetilde{q}_i \frac{x_i^{-\epsilon_i}}{\xi\left(\mathbf{x}\right)^{\epsilon_i + \frac{\widetilde{\delta}-1}{\widetilde{\delta}}}}, \tag{49}$$

where $\xi\left(\mathbf{x}\right)$ is implicitly defined by the condition $\sum_{i=1}^n \widetilde{q}_i x_i^{1-\epsilon_i} \xi^{\frac{1-\widetilde{\delta}}{\widetilde{\delta}}-\epsilon_i} = 1$, with:

$$\varepsilon_i = \frac{1}{\epsilon_i} > 0, \quad q_i = \widetilde{q}_i^{\epsilon_i} \quad \text{and} \quad \delta = 1 - \widetilde{\delta}.$$

Pollak (1972) showed that the underlying preferences can be represented for $\delta \neq 1/2$ by:

$$U\left(\mathbf{x}\right) = \sum_{j=1}^n \frac{\widetilde{q}_j \left(x_j \xi\right)^{1-\epsilon_j}}{1-\epsilon_j} - \frac{\widetilde{\delta}\xi^{\frac{2\widetilde{\delta}-1}{\widetilde{\delta}}}}{2\widetilde{\delta}-1} \quad \text{and} \quad V(\mathbf{s}) = \sum_{j=1}^n \frac{q_j \left(s_j \rho\right)^{1-\varepsilon_j}}{\varepsilon_j - 1} + \frac{\delta \rho^{\frac{2\delta-1}{\delta}}}{2\delta - 1}.$$

with the aggregators defined above. In the special case with $\delta = 1/2 = \widetilde{\delta}$ preferences are given by:

$$U\left(\mathbf{x}\right) = \sum_j \frac{\widetilde{q}_j \left(x_j \xi\right)^{1-\epsilon_j}}{1-\epsilon_j} - \ln \xi \quad \text{and} \quad V(\mathbf{s}) = \sum_j \frac{q_j \left(s_j \rho\right)^{1-\varepsilon_j}}{\varepsilon_j - 1} + \ln \rho,$$

and are homothetic.

Given the inverse and direct demand systems, when firms maximize profits taking as given the aggregators, we immediately obtain the following prices under monopolistic competition:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon_i} = \frac{\varepsilon_i c_i}{\varepsilon_i - 1}, \tag{50}$$

where the idiosyncratic markups are constant as in our additive, power subutility examples. In fact, we can also derive the equilibrium quantities as:

$$\widehat{x}_i = \frac{q_i (\varepsilon_i - 1)^{\varepsilon_i} E^{\varepsilon_i}}{c_i^{\varepsilon_i} \varepsilon_i^{\varepsilon_i} \rho\left(\widehat{\mathbf{s}}\right)^{\frac{\delta-1}{\delta}+\varepsilon_i}}.$$

[34] According to a terminology suggested by Pollak (1972), "*strongly* self-dual" preferences are such that they can be represented both by $U\left(\mathbf{x}\right)$ and by $-U(\mathbf{s})$: see Samuelson (1965) and Houthakker (1965).

These results make this family the natural extension of the power additive preferences. The availability of a homothetic version (for $\delta = 1/2$), with the associated well-defined price and consumption indexes, and the flexibility of the general specification provide interesting advantages for further applications that depart from the CES paradigm.

3 Monopolistic competition with multiple aggregators

In this section we extend the approach to monopolistic competition of the previous section to other types of separable preferences generating demands that depend on common aggregators. In particular, we start with the case where each demand depends on *two* common aggregators, and can be written as:

$$s_i = s_i\left(x_i,\xi\left(\mathbf{x}\right),\psi\left(\mathbf{x}\right)\right) \quad \text{and} \quad x_i = x_i\left(s_i,\rho\left(\mathbf{s}\right),\omega\left(\mathbf{s}\right)\right). \tag{51}$$

Assuming $\partial s_i/\partial x_i$ and $\partial x_i/\partial p_i < 0$, again we obtain that $s_i = x_i^{-1}(x_i, \xi(\mathbf{x}), \psi\left(\mathbf{x}\right))$ where $\xi(\mathbf{x}) = \rho\left(\mathbf{s}\left(\mathbf{x}\right)\right)$ and $\psi\left(\mathbf{x}\right) = \omega\left(\mathbf{s}\left(\mathbf{x}\right)\right)$. We can then keep defining unambiguously monopolistic competition as the environment in which firms adopt their strategies anticipating the correct value of the aggregators but taking them as given. Property (51), which encompasses (25) as a special case, does not identify a specific kingdom of preferences. Nevertheless, below we discuss some relevant types of preferences which satisfy it.

3.1 Separable marginal utility preferences

Property (51) holds for any preferences for which the direct utility generates a marginal utility of each good that can be written in a separable fashion as:

$$U_i\left(\mathbf{x}\right) = f_i\left(x_i, \xi\left(\mathbf{x}\right)\right), \tag{52}$$

with $\partial f_i/\partial x_i < 0$ for given aggregator ξ. Preferences with such a separable marginal utility include the three classes of GAS preferences studied in Section 2.[35] Then, the inverse demand is:

$$s_i\left(\mathbf{x}\right) = \frac{f_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\psi\left(\mathbf{x}\right)}, \tag{53}$$

where $\psi = \widetilde{\mu} = \sum_j x_j f_j$ is the second aggregator. The perceived inverse demand elasticity when both aggregators are taken as given is provided by the function:

$$\epsilon_i\left(x, \xi\right) = -\frac{x_i f_{ii}(x,\xi)}{f_i(x,\xi)} \tag{54}$$

[35] Another example is given by the following generalization of the Gorman-Pollak preferences:

$$U\left(\mathbf{x}\right) = \sum_i u_i\left(x_i, \xi\left(\mathbf{x}\right)\right) - \phi\left(\xi\left(\mathbf{x}\right)\right),$$

where $\xi\left(\mathbf{x}\right)$ is defined by the condition $\phi'\left(\xi\right) \equiv \sum_{j=1}^n x_j \partial u_i\left(x_i,\xi\right)/\partial\xi$, leading to $f_i\left(x_i,\xi\left(\mathbf{x}\right)\right) = \partial u_i\left(x_i,\xi(\mathbf{x})\right)/\partial x_i$.

which depends crucially on the value of the aggregator ξ that affects the marginal utility.

An equilibrium analysis requires the specification of preferences. To show how one can adapt our approach in this environment, we focus on a generalization of well-known symmetric preferences used in models of monopolistic competition. Let us consider the direct utility:

$$U(\mathbf{x}) = \sum_{j=1}^{n} u_j(x_j) - \frac{\eta}{2}\xi(\mathbf{x})^2, \tag{55}$$

where $\eta > 0$ and $\xi(\mathbf{x}) = \sum_{j=1}^{n} x_j$. One can recognize here the non-linear component of the utility specification used by Melitz and Ottaviano (2008) under a common and quadratic subutility for any good. This class of preferences satisfies the separability (52) with $f_i(x_i, \xi(\mathbf{x})) = u_i'(x_i) - \eta\xi(\mathbf{x})$. Within this class we have market shares $b_i = (u_i' - \eta\xi)\, x_i/\psi$, and the corresponding perceived elasticity of inverse demand is given by:

$$\epsilon_i(x, \xi) = \frac{-u_i''(x)\, x}{u_i'(x) - \eta\xi}, \tag{56}$$

which allows one to compute the pricing conditions.

It is useful to consider an example of this class of preferences to sketch how in principle one can apply the solution procedure of Section 2 in the presence of two aggregators. Consider the quadratic subutility $u_i(x) = \alpha_i x - \frac{\gamma_i}{2}x^2$ where the parameter α_i changes across goods. The pricing condition becomes:

$$p_i(x_i) = c_i\left(\frac{\alpha_i - \gamma_i x_i - \eta\xi}{\alpha_i - 2\gamma_i x_i - \eta\xi}\right),$$

and by Hotelling-Wold identity we have:

$$s_i(x_i, \xi, \psi) = \frac{\alpha_i - \gamma_i x_i - \eta\xi}{\psi},$$

where $\psi = \sum_j x_j\left(\alpha_j - \gamma_j x_j - \eta\xi\right)$. Thus we can compute the quantity rules $x_i = (\alpha_i - \eta\xi - \psi c_i/E)/2\gamma_i$ and the pricing rules $s_i = (\alpha_i - \eta\xi + c_i\psi/E)/2\psi$. Using the budget constraint one can solve for the marginal utility of income $\psi(\xi)$ in function of the other aggregator and for the quantity $x_i(\xi, \psi(\xi))$. Using the definition $\xi = \sum_{j=1}^{n} x_j(\xi, \psi(\xi))$ we can solve for $\widehat{\xi}(\mathbf{c}, E)$ and then $\psi(\widehat{\xi}(\mathbf{c}, E))$, and derive explicitly the prices:

$$\hat{p}_i = \frac{c_i}{2} + \frac{\left(\alpha_i - \eta\widehat{\xi}(\mathbf{c}, E)\right)E}{2\psi(\widehat{\xi}(\mathbf{c}, E))}, \tag{57}$$

which generalize (36).[36]

[36] One can also consider an asymmetric version of the quasilinear preferences used originally

In a similar way one can analyze preferences whose indirect utility function features a marginal disutility that can be written in a separable fashion $V_i(\mathbf{s}) = g_i(s_i, \rho(\mathbf{s}))$, with $\partial g_i/\partial s_i > 0$. The corresponding demand satisfies (51) with $\omega = \mu = \sum_i g_i s_i$. Again, preferences of this kind do nest the three classes of GAS preferences of Section 2, but they also include others. Examples have been occasionally examined in the literature, though under symmetric conditions: for instance, the homothetic preferences proposed by Datta and Dixon (2001) and the generalized quadratic indirect utility we have studied elsewhere.

3.2 Implicitly additive preferences

Another interesting type of demands satisfying (51) is delivered by the implicitly additive preferences of Hanoch's (1975). They have been used by Kimball (1995) under homotheticity to study nominal price rigidities in macroeconomics, and by Feenstra and Romalis (2014) to study endogenous qualities in trade.[37] Here we analyze in detail the case of *directly* implicitly additive preferences.

Let us assume that preferences can be represented by a direct utility $U(\mathbf{x})$ which is implicitly defined by:

$$F(\mathbf{x}, U) = \sum_{j=1}^{n} F^j(x_j, U) \equiv 1, \tag{58}$$

where the "transformation function" F satisfies the relevant regularity conditions (F must be monotonic). Then preferences are directly implicitly additive, and of course direct additivity is just a special case of it. They are homothetic if $F^i(x_i, U) = F^i(x_i/U)$ for any i. The marginal utility of commodity i is given by:

$$U_i(\mathbf{x}) = \frac{-F_i^i(x_i, U(\mathbf{x}))}{\varphi(\mathbf{x})},$$

by Melitz and Ottaviano (2008):

$$U = x_0 + \sum_{j=1}^{n} \alpha_j x_j - \frac{1}{2}\sum_{j=1}^{n} \gamma_j x_j^2 - \frac{\eta}{2}\xi^2,$$

with $\alpha_j, \gamma_j, \eta > 0$ and $\xi = \sum_j x_j$. If the outside good 0 is the *numeraire* we have the linear inverse demands $p_i = \alpha_i - \gamma_i x_i - \eta\xi$ which exhibit the GAS property. It is immediate to express the profit (per consumer) of each firms as $\pi_i = [\alpha_i - \gamma_i x_i - \eta\xi - c_i]\, x_i$. Taking ξ as given, profits are maximized producing $x_i = (\alpha_i - \eta\xi - c_i)/2\gamma_i$, which allows one to compute $\widehat{\xi}$ and then the equilibrium price as:

$$\widehat{p}_i = \frac{c_i + \alpha_i}{2} - \frac{\frac{\eta}{2}\sum_{j=1}^{n}\frac{\alpha_j - c_j}{\gamma_j}}{2 + \eta\sum_{j=1}^{n}\gamma_j^{-1}}.$$

[37] Feenstra and Romalis (2014) actually use an expenditure function with an "implicitly additive" functional form. However, by a result of Blackorby *et al.* (1978: Theorem 4.10, p. 149) this is equivalent to direct implicit additivity of preferences under some regularity conditions. Also see the application by Matsuyama (2017) to study the impact of non-homotheticity on structural change.

where $\varphi = \sum_{j=1}^{n} F_U^j$, so that the marginal utility depends now on two aggregators, namely φ and the same utility function U.

The usual Hotelling-Wold identity for utility maximization allows us to solve for the inverse demand system as follows:

$$s_i(\mathbf{x}) = \frac{F_i^i(x_i, \xi(\mathbf{x}))}{\psi(\mathbf{x})}$$

for $i = 1, .., n$, where $\xi = U$ and $\psi = \sum_j F_j^j x_j$ (notice that no aggregator corresponds to the marginal utility of income, and that φ does not appear in the expression of demand). The perceived inverse demand elasticity when aggregators are taken as given is provided by the function:

$$\epsilon_i\left(x, U\right) = -\frac{x F_{ii}^i(x, U)}{F_i^i(x, U)}, \tag{59}$$

which depends on the utility level. As Hanoch (1975) noted, the properties of the "substitution function" $x_i F_{ii}^i / F_i^i$ completely determine the substitutability of good i along an indifference curve,[38] and in our monopolistic competition setting this function determines markups.

A similar analysis can be employed to study *indirectly* implicitly additive preferences. These have an indirect utility $V(\mathbf{s})$ that is implicitly defined by $\sum_{j=1}^{n} G^j(s_j, V) \equiv 1$ where the function G has to satisfy some regularity conditions. This type of preferences nests the indirectly additive one, and also a family of homothetic preferences, which is often used in macroeconomics after the work of Kimball (1994). In this environment the marginal disutility depends on two aggregators, one of which is the utility itself which affects the perceived direct demand elasticity. Finally, for the case of implicit additivity, one can generally prove that to take the aggregators as given is approximately correct when market shares become negligible: see Appendix B. Formally:

Proposition 2. *When preferences are implicitly additive and the market shares become negligible, the perceived demand elasticity does approximate the average Morishima measure.*

Accordingly, also in this case the monopolistic competition equilibrium where firms take aggregators as given approximates the imperfect competition equilibria of Section 1, which in this sense do converge. In general, preferences which are directly implicitly additive differs from those which are indrectly implicitly additive, but they have one interesting family in common that we discuss in the following example. As far as we know this has never been used under monopolistic competition, but it could be useful in both trade and macroeconomic applications.

[38] Useful cases arise when the substitution functions are simple. Among them are CES generalizations mentioned in Section 2: the so-called CRES and CDE. However, it is worth mentioning that the elasticities of substitution to which this terminology refers are not the gross Morishima we use here, but the net Allen-Uzawa measures: see Blackorby and Russell (1981).

Implicit CES preferences Consider the following specification of implicit additivity (Gorman, 1970,a,b, and Blackorby and Russell, 1981):

$$F^i(x_i, U) = q_i(U) x_i^{1-\epsilon(U)}, \tag{60}$$

where ϵ and q_j are constant for a given utility level but can change across indifference curves. Note that all F^i are homogeneous of the same degree $1-\epsilon$ with respect to x_j: then $x_i F_i^i = (1-\epsilon) F^i$ and $\psi = 1-\epsilon$. Thus by the Hotelling-Wald identity the inverse demand of commodity i is given by:

$$s_i(\mathbf{x}) = q_i(U(\mathbf{x})) x_i^{-\epsilon(U(\mathbf{x}))},$$

therefore this family of preferences is actually a member of the GAS preferences. Accordingly, monopolistic competition prices satisfy:

$$p_i = \frac{c_i}{1-\epsilon(U(\mathbf{x}))}, \tag{61}$$

which requires $\epsilon \in (0,1)$ and shows that markups are identical across firms in spite of the differences among goods. Nevertheless, markups possibly vary according to changes in expenditure or costs through their impact on the equilibrium utility, which makes this specification particularly attractive.[39] A useful restricted version of implicit CES preferences can be expressed with direct utility U and indirect utility V defined implicitly as follows:

$$U = \left[\sum_{j=1}^{n} \widetilde{q}_j x_j^{1-\epsilon(U)}\right]^{\frac{1}{1-\epsilon(U)}} \quad \text{and} \quad V = \left[\sum_{j=1}^{n} q_j s_j^{1-\varepsilon(V)}\right]^{\frac{1}{\varepsilon(V)-1}} \tag{62}$$

where $\varepsilon(U) = 1/\epsilon(U)$ and $q_i = \widetilde{q}_i^{\epsilon(U)}$ (and the explicit CES case is clearly nested when the elasticities are constant). Notice that we would obtain an interesting class of symmetric preferences by imposing a common quality index q across goods, satisfying the conditions for the equivalence of free entry equilibria and optimal allocation under homogeneous firms (Bertoletti and Etro, 2016: Prop. 6). Here, however, we are interested in the asymmetric case and its possible applications to trade and macroeconomics.

Concerning trade applications, the implicit CES preferences allow us to endogenize quality differences between heterogeneous firms retaining common markups depending on the equilibrium utility. For instance, if $\varepsilon(\cdot)$ is an increasing function (higher utility makes goods more substitutable), opening up to trade or a trade liberalization exert additional impacts compared to the canonical case of explicit CES preferences. In particular, by increasing utility, these

[39] This family of preferences includes cases where the elasticity ϵ is constant and only q_i changes with utility, as in the "non-homothetic CES" CRES of Hanoch (1975), for which $F^i(x_i, U) = d_i U^{-e_i(1-\epsilon)} x_i^{1-\epsilon}$ with d_i and e_i positive parameters affecting both price and income demand elasticities. Preferences become homothetic when the parameters e_i are identical. See Feenstra and Romalis (2014) and Matsuyama (2017) for other examples of similar preferences.

trade shocks reduce the markups of all the firms, which tends to reduce their profitability: this can generate a stronger selection effects on the set of active firms in a model à la Melitz (2003), and it should also leads to revise the quality choices when these are endogenous (in ways that depend on the costs of quality investment).

Concerning macroeconomics, the advantage of this kind of intratemporal preferences is that they deliver markups that depend on an aggregate variable such as utility and, therefore, the level of aggregate expenditure. This implies that aggregate shocks affect the economy not only through the traditional channels, but also through their impact on markups in flexible price models with monopolistic competition. In particular, countercyclical markups (again when higher utility makes goods more substitutable) tend to magnify the propagation of positive temporary shocks because they induce a temporary reduction in the relative price of the final goods, which boosts consumption, and a temporary increase of the real wages, which incentivizes labor supply. A quantitative analysis of similar substitution effects is in Cavallari and Etro (2017): their preferences generate countercyclical markups under monopolistic competition, which allows the model to outperform a standard Real Business Cycle version with perfect competition in matching moments of the aggregate variables, both in closed and open economy applications.

3.3 Other Separable Preferences

As it should be clear by now, there are other preferences that do not belong to the types analyzed above but do satisfy (51). To exemplify we build an instructive example on the basis of a well-known demand system introduced by Deaton and Muellbauer (1980) and often used in empirical analysis, the Almost Ideal Demand System (AIDS).

Restricted AIDS preferences Consider preferences represented by the following indirect utility:

$$V(\mathbf{s}) = \frac{-\rho(\mathbf{s})}{\zeta(\mathbf{s})}, \tag{63}$$

where the aggregators ρ and ζ are defined by

$$\rho \equiv \alpha_0 + \sum_j \alpha_j \ln s_j + \frac{1}{2} \sum_j \sum_i \gamma_{ij} \ln s_j \ln s_i \quad \text{and} \quad \zeta \equiv \beta_o \prod\nolimits_j s_j^{\beta_j}$$

and we assume $\sum_j \gamma_{ij} = \sum_j \beta_j = 0$ and $\sum_{j=1}^n \alpha_j = 1$ to satisfy the regularity conditions, and $\gamma_{ij} = \gamma_{ji}$ without loss of generality. For any $i, j = 1, .., n$ the marginal disutility is:

$$V_i(\mathbf{s}) = -\frac{\alpha_i + \sum_j \gamma_{ij} \ln s_j - \beta_i \rho(\mathbf{s})}{s_i \zeta(\mathbf{s})},$$

and the direct demand functions can be derived from the Roy's identity as:

$$x_i(\mathbf{s}) = \frac{\alpha_i + \sum_j \gamma_{ij} \ln s_j - \beta_i \rho(\mathbf{s})}{s_i}. \tag{64}$$

Demand system (64) does not depend in general just on one or two common aggregators, thus we cannot use it to study monopolistic competition as an environment where firms set prices taking aggregators as given. However, consider the following additional restrictions that introduce some symmetry between goods:

$$\gamma_{ij} = \gamma\gamma_i\gamma_j \text{ for } i \neq j, \ \sum_{j=1}^{n} \gamma_j = 1, \ \gamma_{ii} = \gamma\gamma_i(\gamma_i - 1)$$

with $\gamma > 0$. In this case, the marginal utility becomes:

$$V_i(\mathbf{s}) = -\frac{\alpha_i + \gamma\gamma_i \left[\omega(\mathbf{s}) - \ln s_i\right] - \beta_i \rho(\mathbf{s})}{s_i \zeta(\mathbf{s})},$$

which is separable in three aggregators, namely ρ, ζ and $\omega = \sum_j \gamma_j \ln s_j$. In this case, the direct demand functions read as:

$$x_i(s_i, \rho(\mathbf{s}), \omega(\mathbf{s})) = \frac{\alpha_i + \gamma\gamma_i \left[\omega(\mathbf{s}) - \ln s_i\right] - \beta_i \rho(\mathbf{s})}{s_i}, \tag{65}$$

where only two aggregators remain. Accordingly, this restricted AIDS specification satisfies (51). The aggregator ρ also disappears when $\beta_i = 0$ for any i, delivering the homothetic translog demand considered in Matsuyama and Ushchev (2017), which has the GAS property. Otherwise, the demand and the perceived elasticity of demand depend on two aggregators. Taking as given the aggregators this elasticity can be computed as

$$\varepsilon_i = 1 + \frac{\gamma\gamma_i}{b_i} \tag{66}$$

and it grows unboundedly when the market share becomes negligible (as in the translog case). This outcome is consistent with what one could obtain using the average Morishima elasticities to study strategic interactions as suggested in Section 2.

3.4 Several aggregators

We conclude this part noting that in principle the approach to monopolistic competition that we have explored when preferences are separable can be extended to other cases in which each demand function depends on *more than two* aggregators. In fact, the associated procedure to determine the equilibrium could be applied to any system of well defined "perceived" demands as soon as the alledged behavioral rules (based on the perceived demand elasticities) were consistent with the demand system, so that firms could be seen as correctly anticipating the actual demands. However, to argue that taking all aggregators as given is approximately profit-maximizing for firms, one should verify that when the market shares become negligble the perceived demand elasticity does converge to the relevant Morishima measure (this basically requires that the impact of each firms on the aggregators vanishes).

4 Conclusion

Following our earlier investigations, we have analyzed imperfect and monopolistic competition when consumers have asymmetric preferences over many differentiated commodities and firms are heterogeneous in costs. Defining monopolistic competition as the market structure which arises when market shares are negligible, we have been able to obtain a well-defined and workable characterization of monopolistic competition pricing. Moreover, we have presented a simple and consistent approach to monopolistic competition for demand functions depending on common aggregators.

We believe that our approach could be usefully employed in trade and macroeconomic applications. Most of the recent research on heterogeneous firms is actually based on symmetric preferences (Melitz, 2003; Melitz and Ottaviano, 2008; Dhingra and Morrow, 2018; Arkolakis *et al.*, 2018), which is hardly realistic, especially to analyze heterogenous quality. Also the macroeconomic applications of monopolistic competition have usually focused on symmetric preference aggregators (Bilbiie *et al.*, 2012; Boucekkine *et al.*, 2017). Departing from these assumptions would allow to examine markup variability among goods and over time and its influence along the business cycle and across countries. Finally, the functional forms for separable preferences that we have considered await for an empirical assessment.

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Appendix

A: Monopolistic competition with GAS preferences. Assume that preferences belonging to the GAS type. Taking as given the relevant aggregator, in a monopolistic competition equilibrium firms compute the perceived (inverse) demand elasticity according to:

$$\epsilon_i = -\frac{\partial \ln s_i\left(x_i, \xi\right)}{\partial \ln x_i}.$$

We now show that, when market shares are indeed negligible, to take the aggregator ξ as given approximately coincides with using the average Morishima measures as the relevant demand elasticities (cross demand effects are approximately zero unless the own demand elasticities are large), and is thus approximately profit-maximizing.

Let us start by computing the MEC between commodities i and j $(i \neq j)$:

$$\epsilon_{ij} = -\frac{\partial \ln\left\{s_i\left(\mathbf{x}\right)/s_i\left(\mathbf{x}\right)\right\}}{\partial \ln x_i} = \frac{\partial \ln s_j\left(x_j, \xi\left(\mathbf{x}\right)\right)}{\partial \ln x_i} - \frac{\partial \ln s_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln x_i}$$

$$= \left[\frac{\partial \ln s_j\left(x_j, \xi\left(\mathbf{x}\right)\right)}{\partial \ln \xi} - \frac{\partial \ln s_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln \xi}\right]\frac{\partial \ln \xi\left(\mathbf{x}\right)}{\partial \ln x_i} - \frac{\partial \ln s_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln x_i}. \tag{67}$$

This implies $(h \neq i \neq j)$

$$\epsilon_{ij} - \epsilon_{ih} = \left[\frac{\partial \ln s_j\left(x_j, \xi\left(\mathbf{x}\right)\right)}{\partial \ln \xi} - \frac{\partial \ln s_h\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln \xi}\right]\frac{\partial \ln \xi\left(\mathbf{x}\right)}{\partial \ln x_i},$$

and:

$$\overline{\epsilon}_i = \left[\sum_{j\neq i}\frac{\partial \ln s_j\left(x_j, \xi\left(\mathbf{x}\right)\right)}{\partial \ln \xi}\frac{b_j(\mathbf{x})}{1-b_i(\mathbf{x})} - \frac{\partial \ln s_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln \xi}\right]\frac{\partial \ln \xi\left(\mathbf{x}\right)}{\partial \ln x_i} - \frac{\partial \ln s_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln x_i}. \tag{68}$$

By differentiating the identity $\sum_j s_i\left(x_j, \xi\right) x_j = 1$ we can compute:

$$\frac{\partial \ln \xi\left(\mathbf{x}\right)}{\partial \ln x_i} = -\frac{\frac{\partial \ln s_i\left(x_i, \xi\left(\mathbf{x}\right)\right)}{\partial \ln x_i} + 1}{\sum_{j=1}^{n}\frac{\partial \ln s_j\left(x_j, \xi\left(\mathbf{x}\right)\right)}{\partial \ln x_j}}\frac{b_i(\mathbf{x})}{\xi\left(\mathbf{x}\right)^2}. \tag{69}$$

Accordingly we have $\overline{\epsilon}_i \approx \epsilon_i \approx \epsilon_{ij}$ when $b_i \approx 0$.[40] Notice that $\overline{\epsilon}_i = \epsilon_i = \epsilon_{ij}$ even when shares are *not* negligible if both preferences and the consumption bundle (and then the price vector) are symmetric (as in Bertoletti and Etro, 2016).

Analogously, one can derive the MES and show that with GAS preferences small market shares imply $\overline{\varepsilon}_i \approx \varepsilon_i \approx \varepsilon_{ij}$ and thus $\overline{\varepsilon}_i \approx \overline{\epsilon}_i^{-1}$. Thus to take the aggregator ρ as given while choosing the own price is approximately correct when market shares are indeed negligible. □

[40] This formally assumes that not all the demand own elasticities *and* the quantity aggregator are too small.

B: Monopolistic competition with implicitly additive preferences. Let us consider first the directly implicitly additive preferences defined by (58). Since $\ln(s_i/s_j) = \ln F_i^i(x_i,\xi) - \ln F_j^j(x_j,\xi)$, it follows that:

$$\epsilon_{ij}(\mathbf{x}) = \left[\frac{F_{jU}^j(x_j,\xi(\mathbf{x}))}{F_j^j(x_j,\xi(\mathbf{x}))} - \frac{F_{iU}^i(x_i,\xi(\mathbf{x}))}{F_i^i(x_i,\xi(\mathbf{x}))}\right] U_i(\mathbf{x})x_i - \frac{F_{ii}^i(x_i,\xi(\mathbf{x}))x_i,}{F_i^i(x_i,\xi(\mathbf{x}))}$$

$$= \left[\frac{F_{iU}^i(x_i,\xi(\mathbf{x}))}{F_i^i(x_i,\xi(\mathbf{x}))} - \frac{F_{jU}^j(x_j,\xi(\mathbf{x}))}{F_j^j(x_j,\xi(\mathbf{x}))}\right] \frac{\psi(\mathbf{x})b_i(\mathbf{x})}{\sum_j F_U^j(x_j,\xi(\mathbf{x}))} + \epsilon_i(\mathbf{x})$$

where we used the fact that $b_i = F_i^i(x_i,\xi)x_i/\psi$. This allows us to compute:

$$\epsilon_{ij}(\mathbf{x}) - \epsilon_{ij}(\mathbf{x}) = \left[\frac{F_{hU}^h(x_h,\xi(\mathbf{x}))}{F_h^i(x_h,\xi(\mathbf{x}))} - \frac{F_{jU}^j(x_j,\xi(\mathbf{x}))}{F_j^j(x_j,\xi(\mathbf{x}))}\right] \frac{\psi(\mathbf{x})b_i(\mathbf{x})}{\sum_j F_U^j(x_j,\xi(\mathbf{x}))},$$

and the average MEC as:

$$\bar{\epsilon}_i(\mathbf{x}) = \left[\frac{F_{iU}^i(x_i,\xi(\mathbf{x}))\psi(\mathbf{x})}{F_i^i(x_i,\xi(\mathbf{x}))} - \frac{\sum_{j\neq i} F_{jU}^j(x_j,\xi(\mathbf{x}))x_j}{1-b_i(\mathbf{x})}\right] \frac{b_i(\mathbf{x})}{\sum_j F_U^j(x_j,\xi(\mathbf{x}))} + \epsilon_i(\mathbf{x}).$$

Once again, $b_i \approx 0$ implies $\bar{\epsilon}_i \approx \epsilon_i \approx \epsilon_{ij}$, and thus that to take the aggregates as given is approximately correct when market shares are negligible. Notice that $\bar{\epsilon}_i = \epsilon_i = \epsilon_{ij}$ even when this is *not* the case if both preferences and the consumption bundle (and then the price vector) are symmetric. Similar results can be shown when preferences are indirectly implicitly additive. □